INVESTIGATION OF PACKED BED CHARACTERISTICS USING SPHERICAL PARTICLES

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by

Mariem Yousif Gadhban

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Rabee el-awal **1429**

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I certify that this thesis entitled **"INVESTIGATION OF PACKED BED CHARACTERISTICS USING SPHERICAL PARTICLES"** was prepared by **Mariem Yousif Gadhban** under my supervision at Nahrain University/College of Engineering in partial fulfillment of the requirements for the degree of Master of Science in Chemical Engineering.

Signature: Latif
Name: **Dr. Mohammed N. Latif**
(Supervisor)
Date: 23 / 8 / 2008

Signature:
Name: **Prof. Dr. Qasim J. Slaiman**
(Head of Department)
Date: 7 / 9 / 2008

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We certify, as an examining committee, that we have read this thesis entitled **"INVESTIGATION OF PACKED BED CHARACTERISTICS USING SPHERICAL PARTICLES"**, examined the student **Mariem Yousif Gadhban** in its content and found it meets the standard of thesis for the degree of Master of Science in Chemical Engineering.

Signature: Latif
Name: **Dr. Mohammed N. Latif**
(Supervisor)
Date: 4/9/2008

Signature:
Name: **Asst. Prof. Dr. Balasim Ahmed Abid**
(Member)
Date: 4/9/2008

Signature: Mousa
Name: **Dr. Khalid M. Mousa**
(Member)
Date: 4/9/2008

Signature: Nada
Name: **Prof. Dr. Nada B. Nakkash**
(Chairman)
Date: 4/9/2008

Approval of the College of Engineering

Signature: M. J. Jweeg
Name: **Prof. Dr. Muhsin J. Jweeg**
(Dean)
Date: 15/9/2008

Abstract

Several semi-empirical modified equations for the pore size and porosity had been suggested depending on the parameters affecting them for sphere particles of mono, binary and ternary packing systems. The parameters affecting the pore size were found to be the particle diameter, the porosity of the bed and the permeability to fluid flow. The parameters affecting the porosity in the packed bed of sphere packing are the particle and bed diameters. Several types of packing materials with different sizes have been used in the packed bed, and each had been studied separately. The velocity used in this work was (0.006-0.03)m/s , the bed diameter was (0.0762-0.1524)m , the particle diameter (0.003-0.01)m , length (0.15-0.65)m and the porosity (0.3-0.5)m.

The results of the porosity modified equations have been compared with Furnas equation of porosity and with results taken from documented literature data; the comparisons show a very good agreement between the porosity modified equations and results.

The calculation results of the modified Lattif for the pore size of ternary systems have been compared with Millington and Quirk equation and with results taken from documented literature data; the comparisons show that both equations are comparable with data results, therefore both equations could be used for ternary systems.

Semi-Empirical equations for pressure drop of water flow through packed bed were developed by modifying Forchheimer's equation to include the effect of pore size and porosity equations which were suggested in the present work, for a certain shape of packing and for any shape.

Different parameters affecting the pressure drop of fluid flow through packed bed have been studied.

The calculation results of the pressure drop for water flow through packed bed have been compared with many documented experimental literatures. This comparison gave a very good agreement. The results from Ergun equation using similar conditions have been represented in the curves for the sake of comparison.

A semi-empirical formula was suggested to evaluate the permeability for each type of packing using experimental data. The permeability equation has been compared with the experimental data results and Carman -Kozeny.

Maxwell equation of tortuosity was modified to satisfy the suggested equations of pore size, porosity and pressure drop.

Semi-empirical equations have been suggested for the friction of the wall by studying the main parameters that cause this friction effect of the wall.

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Notations

Symbols		Notations	units
A	=	The bed cross sectional area	m
a	=	Representation of packing and fluid characteristics at laminar flow.	
a	=	The inclusion sphere radius	m
a_k	=	The ratio between diameter of small and large particle	
a_m	=	The ratio between diameter of medium and large particle	
b	=	Representation of packing and fluid characteristics at turbulent flow.	
c	=	Proportionality parameter which depends on the shape of the channels	
C_E	=	The Ergun constant	
D_r	=	Diameter of the bed	m
D_0	=	Diffusion coefficient in the bulk medium	m
D_e	=	Effective diffusion coefficient	m
$D_{P,g}$	=	The gas diffusion coefficient in soil and	m
$D_{0,g}$	=	The gas diffusion coefficient in air	m
d	=	The diameter of glass bed	m
d_p	=	Diameter of the particle	m
dp_{eff}	=	Effective particles diameter	m
dpore	=	Effective pore diameter	m
d_{pi}	=	Diameter of particle i in mixture	m
d_t	=	Diameter of tube	m
d_m	=	The medium particle diameter	m
d_k	=	The small particle diameter	m
d_g	=	The large particle diameter	m

F = A function of the ratio between the diameter of small and large particle

f_w = Wall effect correction factor.

g = Acceleration due to gravity, 9.81 m/s^2

K = Kozeny's coefficient.

K_C = Kozeny's constant.

k = Permeability coefficient for the bed m^2

k_1 = Darcian permeability depend only on the medium properties

k_2 = Non-Darcian permeability depend only on the medium properties

L = The height of packing in the bed m

L_e = Average length of porous medium m

l = Thickness of the bed m

M = The liquid content

m = A measure of the influence of fluid inertia

Δp = Pressure drop through packed bed, Pa $kg/m.s^2$

R = Reduce of horizontal pipe. m

Δr = An annulus thickness of element. m

r_H = The hydraulic diameter m

S = Specific surface area of the particles m^2/m^3

S_p = Surface area of a particle m^2

S_B = Specific surface area of the bed m^2/m^3

S_c = Surface of the container per unit volume of bed m^{-1}

t = The average survival time s

u = Superficial velocity m/s

u_1 = Average velocity through the pore channels m/s

V = Volume of the fluid flowing through bed in time t. m^3

V_p = Volume of a particle m^3

x_i = The weight fraction of particle i.

Greek Symbols

ε = Porosity of the bed. m

μ = Fluid viscosity kg/m.s

Φ = Sphericity.

ρ = Density of fluid kg/m^3

ρ_p = Density of particle kg/m^3

ρ_b = Bulk density kg/m^3

ρ_t = True density kg/m^3

ρ_1 = The density of the liquid kg/m^3

α = Contact angle

σ = The surface tension of the liquid $N.m^{-1}$

τ = Tortuosity factor.

τ_{rz} = Shear stress.

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Chapter One

1.1 Introduction

Fluid flow through packed bed has many important applications in chemical and other processes in engineering fields such as fixed-catalytic reactor, adsorption of a solute, gas absorption, combustion, drying, filter bed, distillation, extraction, wastewater treatment and the flow of crude oil in petroleum reservoir [1].

Packed beds are consists of a channel or duct which contains some form of porous material or a collection of randomly packed spheres or other non-spherical particle. Their design is simple, consisting of a column filled with packing materials of varying sizes and shapes. Fluid is passed through the bottom of the column. Pressure readings are taken by two sensors, located at the top and bottom of the bed [2]. The packing material may be glass marbles, ceramics, plastics, pea gravel, or mixtures of materials [3]. It should have a large void volume to allow flow of fluid without excessive pressure drop and it should be chemically inert to fluids being processed [4].

The most important factor in concerning the bed from a mechanical perspective is the pressure drop required for the liquid or the gas to flow through the column at a specified flow rate [5]. A simple model for predicting pressure drop through packed columns was developed by **Ergun** in 1952. This model is now commonly referred to as the Ergun equation [6]. Ergun believed that the pressure drop over the length of the packing is dependent upon rate of fluid flow, viscosity and density of the fluid, closeness and orientation of packing, size, shape, and surface of the packing material [8].

The advantage of using packed column rather than just tank or other reaction vessel is that the packing affords a large contacting surface area for fluids to flow [9]. Usually increased surface area provides a high degree of turbulence in the fluids which are achieved at the expense of increased capital cost and/or pressure drop, and a balance must be made between these factors when arriving at an economic design [10].

1.2 Packed Beds

A packed bed is simply a vertical column partially filled with small media varying in shape, size, and density. A fluid (usually air or water) is passed thought this column from the bottom and the pressure is measured by two sensors above and below the packed bed. This packed bed becomes “fluidized” when the fluid flows at such a high velocity that the closely packed particles are freed and the space between the packing increases and the particles appear to float and oscillate slightly in the column so the mixture behaves as though it is a fluid [22]. The pressure drop in packed and fluidized beds depends on the type of packing, the bed void fraction, properties of the fluid, column diameter, and also the flow rate of fluid [17].

Packed systems in industry may be divided into the following classes:

1. Fixed beds
    a. Solid- gas system.
    b. Solid- liquid systems.
2. Moving beds.
3. Solid- liquid- gas system.

Typical example of solid-gas fixed-bed systems are the catalytic reactors which were used by the Germans in the Fischer-Tropsch synthesis retorting of oil Shale, roasting of ores, combustion of coal and coke in fuel beds, and blast furnace operations [54].

The most important solid-liquid fixed-bed applications are water filtration, flow of oil through sand strata, coal washing, and leaching [55].

Moving beds are employed in the FCC (fluidized catalytic cracking) process and CFBC (Circulating Fluidized Bed Combustion) [56].

The solid-liquid-gas system comprises fractionating towers, absorbers, scrubbers, and many other kinds of chemical engineering equipment [54]

The aim of this work is to:

Suggesting semi-empirical equations depending on the literature of the following:

I. Pore diameter of packed bed.
II. Porosity as a function of particle and bed diameter.
III. Pressure drop of fluid flow through packed beds. These equation include the suggested pore size and porosity equation within the term of permeability the pressure drop equation, and studying the effect of different parameters in the pressure drop equation.
IV. Permeability of the beds for fluid flow, and study the effect of pore size, tortuosity and bed porosity on the permeability equations.
V. Tortuosity of packed bed as a function of bed porosity.

Chapter Two

Literature Survey and Theoretical Background

2.1 Literature Survey

The flow of fluids through beds composed either of irregularly shaped materials, or of packing of regular geometrical form has attracted considerable attention from many investigators [11]. **Darcy in 1856**[12] derived a semi-empirical equation describing fluid transport in porous media for single-phase flow; he found that the average velocity measured over the whole area of bed is directly proportional to the pressure drop and inversely proportional to the thickness of the bed. **Blake-Kozeny in 1927** [13] derived an equation that correlated the pressure drop at low fluid flow rates, **Burke-Plummer in 1928** [14] derived an expression for the change in pressure at turbulent flow resulting from kinetic energy loss, **Schoenborn and Dougherty in 1944** [15], studied the flow of air, water, and oil through beds of various commercial ring and saddle packing, **Ergun in 1952** [16] studied the pressure drop through packed beds composed of uniform spherical particles. His model was also used for non-spherical shape and/or the particle size distribution was non-uniform. **Leva in 1959** [17], predicted the pressure drop of single incompressible fluids through an incompressible bed of granular particles. **Dullien** and **MacDonald** addressed the problem of multi-sized particles present in a porous media. **Dullien in 1976** [18] modified Kozeny equation assuming pores with periodic step changes in their diameter. **MacDonald in 1991** [19] generalized the Blake-Kozeny equation for multi sized spherical particles the model gave a good agreement with experimental data. **Bey** and **Eigenberger in 1997** [20] have represented the pressure drop in a packed bed by modifying the Ergun equation for a cylindrical coordinated system. **Gibson and Ashby in 1988, Duplessis in 1994 and Richardson in 2000** [21] studied the influence of several structural parameters, such as porosity, tortuosity, surface area and pore diameter, in predicting the pressure drop through porous medium. **Basu et. al. in 2003**[22] studied the effect of various velocity range and their effect on the packing height and pressure drop in the column. **Harkonen in 1987, Lindqvist in 1994, Lammi in 1996, Wang and Gullichsen in 1999 and Lee and Bennington in 2004** measured the average void fraction and flow resistance through packed columns. They found that the pressure drop of liquid through a packed bed depends on many factors, including the particle species and the type and size distribution of the particles [23]. **Hellström and Lundström in 2006**[24] suggested a model for flow through porous media taking into consideration the inertia-effects. They compared their results with Ergun equation, and it fits well to Ergun equation. **Chung and Long in 2007** [25] studied how the pressure drop of a packed bed is related to the flow rate of the fluid coming into the column , they compared their results to the pressure drop predicted by the Ergun equation**.**

Many investigators described the porosity and found that the packing porosity depends upon the particle size, size distribution, particle shape, surface roughness, method of packing, and the size of the container relative to the particle diameter [4]; **Stanek** and **Szekely in 1972 and in 1973** [26,27] suggested a method to correct the effects of the porosity on flow through packed beds by considering two distinct uniform void fractions. **Kubo et al. in 1978**[28] reported photographic observations on flow patterns in voids of packed beds of equal sized spheres. **Standish** and **Borger in 1979**[29], **Standish** and **Mellor in 1980**[30], **Standish** and **Leyshon in 1981**[31], and **Standish** and **Collins in 1983**[32] they also study experimentally the porosity and permeability of multi component mixtures of uniform and irregular shape particles, the study the porosity of multi component mixtures from the results of binary mixtures. **Ouchiyama** and **Tanka in 1984**[33] proposed a mathematical model to calculate the porosity of particulate mixtures, especially those in the ternary system, from the knowledge of particle sizes involved and their proportion in the mixture. **Standish** and **Yu in 1987**[34] studied the porosities of multi-size mixtures; they measured porosities for ternary systems of uniform and non uniform mixtures of spherical particles. **Fuller and Thompson in 1987**[35] studied the influence of distribution of the particle size upon the density of granular material. **Yu**, **Zou**, **Standish** and **Xu** in **1989**[36] presented a general discussion of the porosity and particle size distribution relation with the commonly used size distribution systems including the discrete binary, ternary and quaternary mixtures and the continuous Gaudin-Schuhmann, log-normal, Rosin-Rammler and Johnson's S_B size distributions. **Moallemi** in **1989, Yu** and **Standish** in **1991**, **Summers** in **1994** and **Ismail** in **2000**[37] studied the local voidage for the mixtures of spheres packing (mono, binary and ternary) and found that the local voidage variations in the axial, radial and angular direction.

Furnas in 1929[38] and **Graton** and **Fraser in 1935**[39] studied the proportion of voids in the region next to the wall and the effect of the wall on pressure drop. **Carman in 1937**[40] and **Coulson in 1949**[11] studied the effect of the porosity near the wall. **Graton and Fraser in 1953**[41] showed that the porosity of the bed is greater in the layers next to the wall, which lead to increase the fluid permeability there. **Leva in 1959**[42] have stated that at high flow rates the wall effect is negligible. **Devendra** and **Martin** in **1960**[43] modified the Ergun equation, to include the wall effect. They found that the pressure drop and flow rate can be calculated more accurately when the friction of the wall surface was taken into account. **Dudgeon in 1964** believed that the wall effect was independent of the flow rate[44]. **Roblee et al**. in **1958**; **Benenati** and **Brosilow** in **1962**; **Ridgway** and **Tarbuck** in **1966**, **1968**; **Thadani** and **Peebles** in **1966**; **Kondelik et al**. in **1968**; **Buchlin et al.** in **1977**; **Stanek** and **Eckert** in **1979**; **Goodling et al.** in **1983** and **Stephenson** and **Stewart** in **1986** observed the presence of strong oscillatory radial variations in the region up to about 5 particle diameters from the wall [45].

Benenati and **Brosilow** in **1962**[46] measured the void fraction variations in randomly packed beds of uniformly sized spheres. The results indicate that the minimum void fraction is observed at half a particle diameter from the wall. **Ridgway** and **Tarbuck** in **1966**[47] investigated radial void fraction variations in packed beds of mono sized and binary mixtures of particles. **Ridgway** and **Tarbuck** in **1968**[48] obtained an expression for the voidage variations along a line perpendicular to the container wall in randomly packed bed of spheres. **Pillai** in **1977** [49] derived an expression for area void fraction near the wall. A system is considered where identical spherical particles are packed in a random fashion in the vicinity of a vertical wall. **Cohen** and **Metzner in 1981** [50] proposed a model for the wall effects in a laminar flow through packed beds of spheres of uniform size. The model accounts for radial variations in the porosity (occurring due to the confining effect of the wall), and the effects of the surface area of the column. They have used the Blake-Kozeny equation to describe the flow. **Nield in 1983**[51] proposed an alternative two and three region model for the wall effects in laminar flow by considering the wall region to be occupied by fluid only and the other part of the bed to be a porous medium where Darcy's law is applicable. **Govindarao** and **Froment** in **1986**[52] analyzed voidage pores in randomly packed beds of uniformly sized spheres by dividing the bed into a number of concentric layers of equal thickness, and by expressing the void fraction in a layer in terms of the contribution to the solid volume by spheres with centers lying in appropriate neighboring layers. **Kubie** in **1988**[53] investigated the distribution of voidage in the wall region of a randomly packed bed of uniform size spheres. A generalized equation relating the local voidage to the distribution of the spheres was derived and was combined with some simple observations of the bed to develop a model of particle packing near container walls.

2.2 Flow in Porous Media

Flow in porous media has received much attention in recent years because of its important role in a large variety of engineering and technical applications, such as filtration units, packed beds, and certain types of chemical reactors [57].

A porous medium consists of pores between some particulate phase, contained within a vessel, or some control volume, as illustrated in the figure 2.1**:**

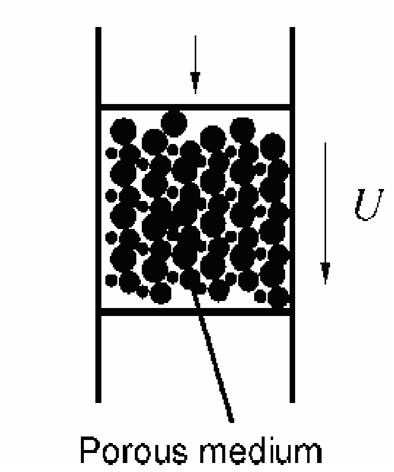


Figure 2.1 Porous media

The existence of particles within the bed will reduce the area available for fluid flow, i.e. to preserve fluid continuity with the entering superficial flow, so the fluid will have to squeeze through a smaller area; hence the velocity within the bed (interstitial velocity) will be greater than the superficial [58].

A material can be defined as a porous medium if the material has the following properties, **Dullien in 1992** [57]:

1- The material must contain relatively small spaces, called pores or voids, imbedded in the solid. The pores usually contain some fluid, such as air, water, etc., or a mixture of different fluids.

2- The fluids should be able to penetrate through one face of the material and emerge from the other side.

Analysis of pore structure and pore radius distribution are necessary in order to construct an effective model for a porous medium [59].

All properties of porous media are influenced by the pore structure. Pore structure parameters represent average behavior of a sample containing many pores. The most important pore structure parameters are the porosity, the

1. Porosity is a measure of the pore space and hence of the fluid capacity of the medium.
2. Tortuosity is a measure of fluid path through bed compared with actual depth of bed.
3. Permeability is a measure of ease with which fluids may traverse the medium under the influence of a driving pressure.
4. Pore size is a measure of the pores diameter between the packing materials.

2.2.1 Porosity of the Bed

The porosity (ε) is defined as the ratio of the void volume to the total volume of the bed (the volume fraction occupied by the fluid phase). [15], i.e:

$$\varepsilon = \frac{Volume\ of\ voids\ in\ a\ bed}{total\ volume\ of\ the\ bed} \quad \ldots (2.1)$$

Other names given to the void fraction are porosity, fractional voidage, or simply voidage. The liquid in a packed bed usually fills this voided volume. For spherical packing, geometric analysis predicts that the void fraction will be constant with consistent packing methods, regardless of the diameter of the spheres [61].

The porosity is the most important property of a porous medium and it affects most of the physical properties of the medium. For a homogeneous porous medium, the porosity may be a constant. But in general, the porosity is a space dependent. The porosity is affected by many variables that may be classified into the categories of particle properties, container properties and packing method [62, 63].

The porosity can be evaluated experimentally using the following equation [30]:

$$\varepsilon = 1 - \frac{\rho_b}{\rho_t} \qquad \dots (2.2)$$

Where ρ_t is the true density of the particles (g/cm^3), ρ_b is the apparent bulk density (g/cm^3).

Furnas [64] proposed equation for the porosity in packed column with sphere packing as function of particle and bed diameter, as shown below:

$$\varepsilon = 0.375 + 0.34\frac{d_p}{D_r} \qquad \dots (2.3)$$

Where d_p is the particle diameter in m and D_r is the bed diameter in m.

Figure 2.2 shows the relationship between bed porosity and the ratio of particles to column sizes for different types of packing materials.

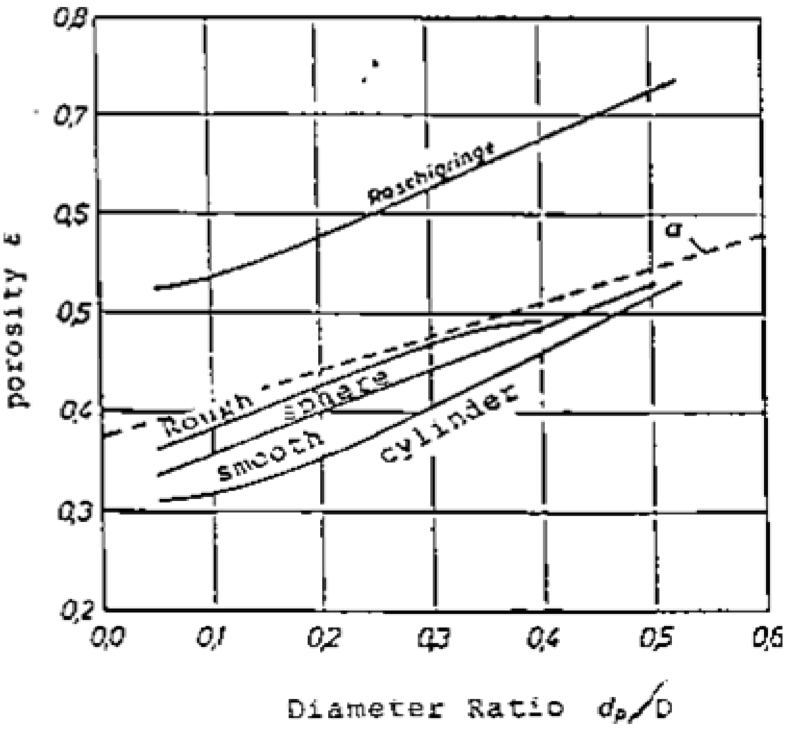


Figure 2.2 The relationship between bed porosity and diameter ratio [78]

The porosity has a great effect on the properties of packed beds. **Leva** [42] found that a 1% decrease in the porosity of the bed produced about an 8% increase in the pressure drop, whilst **Carman** [65] reported a higher value, 10% increase in the pressure drop for every 1% decrease in porosity [26].

Depending on the type of the porous medium, the porosity may vary from near zero to almost unity**. Kaviany** [66] suggested that the normal range of average void fraction was from 0.36 to 0.43. Measurement of porosity is made by using several techniques, such as imbibitions, mercury injection and gas injection methods give an effective porosity value [57].

Feng and **Yu** [67] report an experimental study of the packing of spherical particles with special reference to the effect of liquid addition. It shows that the properties of both particle and liquid affect the packing behaviors significantly. Under given packing conditions, dry based porosity increases to a maximum and then keeps constant with the increase of liquid content. Particle size and surface tension are the main factors in the quantification of this porosity- liquid content relation. Empirical equations have been formulated in terms of dimensionless groups for the purpose of prediction as shown below:

$$\varepsilon = f\left(M, \frac{\sigma}{\rho_p g d^2}, \frac{\rho_1}{\rho_p}, \frac{\mu^2}{\rho_p^2 g d^3}, \alpha\right) \qquad \dots (2.4)$$

2.2.2 Tortuosity Factor

Tortuosity is defined by **Sheidegger** [68] as the ratio of the average pore length (L_e) to the thickness of the medium (L):

$$\tau = \frac{L_e}{l} \quad \text{… (2.5)}$$

Where L_e is the average length of porous medium and l is the bed thickness.

The ratio L_e / l being the tortuosity factor and it is usually represented by τ as described by **Bear and Dullien** [69].

Figure 2.3 shows the effect of tortuosity in a porous medium between the average length and the bed thickness [70]:

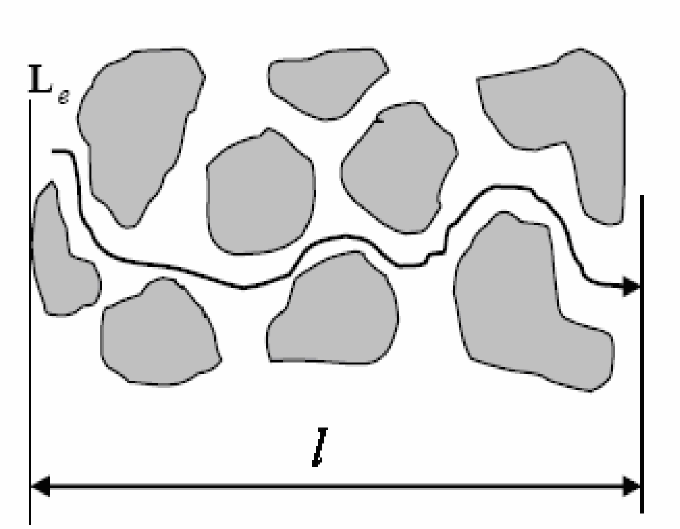


Figure 2.3 The effect of tortuosity in a porous medium showing the average length and the bed thickness [71]

Tortuosity is not a physical constant it depends on other porous media characteristics, such as porosity, pore diameter, channel shape, etc. in general, in granular packing or beds the value of tortuosity lies in the range 1.1-1.7 [69].

It is difficult to determine tortuosity experimentally and in general, tortuosity is calculated by using the porosity and the effective diffusion coefficient or from the Kozeny coefficient [57]. The tortuosity may be expressed as a function of **kozeny's** coefficient K as [70]:

$$\tau = \sqrt{\frac{K}{K_c}} \qquad \dots (2.6)$$

where: K_c is the kozeny's constant.

K is the kozeny's coefficient.

The relationship between tortuosity and porosity were suggested by several empirical equations, such as **Maxwell in 1873, Weissberg in 1963, Comiti and Renaud in 1989 and Boudreau in 1996** [69], as shown below:

$$\tau = 1.5 - 0.5\varepsilon \qquad \textbf{(Maxwell, 1873)} \qquad \dots (2.7)$$

$$\tau = 1 - 0.5\ln(\varepsilon) \qquad \textbf{(Weissberg, 1963)} \qquad \dots (2.8)$$

$$\tau = 1 - 0.41\ln(\varepsilon) \qquad \textbf{(Comiti and Renaud, 1989)} \qquad \dots (2.9)$$

$$\tau = \sqrt{1 - \ln(\varepsilon^2)} \qquad \textbf{(Boudreau, 1996)} \qquad \dots (2.10)$$

Archie in 1942 suggested most frequently relationship between tortuosity and porosity for a mixed bed of particles dependent on the methods applied for packing preparation, as:

$$\tau = \frac{1}{\varepsilon^n} \qquad \dots (2.11)$$

Where n is a numerical value, and depend on the properties of the packing bed. The value of n lies in the range from 0.4 for loose packing to 0.5 for dense packing [72]. Equations all satisfy the condition τ =1 for ε =1, and this consistent with the physical situation observed [69].

Also tortuosity can be calculated from the effective diffusion coefficient D_e, which characterizes mass transfer in porous media, it can be written as [70]:

$$\tau = \frac{D_{\circ}}{D_e}\varepsilon \qquad \dots (2.12)$$

Sen in 1981 and Yun in 2005 showed that for an isotropic medium with spherical particles the tortuosity of porous and granular media decreases with increasing bed voidage [69].

2.2.3 Permeability of the Bed

The permeability, *k*, is the measure of the flow conductance of the porous medium and it is defined by the **Darcy's** law, as shown below[57]:

$$k = \left(\frac{\varepsilon}{\tau}\right)^2 \frac{\varepsilon\, dp_{eff}^2}{36(1-\varepsilon)^2 K_c} \qquad \dots (2.13)$$

Where ε is the porosity of the porous media, τ the tortuosity factor, dp_{eff} is the effective particle diameter and $K_{c,}$ is the **kozeny's** constant, and it is dependent of the porosity for packing [57].Figure 2.4 shows the variation of **Kozeny's** constant with porosity for different shaped particles [8].

The permeability depends on [77]:

1- Characteristic dimension of solid phase $d_{p.}$

2- Geometry of solid phase (e.g. spheres, fibers, foams …).

3- Porosity ε.

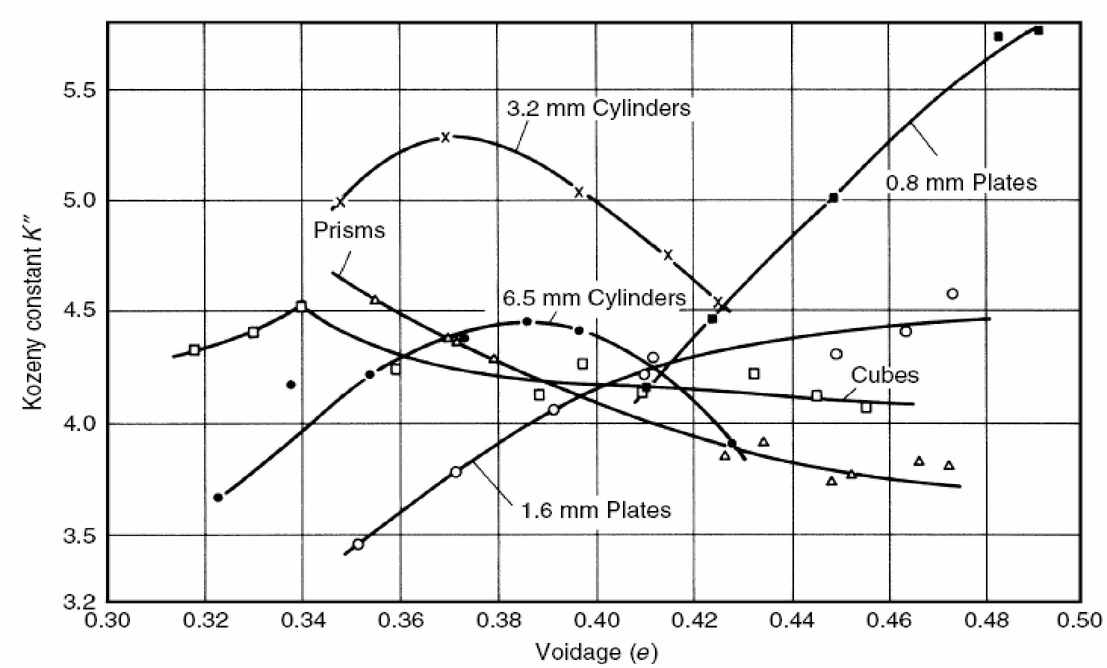


Figure 2.4 Variation of **Kozeny's** constant with porosity for different shaped particles [8].

One very well known equation which relates the permeability k to porous media properties was derived by **Kozeny** [13]. He viewed the porous bed as an assemblage of channels of various cross sections and expressed the permeability as:

$$k = c\frac{\varepsilon^3}{S^2\tau} \qquad \ldots (2.14)$$

where ε is the porosity of the porous media, S the specific surface of the channel, τ the tortuosity factor and c a proportionality parameter which depends on the shape of the channels. The tortuosity factor takes into account the complexity of the channels in the porous media [73,75].

The Kozeny equation has been largely applied and also modified by other researchers. Carman introduced the specific surface exposed to the fluid S_B (S_B=(S-1)) and set the constant c to 1/5 which gave the best fit to his experimental results. The result is known as the Carman -Kozeny equation [75]:

$$k = \frac{\varepsilon^3}{5S_B^2(1-\varepsilon)^2} \qquad \ldots (2.15)$$

A more recent modification of the **Carman -Kozeny** equation is due to Blake [74,19], who related permeability to the void fraction and primary particle size d_p and introduced a correction factor derived from experimental results. In this case the permeability k is written as [76]:

$$k = \frac{d_p^2 \varepsilon^3}{180(1-\varepsilon)^2} \qquad \ldots (2.16)$$

This equation is considered to be valid for media consisting of individual particles [75].

Stokes [77] made an equation for the permeability of dilute bed of spheres, as shown below:

$$k = \frac{2}{9(1-\varepsilon)} a^2 \qquad \ldots (2.17)$$

Where a is the inclusion sphere radius. The stokes law gives estimates that are greater than all other data

While **Happel-Brenner** [77] approximation:

$$k = \left(\frac{2}{9\gamma}\right)\left(\frac{3-(9/2)\gamma+(9/2)\gamma^5-3\gamma^6}{3+2\gamma^5}\right)a^2 \qquad \ldots (2.18)$$

Where a is the inclusion radius, and $\gamma^3 \equiv (1-\varepsilon)$.

Torquato - kim [79] define the permeability as shown below:

$$k = \frac{2}{9t}\frac{\varepsilon}{(1-\varepsilon)} a^2 \qquad \ldots (2.19)$$

Where t is the average survival time, the data for t are taken from [80].

Koponen in **1998** defined the permeability for fiber bed as follows:

$$k = \frac{5.55 d_s^2}{\exp(10.1(1-\varepsilon))-1} \qquad \ldots (2.20)$$

Calmidi in **1998** defined the permeability for metal foams as follows:

$$k = 7.3 * 10^{-4} d_s^2 (1-\varepsilon)^{-0.224} \left(\frac{d_s}{d_{pore}} \right)^{-1.11}$$ … (2.21)

where:

$$\frac{d_s}{d_{pore}} = 1.18 \sqrt{\frac{1-\varepsilon}{3\pi}} \frac{1}{1 - \exp\left(-\frac{1-\varepsilon}{0.04} \right)}$$ … (2.22)

$$d_s = \frac{\varepsilon}{1-\varepsilon} d_p$$ …

(2.23)

$$d_{pore} = \frac{2}{3} \frac{\varepsilon}{1-\varepsilon} d_p$$ … (2.24)

Where ε is the porosity, d_p is the particle diameter in m and d_{pore} is the pore diameter in m [81].

2.2.4 Pore Size

For spherical packing, pore size is conveniently measured by pore diameter so that whenever we say "pores size "means diameter. This pore size or pore diameter represents the diameter of the particle, which may be able to pass through in the pore. Pore size measurements have been made on a number of Indiana shale's **Kaneuji** , **Winslow**, **Dolch, Surendra** [82,83]

The study of pore size is necessary to study the packing of a porous medium. Each void in the porous medium is connected to more than one other pore (through pore or interconnected), connected only to one other pore (blind pore or dead end), or not connected to any other pore (closed pore or isolated) and fluid flows through the interconnected pores [57,84]. Figure 2.5 shows the three possible kinds of pores.

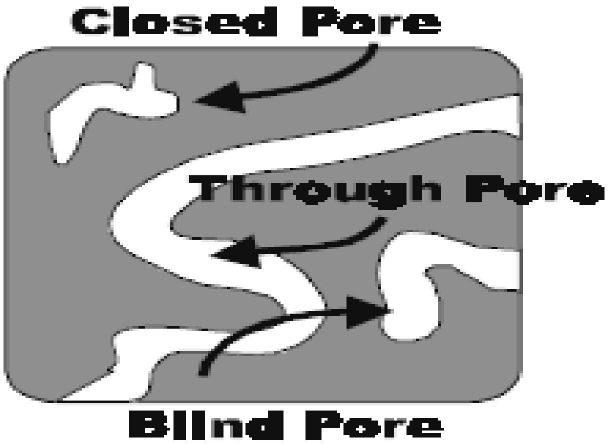


Figure 2.5 Three possible kinds of pores [84]

The classification of pores according to size has been under discussion for many years, but in the past the terms micro pore {pore of internal width greater than 2 mm}.Macro pore {pore of internal width greater than 50 mm}, has been applied in different ways by physics and chemists and some other scientists in an attempt to clarify this situation .Mesopores {pore of Internal width between 2mm and 50 mm} are especially important in the Context of adsorption [85].

Millington and **Quirk** [86] equation is for the determination of the pore size of soil in gas based on the definition of the porous medium and by combining Ficks' law for diffusive transport with Poiseuille's law for convective fluid transport, as shown below:

$$d_{pore} = \sqrt[2]{\frac{8k}{D_{P,g} / D_{0,g}}} \qquad \ldots (2.25)$$

$$\frac{D_{P,g}}{D_{0,g}} = \varepsilon^{1.5} \qquad \ldots (2.26)$$

Where $D_{P,g}$ is the gas diffusion coefficient in soil and $D_{0,g}$ is the gas diffusion coefficient in air[87].

latif [82] proposed a theoretical method for the prediction of pore diameter for three spheres of different size particles as shown below:

$$d_{pore} = \frac{K_1 - (K_4^2 - 4K_5)^{1/2}}{2} \; d_g \qquad \ldots (2.27)$$

Where

$$K_1 = \left[\frac{(a_m + 1)}{(a_m - 1)}\right]^2 \qquad \ldots (2.28)$$

$$K_2 = \frac{4a_m}{(1 + a_m)} \qquad \ldots (2.29)$$

$$K_3 = \left(a_k^2 + a_k K_2\right)^{1/2} - a_k \qquad \ldots (2.30)$$

$$K_4 = (K_2 + 2K_3) * K_1 \qquad \ldots (2.31)$$

$$K_5 = K_3^2 * K_1 \qquad \ldots (2.32)$$

$$a_k = \frac{d_k}{d_g} \qquad \ldots (2.33)$$

$$a_m = \frac{d_m}{d_g} \qquad \ldots (2.34)$$

Where a_k is the ratio between diameter of small and large particle, a_m is the ratio between diameter of medium and large particle, d_g is the large particale diameter, d_k is the small particle diameter, d_m is the medium particale diameter and d_{pore} is the pore daimeter.

In case of three equal sizes of particles latif made the following equation:

$$d_{pore} = 0.155 * d_g \qquad \text{… (2.35)}$$

Fawaz [88] calculated the pore size for three spheres of different size using theoretical method suggested by **latif** [82].He found the relation between mean pore diameter and percentage output of impurirties. **Omar** [89]calculated the pore size for binary system using theoretical method suggested by latif and experimentally he found the percentage output of impurities which passed through the packed bed, and compared it with experimental results obtained by **Kreutz.**

2.3 Factors Affecting Pressure Drop through Packed Bed

The flow of single phase through a packed bed extensively for many chemical engineering applications, particularly for the design of fixed catalytic beds and therefore expressions are needed to predict pressure drop across beds [90]. There are several factors affected on the pressure drop, some of it related to the physical properties of fluid such as viscosity and density, and others consist the Rate of fluid flow, Closeness and orientation of packing, Size, shape and surface roughness of particles[3].

2.4 Specific Surface Area

The specific surface area of a particle is used through most of the equations or formulas of fluid flow through packed bed, and it is defined as follows:

$$S = \frac{S_p}{V_p} \qquad \text{… (2.36)}$$

Where S is specific surface area of a particle in m^{-1}, S_p is the surface area of a particle in m^2 and V_p is the volume of a particle in m^3. Therefore for spherical particle:

$$S = \frac{\pi\, d_p^{\;2}}{\pi \left(d_p^{\;3}/6\right)} = \frac{6}{d_p} \qquad \ldots (2.37)$$

where d_p is the particle diameter in m [91, 8].

For beds consisting of a mixture of different particle diameters, the effective particle diameter (dp_{eff}) can be used instead of d_p as [92]:

$$dp_{eff} = \frac{1}{\sum_{i=1}^{n} \frac{x_i}{d_{pi}}} \qquad \ldots (2.38)$$

where: x_i is the fractional weight of spherical particle.

d_p is the diameter of spherical particle.

2.5 The Shape of the Particles

For fluid flow through packed beds many particles of irregular shapes usually used. To treat this problem the particles are considered as spheres by introducing a factor called sphericity Φ which allows calculation of an equivalent diameter [93].

The sphericity of a particle is the ratio of the surface area of this sphere having the same volume as the particle to the actual surface area of the particle, as shown below:

$$\Phi = \frac{a_{sphere}}{a_{particale}} = \frac{6/d_p}{S_{particle}/V_{particle}} \qquad \ldots (2.39)$$

For a sphere, the surface area $S_p = \pi d_P^2$ and the volume is $V_p = \pi d_p^3/6$. Table 2.1 below shows the shape factor for different packing geometries [61].

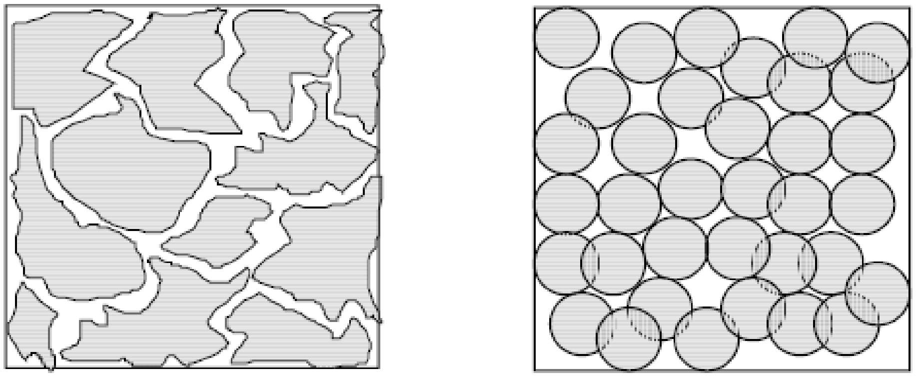
Table 2.1 Shape factor for different particles [92, 94]

Material	*Shape Factor*	*Material*	*Shape Factor*
Spheres	1.0	Flint sand	0.65
Cubes	0.81	Crushed glass	0.65
Cylinders, d_p=L (length)	0.87	Coal dust	0.73
Berl saddles	0.3	Mica flaskes	0.28
Rasching rings	0.3	Rounded sand	0.83
Sands, average	0.75	Ottawa sand	0.95

The shape factor is difficult to evaluate when dealing with small irregular shapes. The particle shape affects the packed bed resistance in two ways [44]:

i) The fluid paths in beds of irregular particles are more tortuous than those in similar beds of spheres (Fig.2.6).

ii) It have voids differing in both size and shape from those of similar beds consisting spheres.



a. irregular particles

b. regular particles

Figure 2.6 Different shapes of particles [57]

Zou and **Yu** [95] proposed an empirical equation (referred to as ZY model) to quantify the relationship between the porosity and sphericity of cylindrical particles in dense random packing. This relation was based on the experiments of wood cylinder packing. For cylinders the length-to-diameter of the bed is more than 1, so that the equation of dense packing will be [98]:

$$\ln\varepsilon = \Phi^{6.74}\exp[8(1-\Phi)]\ln 0.36 \qquad \text{… (2.40)}$$

And for disc where the length to diameter of the bed is less than 1, the equation will be:

$$\ln\varepsilon = \Phi^{0.63}\exp[0.64(1-\Phi)]\ln 0.36 \qquad \text{… (2.41)}$$

The sphericity of a cylinder is related to the ratio of length to diameter of the bed by [98]:

$$\Phi = 2.621\frac{(L/D_r)^{\frac{2}{3}}}{1+2(L/D_r)} \qquad \text{… (2.42)}$$

Rahli et al. [96] also proposed an empirical equation fore the porosity (referred to as RTB model), as:

$$\varepsilon = 1-\frac{11}{2\frac{L}{D_r}+6+\frac{\pi D_r}{2L}} \qquad \text{… (2.43)}$$

Parkhouse and Kelly [97] gave an equation for the relationship between the porosity and the ratio of length to diameter of the bed using more than 7 cylinders, based on the statistical approach to the distribution of the pores in the stacks, as shown below:

$$\varepsilon = 1-2\frac{D_r\ln(L/D_r)}{L} \qquad \text{… (2.44)}$$

Figure 2.7 below shows the porosity as a function of the ratio of the length to diameter of the bed. It is clear that the porosity increases with this ratio [98]

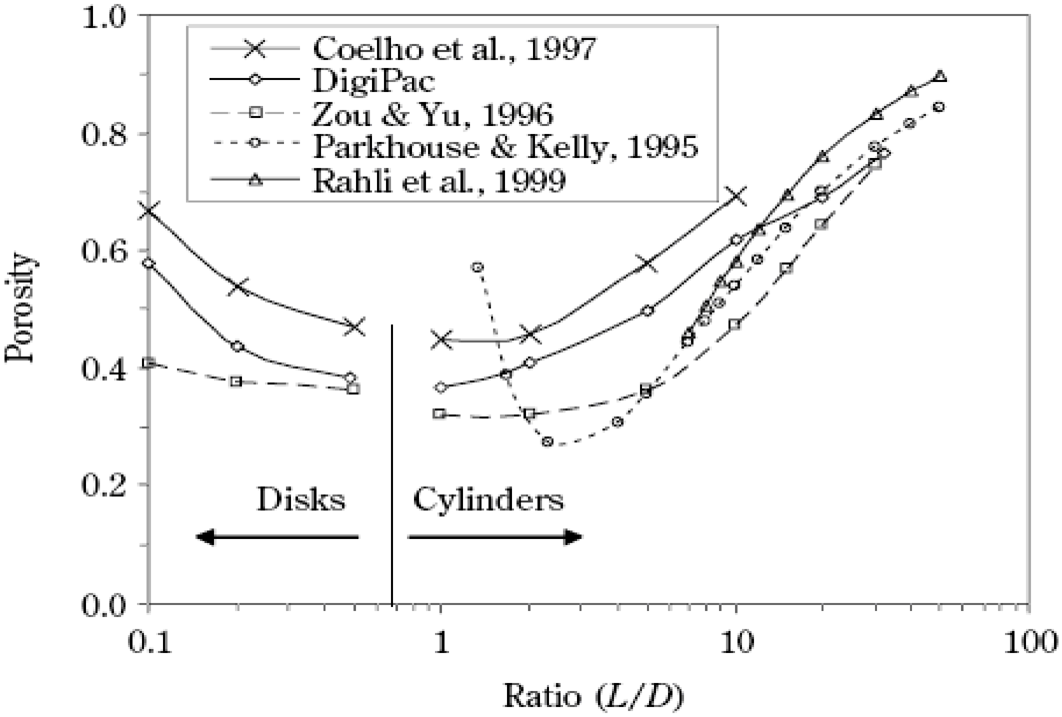


Figure 2.7 Porosity versus ratio of length to bed diameter for the packing of cylinders [98]

Mohammad Asif [99] study the sphericity in a packed bed of plastic cylinder particles. He found the following relationship:

$$\Phi = \frac{1.31\left(d_p L^2\right)^{1/3}}{L + 0.5 d_p} \qquad \dots (2.45)$$

2.6 Container Wall Effect

In a randomly packed bed of particles such as spheres, the layer of particles nearest to the wall tends to be highly ordered with most of them having a pint contact with the wall. The next layer builds up on the surface of the first in a less ordered fashion. The subsequent layers are less and less ordered until a fully randomized arrangement is obtained in regions far away from the wall. Thus, the wall has a confining effect on the location of the particles and therefore on the structural characteristics such as the void fraction and specific lateral surface area. In beds of spheres, for example, the confining effect of the wall gives rise to heavily damped oscillations in the void fraction. The void fiaction starts from a value of unity at the wall, falls to a minimum value of about *0.2* at a distance of about half a particle diameter, and attains a constant value at a distance of about four to five particle diameters from the wall. Thus, for beds of ratio of bed to particle diameter less than 10 packed with spheres, the variations are spread over the entire cross section of the bed [45]

Figure 2.8 shows the fluctuation of porosity in a bed of spheres and cylinders [8,45].

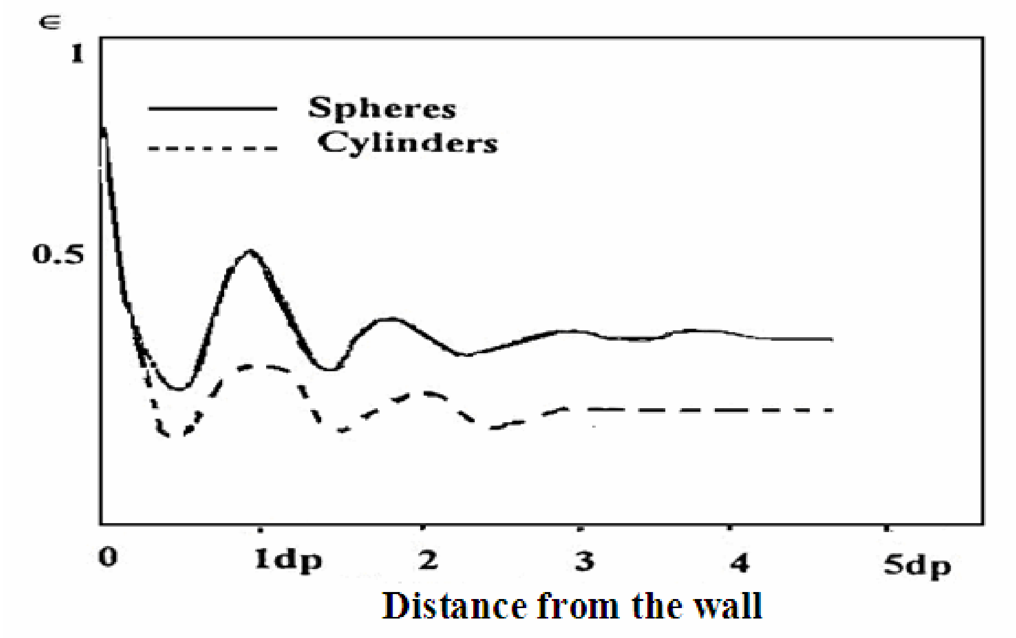


Figure 2.8 The fluctuation of porosity in a bed of spheres and cylinders [8,45]

The wall effects could arise due to [45]:

1- The well defined variations in porosity and lateral surface area brought about by the confining nature of the wall.

2- Resistance of the wall to flow, because the walls create an additional surface area providing additional resistance to flow.

3- The viscous effects in the region close to the wall.

4- The particles adjacent to the walls pack more loosely than those more remote from them thus increasing the porosity of the zone near the walls.

As the surface of the column offers considerable resistance to flow, which is particularly appreciable when viscous effects in the region close to the wall are significant, it is important to make a correction for the friction of the column wall [45].

To decrease wall effects, the particle diameter should be small in comparison with the column diameter in which the packing is contained [72]. **Furnas** [38] studied the wall effect and found that when the ratio of the diameter of the container (D_r), to that of the particle (d_p), is greater than 10:1, the wall effect can be neglected [52].

A wall effect correction factor *fw* for velocity though packed bed has been determined experimentally by **Coulson and Richardson** [8] as:

$$f_w = \left(1 + \frac{1}{2}\frac{S_c}{S}\right)^2 \qquad \text{… (2.46)}$$

Where: S_c is the surface of the container per unit volume of bed.
S is the specific surface area of the particles.

Carman [40] has suggested correcting for the friction of the wall by including the lateral surface area of the column wall. This is given by:

$$f_w = \frac{S}{S + S_C} \qquad \text{… (2.47)}$$

Sullivan and Hertel [102] have proposed the following correction factor.

$$f_w = \frac{S}{S + 2S_C^2/3} \qquad \dots (2.48)$$

Coulson [11] has suggested introducing the following correction factor:

$$f_w = \frac{S}{S + S_C^2/2} \qquad \dots (2.49)$$

Machac and Hlhvsova [100] have suggested the following simple equation for the correction factor:

$$fw = 10^{1/a} \qquad \dots (2.50)$$

Where a is the aspect ratio which is the ratio of bed to particle diameter.

To account the wall-effects, **Liu and Masliyah** [101] suggested the following corrections:

$$f_w = 1 + \frac{\pi d_p}{6(1-\varepsilon)D_r} \qquad \dots (2.51)$$

$$f_w = 1 - \frac{\pi^2 d_p}{24 D_r}\left[1 - \frac{d_p}{2D_r}\right] \qquad \dots (2.52)$$

Chapter Three

Suggested Semi-Empirical Equations

3.1 Introduction

The pore size (diameter) and porosity have a great effect on the properties of packed beds. There is no doubt that any small change in the porosity of the packed bed leads to a big change in the pressure drop required for the liquid or the gas to flow through the packed bed.

This chapter deals with suggesting modified equations for the pore size and porosity in a packed bed of sphere packing, and implements these equations in fluid flow through packed beds equation.

Several modified suggested equations for the pore size and porosity had been attempted depending on the parameters affecting them for sphere particles of mono, binary and ternary packing systems. The considered parameters affecting the pore size (diameter) are diameter of the particle, the porosity of the bed and the permeability to fluid flow. The considered parameters affecting the porosity in the packed bed of sphere packing are the particle and bed diameters. General equations for pore size and porosity that can be used for any type of packing systems were also suggested in the present work.

3.2 The Pore Size suggested Equations

The pores are tortuous and interconnected, with a distribution of different sizes and shapes, it is convenient to assume that the pore diameter (d_{pore}) represents a cylindrical form of the hydraulic diameter (r_H) which is the cross sectional area perpendicular to fluid flow divided by the wetted perimeter, this can be represented as [103]:

$$r_H \ \ for\ pourous\ medium = \frac{volume\ open\ to\ flow}{total\ watted\ surface} \qquad \dots (3.1)$$

$$r_H = \frac{volume\ of\ bed * \varepsilon}{No\ of\ spherical\ particles * surface\ area\ of\ one\ particle} \qquad \dots (3.2)$$

But:

$$No.of\ particles = \frac{Volume\ of\ bed * (1-\varepsilon)}{Volume\ of\ one\ particle} \qquad \dots (3.3)$$

Therefore:

$$r_H = \frac{volume\ of\ bed * \varepsilon}{volume\ of\ bed * (1-\varepsilon) * \frac{surface\ area}{volumeofparticle}} \qquad \dots (3.4)$$

i.e.

$$r_H = \frac{\varepsilon}{(1-\varepsilon)\left(\pi d_p^2 / \frac{\pi}{6} d_p^3\right)} \qquad \dots (3.5)$$

i.e.

$$r_H = \frac{d_p}{6}\left(\frac{\varepsilon}{1-\varepsilon}\right) \qquad \dots (3.6)$$

While the pore diameter (equivalent to the hydraulic diameter) is defined as four times the cross sectional area per wetted perimeter [104], therefore:

$$d_{pore} = 4r_H = \frac{2}{3}\frac{\varepsilon}{(1-\varepsilon)}d_p \quad \dots (3.7)$$

Equation (3.7) can be taken as experimental value of pore diameter where the particle diameter and the porosity are determined experimentally.

Many theoretical semi-empirical equations for the pore diameter have been used before, one of them is **Latif** (equation 2.27) [82]. This equation has been modified in this work to get more accurate results for the pore diameter of multi size packing. The equation takes the pore diameter to be proportional to the largest particle diameter in the packing (d_g), as:

$$d_{pore} \propto d_g \quad \dots (3.8)$$

Where the proportionality constant is F:

$$d_{pore} = Fd_g \quad \dots (3.9)$$

It is found that F is a function of the ratio between the diameter of small and large particle (a_k), and the ratio between the diameter of medium and large particle (a_m), as shown below:

$$F = f(a_m, a_k) \quad \dots (3.10)$$

Experimental data were used to write an empirical formula to the proportionality constant:

$$F = b_1 \frac{a_m^{b_2} * a_k^{b_3}}{b_4 a_m^{b_5} - b_6 a_k^{b_7}} \quad \dots (3.11)$$

where b_1, b_2,… b_7 are constants which can be evaluated from experimental data taken from literature by statistical fitting.

Then the first modified equation for the pore diameter can be written after the substitution of equation (3.11) in (3.9), as shown below:

$$d_{pore} = \left(b_1 \frac{a_m^{b_2} * a_k^{b_3}}{b_4 a_m^{b_5} - b_6 a_k^{b_7}} \right) * d_g \qquad \ldots (3.12)$$

Equation (3.12) can be used only for multi sized packing system, because the multi size packing includes different sizes of particles.

Another equation for the determination of the pore diameter was developed, but this time for a certain type of packing system (mono, binary, ternary,..., multi sized packing systems). This equation can be considered as a modification of **Millington** and **Quirk** equation (equation 2.25) [86], it can be used for pore size of sphere packing in water. In the modification, it was suggested that the pore diameter is a function of permeability and porosity of the packed bed, and can be written as follows:

$$d_{pore} = f(k, \varepsilon) \qquad \ldots \quad (3.13)$$

Three forms for equation (3.13) were proposed. These forms are shown below:

$$d_{pore} = c_1 \frac{k^{c_2}}{\varepsilon^{c_3}} \qquad \ldots (3.14a)$$

$$d_{pore} = \left(\frac{c_4 k}{\varepsilon^{c_6}} \right)^{c_5} \qquad \ldots (3.14b)$$

$$d_{pore} = c_7 \frac{k^{c_8}}{(1-\varepsilon)^{c_9}} \qquad \ldots (3.14c)$$

Where c_1, c_2, …c_9 are constants and can be evaluated from experimental data taken from literature by using statistical fitting. After the substitution of the constants it was found that equation (3.14a) gives the best results after using experimental data from literature [1,3,105,113,114,115] for water flow through packed bed of sphere packing. This modified equation will be used in the present work.

3.3 The Porosity suggested Equations

The porosity has a great effect on the properties of packed beds. Several attempts were made to simulate the porosity in packed beds [11,26,27,33,40]. Semi-empirical equations were developed in the present work by modifying Furnas equation of porosity (equation 2.3) [83]. The new forms of the suggested equations of porosity depend on particle diameter (d_p) and bed diameter (D_r). Experimental data were used to get the new forms of porosity. The equation which gives the best results compared with experimental results taken from literature was used then in the present work. The proposed equations of porosity can be written as follows:

$$\varepsilon = i_1 + i_2 \left(\frac{d_p}{D_r} \right)^{i_3} \qquad \dots (3.15a)$$

$$\varepsilon = i_4 + i_5 \frac{d_p}{D_r} \qquad \dots (3.15b)$$

$$\varepsilon = i_6 \left(\frac{d_p}{D_r} \right)^{i7} \qquad \dots (3.15c)$$

$$\varepsilon = i_8 \frac{d_p^{i_9}}{D_r^{i_{10}}} \qquad \dots (3.15d)$$

$$\varepsilon = i_{11} - i_{12} d_p + i_{13} D_r^{i_{14}} \qquad \dots (3.15e)$$

Where i_1, i_2,... i_{14} are constants and can be evaluated from experimental data taken from literatures [1,3,112,113,114,115,116,117] by using statistical fitting. After the substitution of the constants in the above equations it was found that equation (3.15a) was the best one compared with experimental data for water flow through packed bed of sphere packing. This equation was considered in the present work as a proposed porosity equation.

3.4 The Pressure Drop Semi-Empirical Equations

Semi-empirical equations for modeling fluid flow through packed bed have been proposed, which include the suggested pore size and porosity equation.

Different models have been developed to characterize the flow of fluids through porous medium. The first attempts can be traced back to the publication of **Darcy** [12] where he established his known Darcy's Law, which states that the pressure drop per unit length for a flow through a porous medium is proportional to the product of the fluid velocity and the dynamic viscosity [105].

$$\frac{-\Delta P}{l} = \frac{\mu}{K} u \qquad \dots (3.16)$$

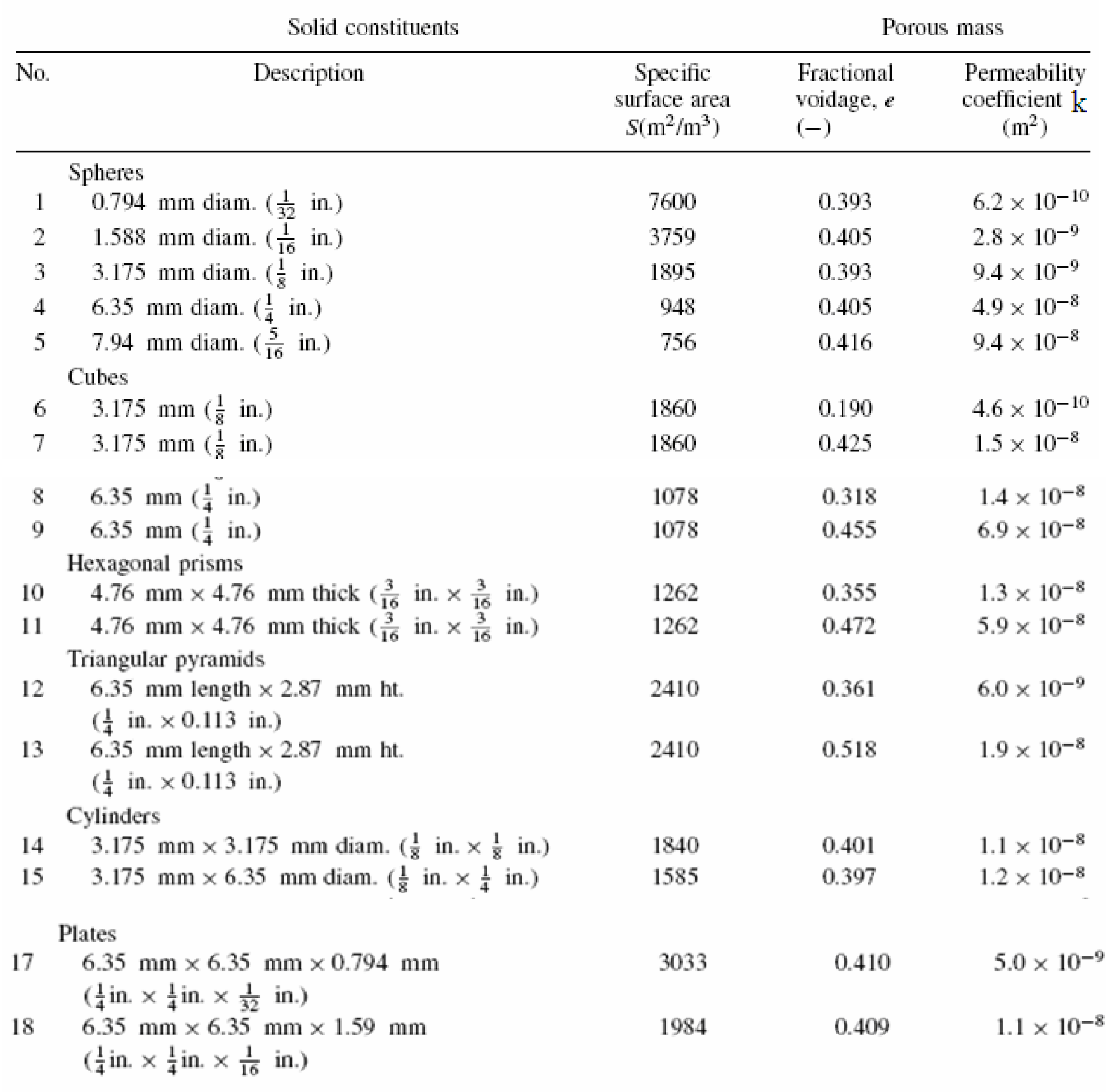
where $-\Delta P$ is the pressure drop across the bed, l is the thickness of the bed, u is the average velocity of flow of the fluid, defined as (1/A) (dV/dt), A is the total cross sectional area of the bed, V is the volume of fluid flowing in time t, and K is a constant depending on the physical properties of the bed and fluid.

Darcy's law has subsequently been confirmed by a number of workers, and they added an inverse proportionality constant to the Darcy Law known as the fluid permeability (k), which is a measure of the resistance the fluid undergoes when passing through a porous medium. The resistance to flow then arises mainly from viscous drag [105]. Equation 3.16 can then be expressed as:

$$\frac{-\Delta P}{l} = \frac{\mu}{k} u \qquad \dots (3.17)$$

where μ is the viscosity of the fluid and k is termed the permeability coefficient for the bed, and depends only on the properties of the bed. The value of the permeability coefficient is frequently used to give an indication of the case with which a fluid will flow through a bed of particles [105]. The values of k for laminar flow region for various types of packing are shown in table 3.1 [8].

Table 3.1 Properties of beds of some regular-shaped materials [8]



Solid constituents			Porous mass	
No.	Description	Specific surface area $S(m^2/m^3)$	Fractional voidage, e (−)	Permeability coefficient k (m^2)
	Spheres			
1	0.794 mm diam. ($\frac{1}{32}$ in.)	7600	0.393	6.2×10^{-10}
2	1.588 mm diam. ($\frac{1}{16}$ in.)	3759	0.405	2.8×10^{-9}
3	3.175 mm diam. ($\frac{1}{8}$ in.)	1895	0.393	9.4×10^{-9}
4	6.35 mm diam. ($\frac{1}{4}$ in.)	948	0.405	4.9×10^{-8}
5	7.94 mm diam. ($\frac{5}{16}$ in.)	756	0.416	9.4×10^{-8}
	Cubes			
6	3.175 mm ($\frac{1}{8}$ in.)	1860	0.190	4.6×10^{-10}
7	3.175 mm ($\frac{1}{8}$ in.)	1860	0.425	1.5×10^{-8}
8	6.35 mm ($\frac{1}{4}$ in.)	1078	0.318	1.4×10^{-8}
9	6.35 mm ($\frac{1}{4}$ in.)	1078	0.455	6.9×10^{-8}
	Hexagonal prisms			
10	4.76 mm × 4.76 mm thick ($\frac{3}{16}$ in. × $\frac{3}{16}$ in.)	1262	0.355	1.3×10^{-8}
11	4.76 mm × 4.76 mm thick ($\frac{3}{16}$ in. × $\frac{3}{16}$ in.)	1262	0.472	5.9×10^{-8}
	Triangular pyramids			
12	6.35 mm length × 2.87 mm ht. ($\frac{1}{4}$ in. × 0.113 in.)	2410	0.361	6.0×10^{-9}
13	6.35 mm length × 2.87 mm ht. ($\frac{1}{4}$ in. × 0.113 in.)	2410	0.518	1.9×10^{-8}
	Cylinders			
14	3.175 mm × 3.175 mm diam. ($\frac{1}{8}$ in. × $\frac{1}{8}$ in.)	1840	0.401	1.1×10^{-8}
15	3.175 mm × 6.35 mm diam. ($\frac{1}{8}$ in. × $\frac{1}{4}$ in.)	1585	0.397	1.2×10^{-8}
	Plates			
17	6.35 mm × 6.35 mm × 0.794 mm ($\frac{1}{4}$in. × $\frac{1}{4}$in. × $\frac{1}{32}$ in.)	3033	0.410	5.0×10^{-9}
18	6.35 mm × 6.35 mm × 1.59 mm ($\frac{1}{4}$in. × $\frac{1}{4}$in. × $\frac{1}{16}$ in.)	1984	0.409	1.1×10^{-8}

Forchheimer [110] proposed a quadratic equation for the non-linear flow region:

$$\frac{\Delta P}{L} = a u + b u^2 \qquad \dots \quad (3.37)$$

where ***a*** and ***b*** are factors which depend on both fluid and porous medium properties. The expression for *a* and *b* has been studied by many investigators [16,73,40,111].The most widely used expression for *a* and *b* is that given by **Ergun** in 1952 [16].

Forchheimer [111] might be the first to point out that the departure of predictions by Darcy's law from measurements may be due largely to the kinetic effect of fluid which is not included in the models for small- Reynolds-number flows. For this reason, he suggested that a term representing the kinetic energy of fluid (ρu^2) must be included in equation (3.17), i.e.

$$\frac{-\Delta P}{L} = \frac{\mu}{k} u + a \ \rho \ u^2 \qquad \dots (3.38)$$

This added term is often referred to as the Forchheimer term in the literatures and, accordingly, the parameter *a* is called the Forchheimer constant or parameter.

It may be relevant to note that the Forchheimer term was also expressed in the form $a\rho u^m$, as

$$\frac{-\Delta P}{L} = \frac{\mu}{k} u + a\rho u^m \qquad \dots (3.39)$$

Where a is a property of the porous media and m is a measure of the influence of fluid inertia, where m=1.6-2.0 [16,73,40,111].The most widely used expression for a is that given by Ergun [16]: $a=C_E/k^{1/2}$, where C_E is the so-called Ergun constant. Although the Ergun constant is dimensionless, it is not a universal constant and is often found to vary with changes in porosity and structure of the porous medium. Ergun's version of equation (3.38) is [105]

$$\frac{\Delta \mathrm{P}}{L}=\frac{\mu}{k}u+\frac{C_E}{\sqrt{k}}\rho u^2 \qquad \ldots (3.40)$$

Forchheimer's equation has been successfully employed to predict the permeability of granular media. Such equation states that for an uncompressible fluid, the pressure drop through a porous medium is given by [104]:

$$\frac{\Delta \mathrm{P}}{L}=\frac{\mu}{k_1}u+\frac{\rho}{k_2}u^2 \qquad \ldots (3.41)$$

where k_1 and k_2 are constants depend only on the medium properties, known respectively as Darcian and non-Darcian permeabilities [104].

Ergun [16] proposed expressions to describe k_1 and k_2 for packed columns made of spheres, cylinders, tablets, nodules, round sand and crushed materials (glass, coke, coal, etc.) as follows:

$$k_1=\frac{\varepsilon^3\phi^2 d_p^2}{150(1-\varepsilon)^2} \qquad \ldots (3.42)$$

$$k_1=\frac{\varepsilon^3\phi\ d_p}{1.75(1-\varepsilon)} \qquad \ldots (3.43)$$

Therefore equation (3.41) could be written as shown below:

$$\frac{\Delta P}{L}=150\ \frac{\mu u}{\phi^2 d_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}+1.75\ \frac{\rho u^2}{\phi d_p}\frac{(1-\varepsilon)}{\varepsilon^3} \qquad \ldots (3.44)$$

Where ΔP, ε, ρ, d_p, Φ, u, L, and μ are the pressure drop, void fraction of the bed, density of the fluid, particle diameter, sphericity of the particle, fluid velocity, height of the bed, and the fluid viscosity respectively [105].

Equation (3.44) is called Ergun equation. His equation is a unique among many equations because it covers any flow type and condition (laminar, transitional and turbulent) [7].

Semi-Empirical equations for fluid flow through packed bed were developed in our work by modifying Forchheimer's equation (3.40) to include the effect of pore size and porosity formulas which were proposed in equations (3.12) and (3.15a). Although pore size and porosity are not included directly in the modified Forchheimer's law, they are included in the permeability of the flow. Three semi-empirical modified equations for fluid flow were proposed for the different types of packing systems, which can be written as follows:

$$\frac{\Delta P}{L} = j_1 \frac{\mu u}{k^{j_2}} + j_3 \frac{\rho u^2}{k^{j_4}} \qquad \text{… (3.45a)}$$

$$\frac{\Delta P}{L} = \frac{\mu}{k} u + \frac{j_5}{\sqrt{k}} \rho u^2 \qquad \text{… (3.45b)}$$

$$\frac{\Delta P}{L} = j_6 \frac{\mu}{k} u + \frac{j_7}{\sqrt{k}} \rho u^2 \qquad \text{… (3.45c)}$$

Where $j_1, j_2, \ldots j_7$ are constants which can be evaluated from experimental data taken from literature [1, 3, 112, 113, 114, 115, 117] by statistical fitting. After the substitution of the constants in the above equations, it was found that equation (3.45a) gives the best results compared with experimental data for water flow through packed bed of sphere packing and was considered in the present work.

Pore diameter, bed porosity and tortuosity are the main parameters of the permeability. A semi-empirical formula was proposed to evaluate the permeability for each type of packing using experimental data, by analyzing the parameters affecting the permeability in the packed beds, as shown below:

$$k = n_1 \frac{d_{pore}^{n_2}\ \varepsilon^{n_3}}{\tau^{n_4}\left(1-\varepsilon\right)^{n_5}} \qquad \ldots (3.46)$$

Where n_1, n_2,… n_5 are constants which can be evaluated from experiments data taken from literature by statistical fitting, τ is the tortuosity which is one of the most important parameter.

The tortuosity can be calculated theoretically using **Maxwell** [69] equation:

$$\tau = 1.5 - 0.5\varepsilon \qquad \ldots (2.7)$$

Equation (2.7) can be modified to satisfy the proposed equations of pore size, porosity and pressure drop. Several forms of tortuosity equations have been suggested. Experimental data were used to get the best form. These forms are as follows:

$$\tau = m_1 - m_2\varepsilon^{m_3} \qquad \ldots (3.47a)$$

$$\tau = m_4 - m_5\varepsilon \qquad \ldots (3.47b)$$

$$\tau = \frac{m_6}{\varepsilon^{m_7}} \qquad \ldots (3.47c)$$

Where m_1, m_2,…m_7 are constants that can be evaluated from experimental data taken from literature [1, 3, 112, 113, 114, 115, 117] by statistical fitting. The more accurate formula was taken to be used in the present work. This formula gives the smallest average percentage error and gives the value of one when the porosity is one. After the substitution of the constants it is found that the best equation is equation (3.47a).

3.5 Wall Correction Factor

A wall effect correction factor *fw* for velocity though packed bed has been determined experimentally by many authors like **Coulson** [8]**.** Semi-empirical equations have been suggested in the present work for the friction of the wall by studying the main parameters that cause this friction effect of the wall. It have been seen that the friction is due to the specific surface area of the particles (S) and the surface area of the container per unit volume of bed (S_c), i.e.:

$$f_w = f(S_C, S) \quad \text{… (3.48)}$$

Or it can be written in terms of the main parameters which are the particle diameter, bed diameter and bed length, as follows:

$$f_w = f(d_p, D_r, L) \text{… (3.49)}$$

From the above relations the correction factor can be written as follows:

$$f_w = y_1 + y_2 \frac{S_C}{S} \quad \text{… (3.50a)}$$

$$f_w = \left(y_3 + y_4 \frac{S_C}{S} \right)^{y_5} \quad \text{… (3.50b)}$$

$$f_w = y_6 \left(\frac{L}{d_p + D_r} \right)^{y_7} \quad \text{… (3.50c)}$$

Where y_1, y_2,…y_7 are constants, which can be evaluated from experimental data taken from literature by statistical fitting. After the substitution of the constants it was found that the best equation to be used is equation (3.50a).

Chapter Four

Results and Discussions

The present chapter deals with the results and discussions of the suggested semi-empirical equations. These results depend on values of pore diameter, porosities, velocities, bed length and other parameters taken from literature work. For each type of packing system, semi-empirical equations were suggested. A general form for multi sized packing systems was also written. This chapter also contains the discussions of the suggested equations results, and the comparisons between these results and experimental results taken from literatures, as well as comparisons were made between all these results and similar results taken from theoretical equations (such as Furnas equation for porosity, Ergun equation for the pressure drop,… etc).

4.1 The Pore Size Suggested Equations

4.1.1 The Equations Constants

Semi-empirical equations for pore size equation (3.14a) were fitted using experimental data obtained from literatures to calculate the different constants in it. This had been done for water flow through packed bed for mono, binary and ternary packing systems. The same thing had been done for a general equation to be used for all types of packing systems.

Table 4.1 Constants of equation 3.14a for water flow through packed bed

System type	c_1	c_2	c_3
Mono	4.2393	0.521	-0.2127
Binary	2.0872	0.2925	-0.3688
Ternary	2.1612	0.2925	-0.4078
General	11.767	0.398	0.18

Table 4.2 Constants of equation 3.12 for water flow through packed bed

System type	b_1	b_2	b_3	b_4	b_5	b_6	b_7
Ternary	0.6371	0.1681	0.9911	1.1561	-0.145	0.1621	0.1591

4.1.2 Studying The Effect of Different Parameters on Pore Size

This section shows the effect of different parameter on pore size using equation 3.14a after the substitution of the constants for the suggested general equation of multi-sized particle systems. The system includes all different types of packing systems (mono size, binary sized and ternary sized spherical particles system). A certain range for each parameter was taken in this study according to the available experimental data from literatures.

4.1.2.1 Effect of porosity on pore size

Figure 4.1 shows that the pore diameter increases with increases the porosity, this is because when the void fraction between particles increases this leads to a large space between particles.

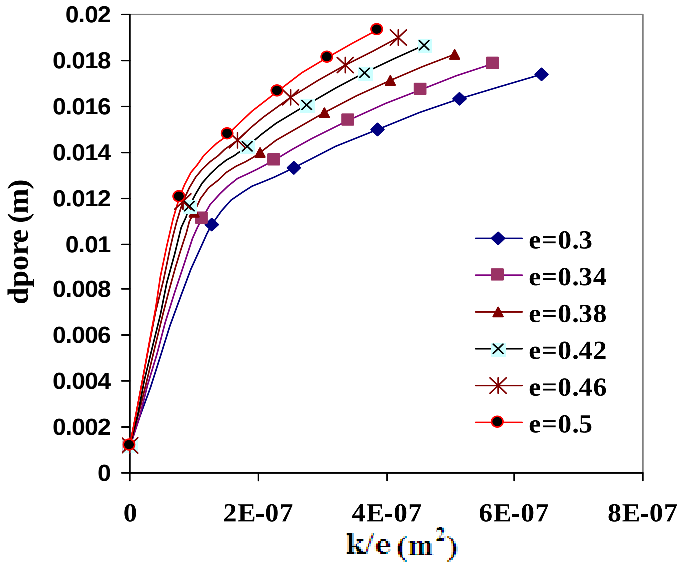


Figure 4.1 Pore size versus the ratio of permeability to porosity

4.1.2.2 Effect of permeability on pore size

Figure 4.2 indicates that increasing the permeability causes an increase in the pore diameter.

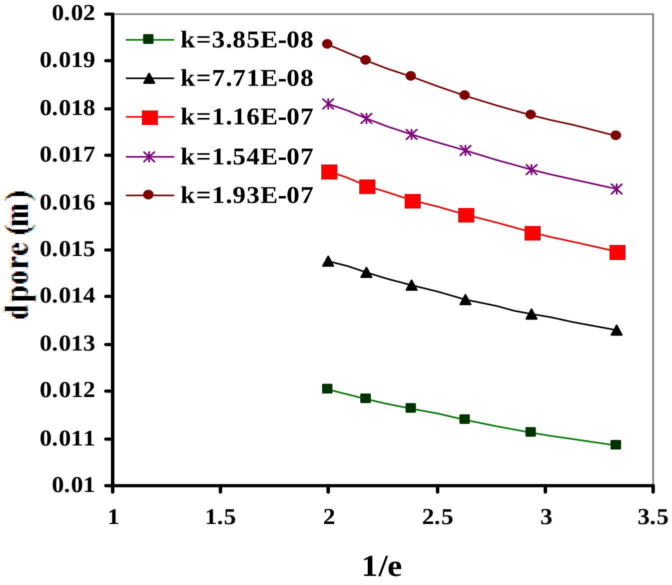


Figure 4.2 Pore size versus the porosity inverse

4.1.3 Comparisons Between Proposed Equations and Experimental Results For Water Flow Through Packed Bed

The pore size modified models have been tested. This test was with experimental data available. Comparisons were made for different types of packing system and also comparisons were made for the general equation after substitution of the constants of the modified equation.

4.1.3.1 Mono Size Spherical Particle System

The modified equation (3.14a) was fitted for water flow through packed beds of mono-sizes spherical particles. 23 experiments from the literature [1,3,105,113,114,115] were used for fitting. The suggested equation was:

$$d_{pore} = 4.2393 \frac{k^{0.521}}{\varepsilon^{-0.2127}} \qquad \ldots (4.1)$$

4.1.3.2 Binary Sized Spherical Particles System

In the packing of binary size particles the mixture contains two sizes of sphere particles. The percentage of each size is equal 50% from the total packing.

The modified equation (3.14a) was fitted for this case using 25 experiments from literatures [3,114,116,117], and can be written as follows:

$$d_{pore} = 2.0872 \frac{k^{0.2925}}{\varepsilon^{-0.3688}} \qquad \ldots (4.2)$$

4.1.3.3 Ternary Sized Spherical Particles System

In the packing of ternary size particles the mixture contains three sizes of sphere particles. The percentage of each size is equal 1/3 from the total packing.

The modified equation (3.14a) was fitted for this case using 19 experiments from literatures [116,117], and can be written as follows:

$$d_{pore} = 2.1612 \, \frac{k^{0.2925}}{\varepsilon^{-0.4078}} \qquad \ldots (4.3)$$

The pore size proposed equation results are shown in table A.2 (appendix A), the tables shows the experimental values of the porosity, permeability and particle diameter taken from literatures [116,117]. The experimental values of the pore size were determined by using equation (3.7).

The semi-empirical equation (3.12) that has been suggested for the pore diameter for multi size was fitted using 20 experiments from literatures [116,117] to evaluate the constants of the equation. The suggested equation was found to be as follows:

$$d_{pore} = \left(0.6371 \frac{a_m^{0.1681} * a_k^{0.9911}}{1.1561 \, a_m^{-0.145} - 0.1621 \, a_k^{0.1591}} \right) * d_g \qquad \ldots (4.4)$$

4.1.3.4 General Equation Results

The modified equation (3.14a) was fitted for all systems considered in the present work, using 62 experiments from the literatures [1, 3, 105, 113, 114, 115, 116, 117], and can be written as follows:

$$d_{pore} = 2.1057 \, \frac{k^{0.294}}{\varepsilon^{-0.208}} \qquad \ldots (4.5)$$

Table A.2 (appendix A) show that the suggested general equation gives a very good fitting to the experimental results; therefore it can be used with confidence for any type of packing systems. The general equation gives very good results much better than the equations written for a certain type of packing.

4.2 The Porosity Proposed Equations

4.2.1The Equations Constants

Equation (3.15a) was fitted using experimental data obtained from literatures [1,3,112,113,114,115,116,117], in order to calculate the different constants in it. This had been done for water flow through packed bed for different types of packing. The resulted constants are presented in tables 4.9 below. The same thing had been done for the general equation that will be used for all types of packing systems.

Table 4.3 Constants of equation 3.15a for water flow through packed bed

System type	i_1	i_2	i_3
Mono	0.3651	0.0813	0.5088
Binary	0.3846	-0.0338	0.8413
Ternary	0.3046	0.7401	0.8797
General	0.3734	0.0688	0.725

4.2.2 Studying The Effect Of Different Parameters on Porosity

The porosity is affected by many variables. The main two are particle diameter and bed diameter. A certain range for each parameter was taken in this study according to the available experimental data from literatures.

4.2.2.1 Effect Of Particle Diameter On Porosity

Figure 4.4 indicates that any increase in the particle diameter causes increase in the bed porosity for the same bed diameter range.

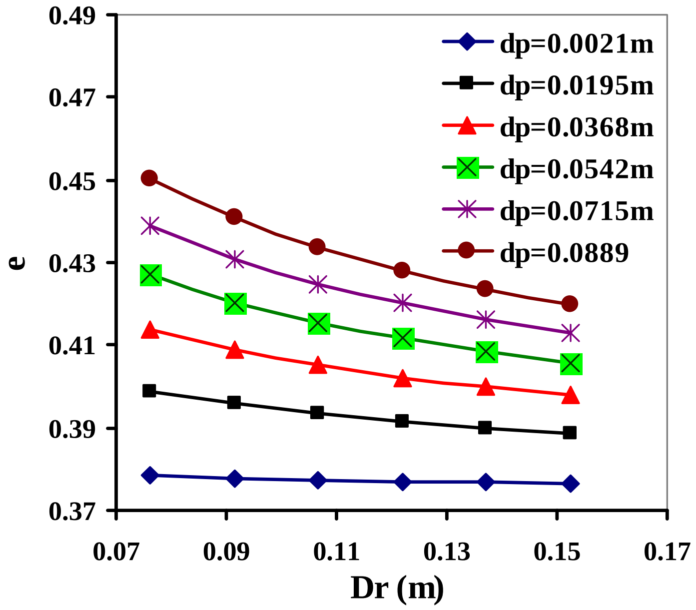


Figure 4.4 Porosity versus bed diameter

4.2.2.2 Effect of Bed Diameter on Porosity

Figure 4.5 shows that when the bed diameter was increased the porosity decreased for the same particle diameter.

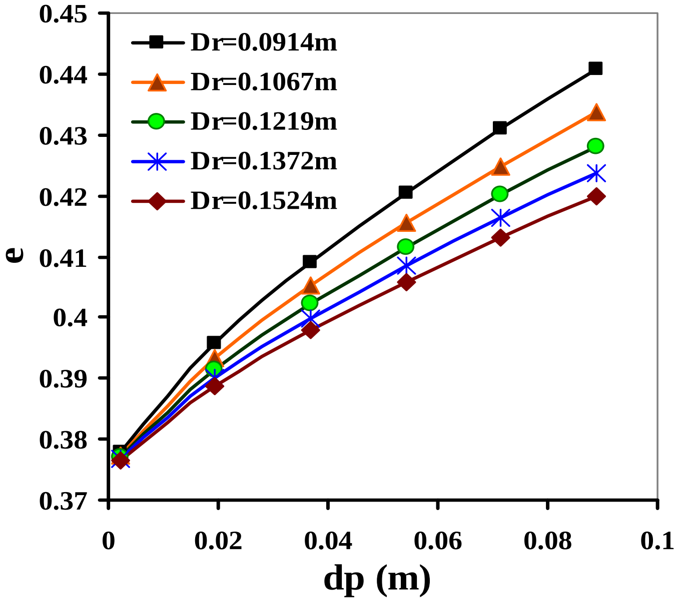


Figure 4.5 Porosity versus particle diameter

4.2.3 Comparisons Between Proposed Equation, Furnas Equation and Experimental Results For Water Flow Through Packed Bed

Comparisons have been made for different types of packing system and also for the general equation after the substitution of the constants in the modified equation (3.15a).

4.2.3.1 Mono Size Spherical Particle System

Equation (3.15a) was fitted for this case using 44 experiments from the literatures [1,3,112,113,114,115,116,117]. The equation will be as follows:

$$\varepsilon = 0.3651 + 0.0813\left(\frac{d_p}{D_r}\right)^{0.5088} \qquad \dots (4.6)$$

Table A.3 show a very good agreement between the porosity obtained by using the proposed equation and the experimental data, while results from Furnas equation for porosity was far away from the experimental data.

4.2.3.2 Binary Sized Spherical Particles System

The modified equation (3.15a) was fitted for this case using 27 experiments from literatures [3,114,116,117], and can be written as follows:

$$\varepsilon = 0.3846 - 0.0338\left(\frac{d_p}{D_r}\right)^{0.8413} \qquad \dots (4.7)$$

The results of the semi-empirical equation for porosity (equation 4.7) are shown in table A.4. This table also show comparisons between the porosity modified equation results, experimental values and theoretical values obtained by using Furnas equation (equation 2.3).

From tables A.4 (appendix A), it can be seen that the modified model gave a good fitting to the experimental data results rather than Furnas equation; this was expected because of the difference between the packing materials that Furnas used and the modified equation used.

The most noticeable effect for mixing two sizes of particles is the decrease in porosity with respect to mono sized particles. This is because for binary systems, the particles with smaller sizes tend to fill the voids between the larger sizes particles.

4.2.3.3 Ternary Sized Spherical Particles System

In the packing of ternary size particles the mixture contains three sizes of sphere particles. The percentage of each size is equal 1/3 from the total packing.

The modified equation (3.15a) was fitted for this case using 20 experimental data from literatures [116,117], which can be written as follows:

$$\varepsilon = 0.3046 + 0.7401\left(\frac{d_p}{D_r}\right)^{0.8797} \qquad \ldots (4.8)$$

The porosity proposed equation results are shown in tables A.5 (appendix A). The tables also shows comparisons between the porosity modified equation results, experimental values taken from literatures [116,117] and theoretical values (Furnas equation).

From tables A.5, it can be seen that the porosity of ternary sized packing are generally close to each other, since the voids were filled with different sizes of particles.

4.2.3.4 General Equation Results

The modified equation (3.14a) was fitted for all systems considered in the present work, using 77 experimental data from literatures [1, 3, 105, 113, 114,115,116,117,118]. This can be written as follows:

$$\varepsilon = 0.3734 + 0.0688\left(\frac{d_p}{D_r}\right)^{0.725} \qquad \ldots (4.9)$$

The porosity proposed equation results are shown in tables A.6. The tables also show comparisons between the porosity modified equation results, experimental data results and theoretical results (Furnas equation).

From tables A.6 is can be seen that, the modified equation results satisfied the experiment results rather than Furnas equation.

The general equation can be used for any packing system, while the equation written for a certain type of packing can be used only for one types of packing which was written for it.

4.3 The Pressure Drop Proposed Equations

4.3.1 The Equations Constants

Equation 3.45a was fitted with experimental data obtained from literatures, to calculate the different constant. This had been done for water flow through packed bed for different types of packing. The resulted constants are presented in table 4.14. The same thing has been done for the general equation that will be used for all types of packing systems.

Table 4. 4 Constants of the pressure drop equation 3.45a

System type	j_1	j_2	j_3	j_4
Mono	0.1309	-0.0492	0.0029	0.6448
Binary	-0.5194	-0.7012	0.0116	0.5627
Ternary	-0.8265	-3.8787	0.0025	0.6296
General	-13.0582	-29.2316	0.1493	0.4243

The permeability values used in equation 3.45a was taken from equation 3.46, after fitting it for water flow through packed bed. The resulting constants are written in table 4.15 below.

Table 4. 5 Constants of permeability equation 3.46

System type	n_1	n_2	n_3	n_4	n_5
Mono	0.00014	1.9264	0.5593	0.219	0.3916
Binary	0.0005	2.6349	-1.1061	0.0869	1.529

Ternary	0.0005	2.3369	0.9209	-0.2841	1.3469
General	0.3109	3.2354	0.6162	0.7682	1.6322

The tortuosity used in equation 3.46 was taken from equation 3.47a after fitting it for water flow through packed bed. The constants are shown in table 4.16 below for different types of packing systems.

Table 4.16 Constants of tortuosity equation 3.47a

System type	m_1	m_2	m_3
Mono	1.5312	0.5268	0.8984
Binary	1.4486	0.4336	1.1777
Ternary	1.3906	0.3107	1.3818
General	0.5787	-0.5264	-0.3384

The semi-empirical equation (equation 3.50a) that have been suggested in the present work for the wall effect correction factor *fw* for velocity though packed bed of large column diameter was fitted using experimental data obtained from literatures, in order to calculate the different constant. The constants are shown in table 4.17 for different types of packing systems.

Table 4. 7 Constants of equation 3.50a

System type	y_1	y_2
Mono	1	0.2495
Binary	1.00025	-3.572
Ternary	1	0.2495
General	1	-0.2861

4.3.2 Studying The Effect Of Different Parameters On Pressure Drop

This section shows the effect of different parameter on pressure drop using equation 3.45a for water flow through packed bed, after the substitution of the constants for the suggested general equation of pressure drop. The general equation includes all different types of packing systems used in the present work.

The important parameters affecting the pressure drop in the equation was found to be the permeability, pore diameter, porosity, tourtosity and bed length. Although pore size, tourtosity and porosity are not included directly in the suggested equation, they are included in the permeability term.

The physical properties of fluid (density and viscosity) were taken for water flow through packed bed at 25°C. The fluid velocity used was taken with in the fixed region.

4.3.2.1 Effect of Pore Diameter on Pressure Drop

Figure 4.6 indicates that an increase in pore diameter causes a decrease in pressure drop; this is due to the fact that when the pore diameter increases the resistance of fluid flow decreases, which leads to decrease in pressure drop.

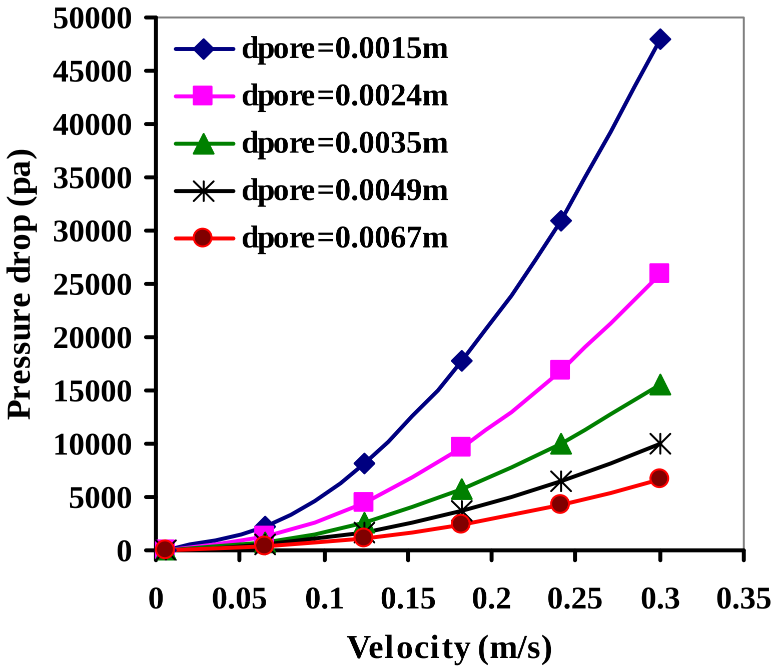


Figure 4.6 Pressure drop versus velocity for the conditions tortuosity of 1.337, porosity 0.34, bed length 0.15 m, column diameter 0.0914m, at different pore diameters of particles.

4.3.2.2 Effect Of Porosity On Pressure Drop

Figure 4.7 show that the pressure drop in the bed is inversely proportional to bed porosity for the same velocity of the fluid entering the bed [92]. This is due to the fact that when the void fraction between particles becomes larger less resistance will affect the fluid flow through the bed.

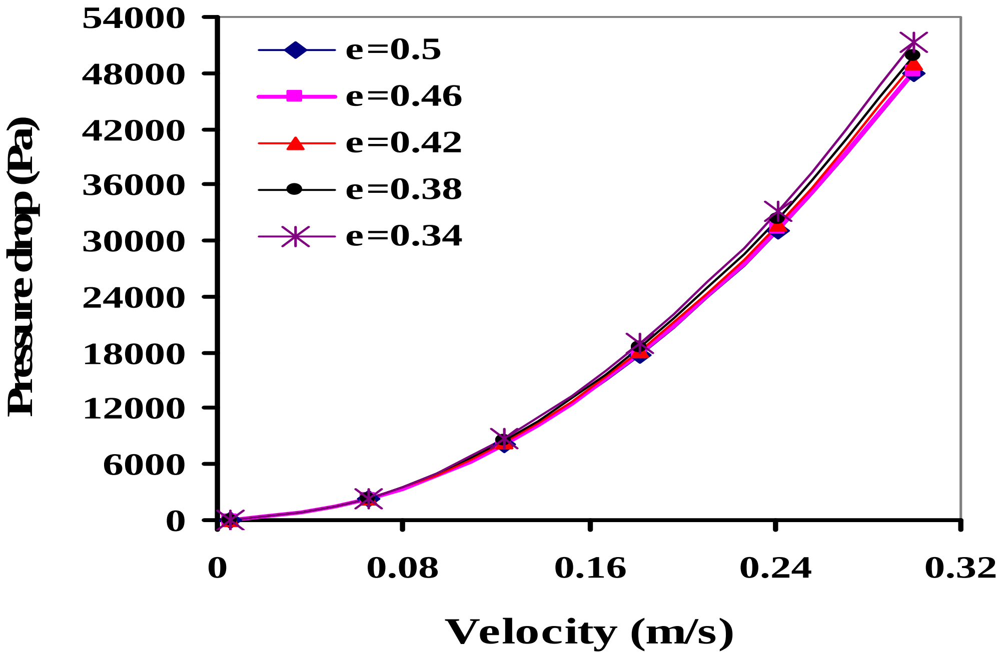


Figure 4.7 Pressure drop versus velocity for the conditions tortuosity of 1.337, bed length 0.15 m, column diameter 0.0914m, pore diameter 0.0015m, at different porosities.

4.3.2.3 Effect of Bed Length on Pressure Drop

Figure 4.8 show that whenever the length of the packing height increases the fluid flow resistance increases. This leads to an increase in pressure drop, this agree with Coluson 1949 [11].

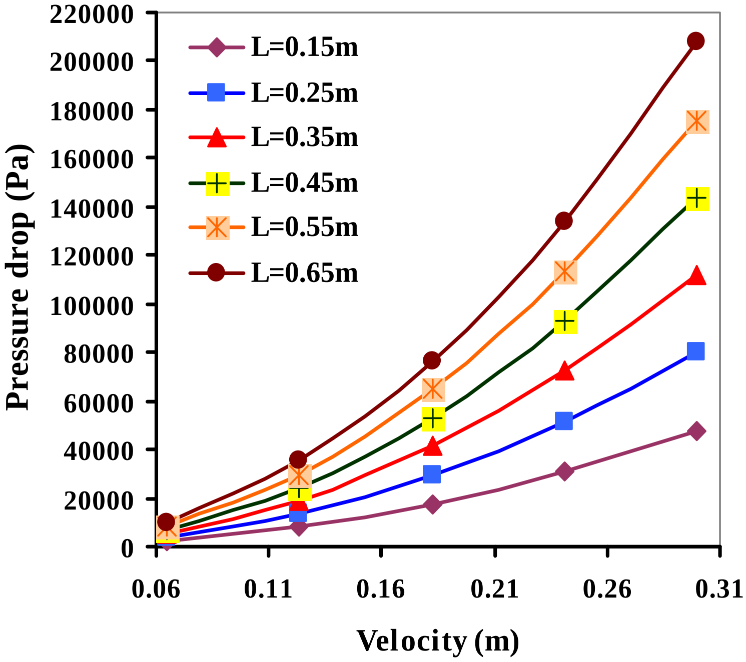


Figure 4.8 Pressure drop versus velocity for the conditions tortuosity of 1.337, column diameter 0.0914m, pore diameter 0.0015m, permeability of 4.86E-11, porosity 0.34, at different bed lengths.

4.3.2.4 Effect of Permeability on Pressure Drop

Figure 4.9 below shows that when the permeability to flow increases the pressure drop decreases, this is because the void fraction between particles become larger which leads to less resistance to fluid flow through the bed.

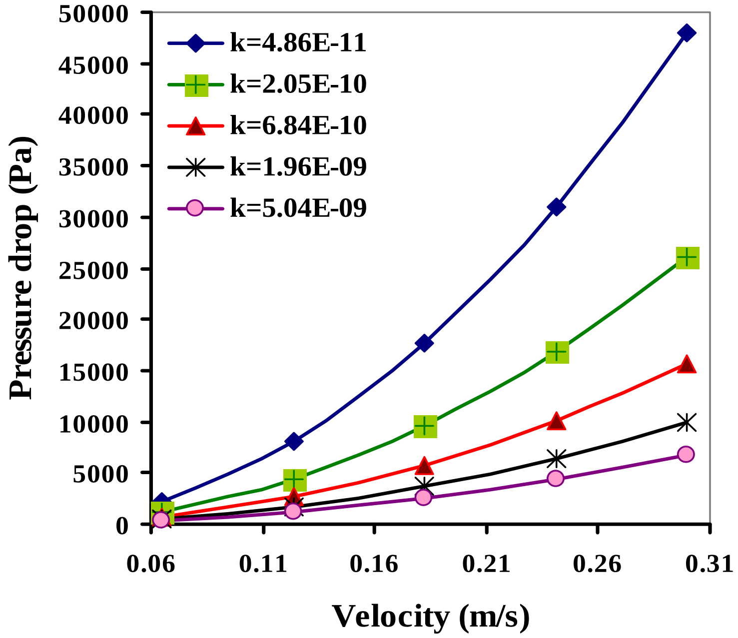


Figure 4.9 Pressure drop versus velocity for the conditions tortuosity of 1.337, column diameter 0.0914m, pore diameter 0.0015m, bed length of 0.15 m, porosity 0.34, at different permeabilitys.

4.3.3 Comparisons Between Proposed Equations, Theoretical Equations and Experimental Results For Water Flow Through Packed Bed

The pressure drop, permeability and tortousity semi-empirical modified equations have been tested in this section by comparing the results of these equations with results of experimental data and theoretical calculations. These comparisons have been made for different types of packing system. The same thing have been done for the modified general equation after the substitution of the constants in it.

4.3.3.1 Mono Size Spherical Particle System

The pressure drop equation (3.45a) was fitted for water flow through packed beds of mono-sizes spherical particles. In this fitting 30 sets of data from literatures [1, 3, 112, 113, 114, 115, 117] were used. In these sets 318 values of pressure drop versus velocity were involved. Equation (3.45a) becomes:

$$\frac{\Delta P}{L} = 0.1309 \frac{\mu u}{k^{-0.0492}} + 0.0029 \frac{\rho u^2}{k^{0.6448}} \qquad \ldots (4.10)$$

The permeability used in equation 4.10 was taken from equation 3.46 which include the pore size and porosity modified equations. The permeability equation (3.46) was fitting for this case using experimental data from literatures, and can be written as follows:

$$k = 0.00014 \frac{d_{pore}^{1.9264} \varepsilon^{0.5593}}{\tau^{0.219} (1-\varepsilon)^{0.3916}} \qquad \ldots (4.11)$$

The tortousity used in equation 4.11 was proposed from best fitting of experimental data. This tortousity equation used can be represented in the following equation:

$$\tau = 1.5312 - 0.5268 \varepsilon^{0.8984} \qquad \ldots (4.12)$$

The permeability used in equation 4.10 has been tested. This test was by comparing the results of this equation with results of experimental data and Carman equation [75]. The results show very good agreement between the permeability equation results and the experimental data, while the theoretical equation of Kozeny-Carman equation was far away from the experimental data results. So the suggested permeability equation can be used with confidence with any type of packing system.

The results of pressure drop versus velocity for water flow through packed beds of mono size particles are plotted in figures 4.10 to 4.14.

In the suggested equation several types of packing were used, which were Pea Gravel, Marbles, Glass Marbles, Black Marbles, Clear Marbles, Acrylic balls and Glass spheres.

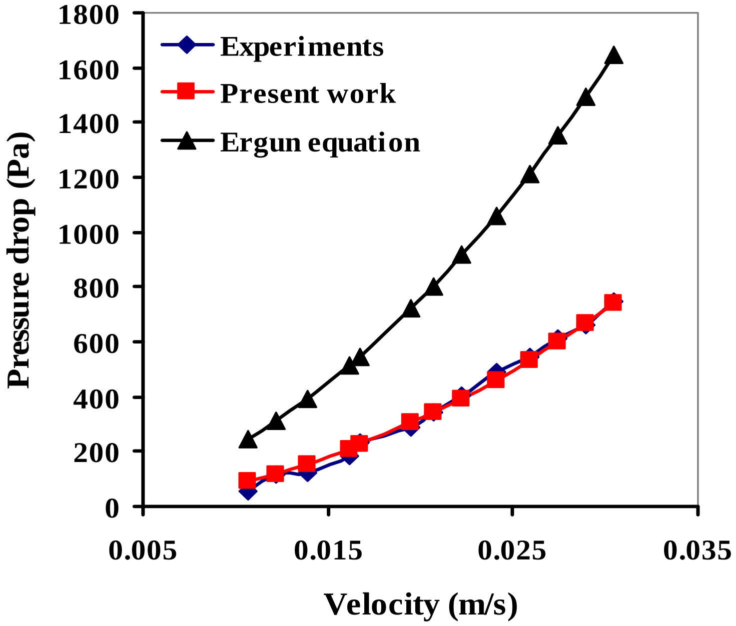


Figure 4.10 Pressure drop versus velocity for pea gravel spherical particles diameter of 1.27 cm, pore diameter of 0.5189cm, bed porosity of 0.395, packing height of 52.07cm, bed diameter of 8.89cm, tortousity of 1.3024 and permeability of 3.82E-09 m^2 [112],(Table 4.19)

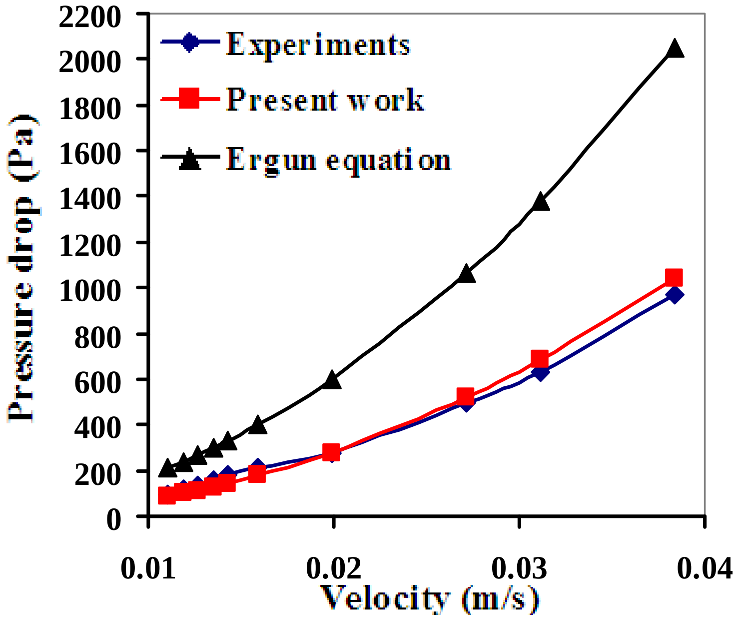


Figure 4.11 Pressure drop versus velocity for acrylic balls of particles diameter of 1.27 cm, pore diameter of 0.571 cm, bed porosity of 0.3969, packing height of

48.26 cm, bed diameter of 8.001 cm, tortousity of 1.3016 and permeability of 4.61E-09 m^2 [114] (Table 4.20)

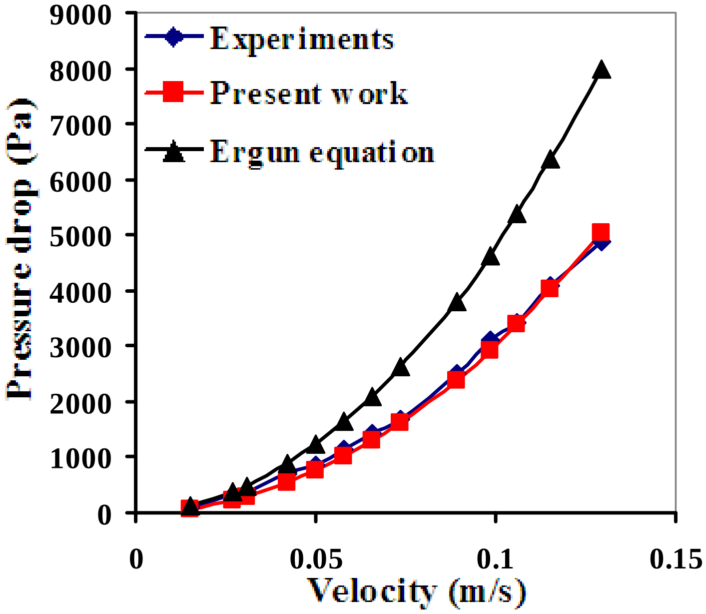


Figure 4.12 Pressure drop versus velocity for black marbles of particles diameter of 1.905 cm, pore diameter of 1.1262cm, bed porosity of 0.4022, packing height of 45.72cm, bed diameter of 8.89cm, tortousity of 1.2988 and permeability of 1.73E-08m^2 [1] (appendix A.7)

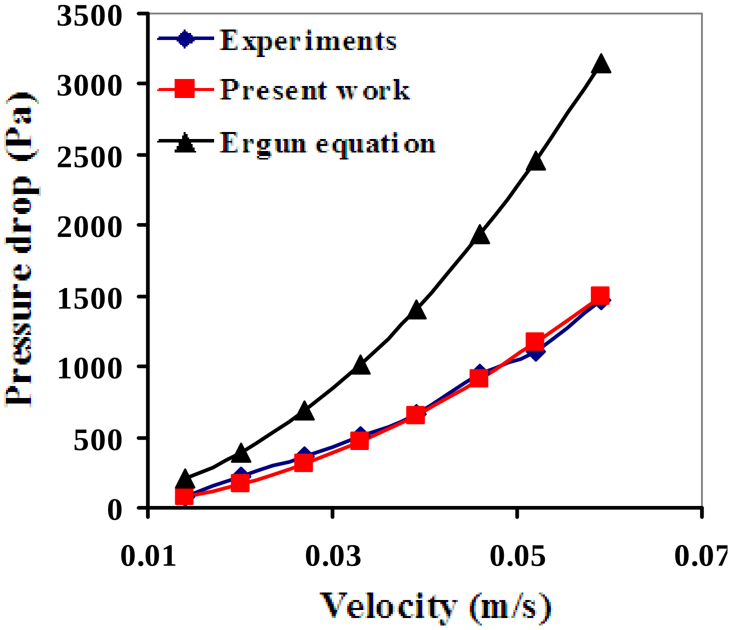


Figure 4.13 Pressure drop versus velocity for black marbles of particles diameter of 1.9 cm, pore diameter of 0.84 cm, bed porosity of 0.4, packing

height of 67.2 cm, bed diameter of 8.89 cm, tortousity of 1.29 and permeability of 9.92E-09m^2 [115],(appendix A.8)

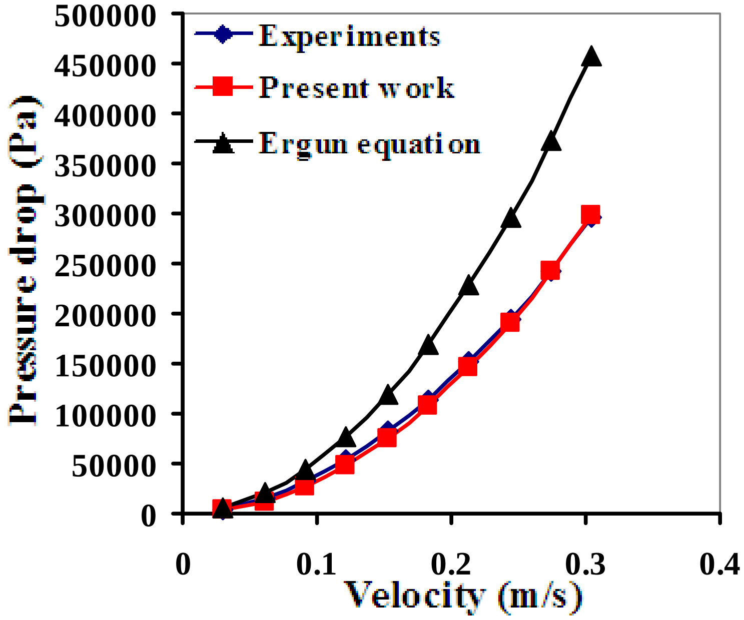


Figure 4.14 Pressure drop versus velocity for glass of particles diameter of 0.42 cm, pore diameter of 0.171 cm, bed porosity of 0.3837, packing height of 20 cm, bed diameter of 7.62 cm, tortousity of 1.308, permeability of 4.4E-10 m^2 [117],(appendix A.9)

From the above figures, it can be seen that the suggested equation gave a good fitting to the experiments results rather than Ergun equation; this was expected because of using different properties for the packing materials, which lead to a large effect on Ergun equation's prediction of pressure drop.

The wall affect on bed porosity increases the porosity, this appears clear in figures 4.10 where the bed porosity increased to a value of 0.395, for a bed diameter of 8.89 cm, and a particle diameter of 1.27cm. The effect of the wall on porosity may be due to the reduction in the ratio of bed diameter to particle diameter than the supposed ratio ($(D_R/d_P) \geq 10$) [38].

For large particles in small column (Fig.4.11), the wall presents an artificial boundary that alters the void fraction, which appears to be smaller than its true value, and the data appears lower than its true value [113].

The wall effect correction factor equation (3.50a) can be found after fitting using experimental data obtained from literatures. The wall effect correction factor for the velocity can be represented in the following equation:

$$f_w = 1 + 0.2495 \frac{S_C}{S} \qquad \text{... (4.13)}$$

The results of the wall correction factor equation (equation 4.13) are shown in tables A.12 (appendix A). The table also show experimental values of particle diameter, bed diameter and bed length. From these tables it could be seen that the values of the correction factor were small. This is may be due to the fact that the ratio of bed diameter to particle diameter of most of the experiments is less than ten.

4.3.3.2 Binary Sized Spherical Particles System

The pressure drop equation (3.45a) was fitted for water flow through packed beds of binary-sized spherical particles. In this fitting 24 sets of data from literatures [114,116,117] were used. In these sets 280 values of pressure drop versus velocity were taken. Equation (3.45a) because:

$$\frac{\Delta \mathrm{P}}{L} = -0.5194 \frac{\mu u}{k^{-0.7012}} + 0.0116 \frac{\rho u^2}{k^{0.5627}} \qquad \text{… (4.14)}$$

The permeability used in equation 4.14 was taken from equation 3.46, which include the pore size and porosity modified equations. The permeability equation (3.46) was fitting for this case using experimental data from literatures, and can be written as follows:

$$k = 0.0005 \frac{d_{pore}^{2.6349}\ \varepsilon^{-1.1061}}{\tau^{0.0869} (1-\varepsilon)^{1.529}} \qquad \text{… (4.15)}$$

The tortousity used in equation 4.15 was proposed from best fitting of experimental data. This tortousity can be represented in the following equation:

$$\tau = 1.4486 - 0.4336\ \varepsilon^{1.1777} \qquad \text{... (4.16)}$$

The suggested permeability equation results used inside equation 4.14 has been compared with experimental data results [114,116,117] and Carman equation (2.16) as shown in tables A.13. It can be seen that the permeability proposed equation results gave a good fit to the experiment rather than Carman -Kozeny equation; the reason of this difference may lie on the basis of Carman -Kozeny equation itself [75].

Comparisons between the pressure drop proposed equation results using equation 4.14, experimental data results and Ergun equation have been made and shown in table 4.23 and in figures 4.15 to 4.19.

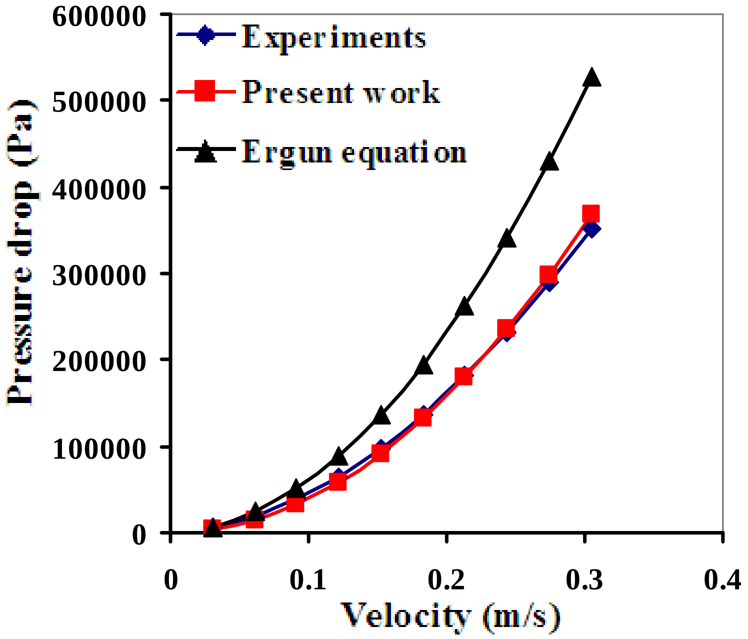


Figure 4.15 Pressure drop versus velocity for Glass spheres of particles diameter (dp_1=0.42 and dp_2=0.51with dp_{eff}=0.46cm),fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.1689cm, bed porosity of 0.3809, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.3094, permeability of 1.46E-10 m^2 [117] (Table 4.23)

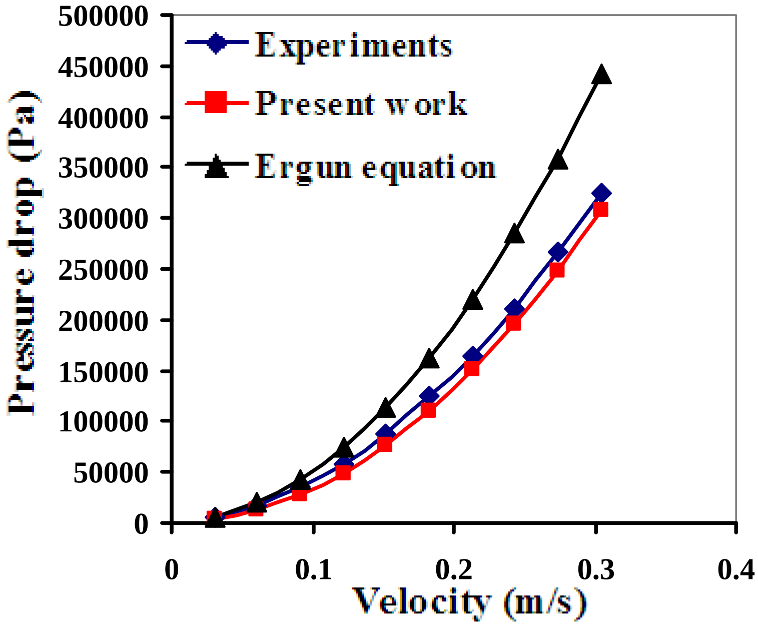


Figure 4.16 Pressure drop versus velocity for Glass spheres of particles diameter (dp_1=0.42 and dp_2=0.61with dp_{eff}=0.4975cm),fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.1906cm, bed porosity of 0.3807, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.3096, permeability of 2.02E-10 m^2 [117] (Appendix A.14)

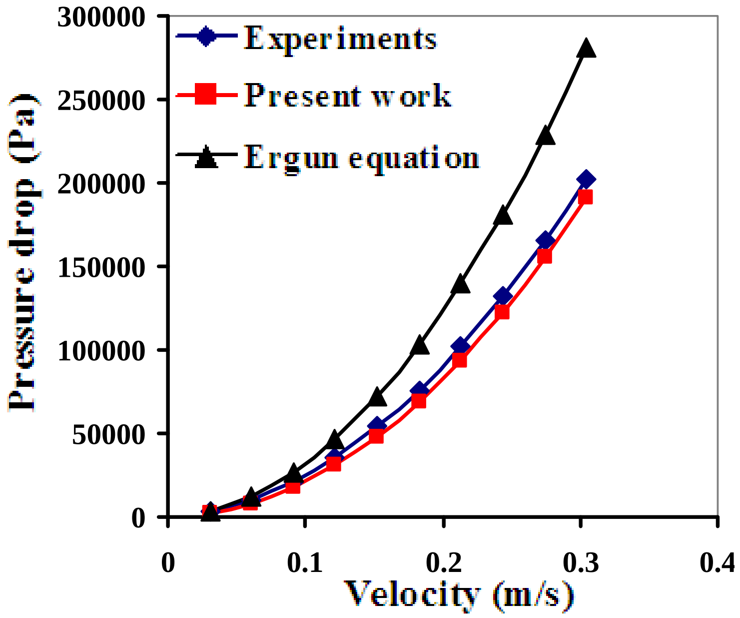


Figure 4.17 Pressure drop versus velocity for glass of particles diameter (dp_1=0.51,dp_2=0.79 with dp_{eff}=0.6198cm), fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.26cm, bed porosity of 0.379, packing height of 20cm, bed diameter of 7.62cm,tortousity of 1.3098, permeability of 4.67E-10 m^2[117],(Appendix A.15)

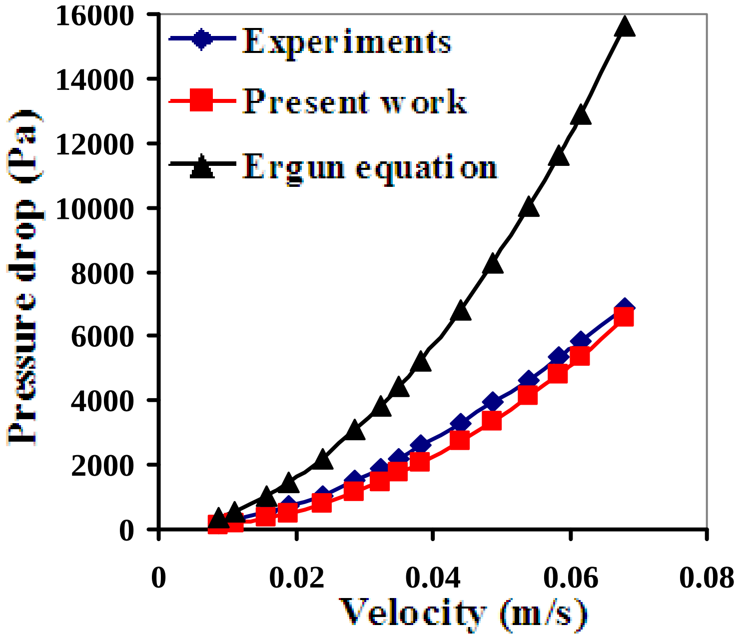


Figure 4.18 Pressure drop versus velocity for Acrylic balls of particles diameter (dp_1=0.655, dp_2=1.27 with dp_{eff}=1.016 cm), fractions of (x_1=0.25,x_2=0.75), pore diameter of 0.338 cm bed porosity of 0.3778, packing height of 49.53 cm, bed diameter of 8 cm, tortousity of 1.31, permeability of 9.1E-10m^2 [114], (AppendixA.16)

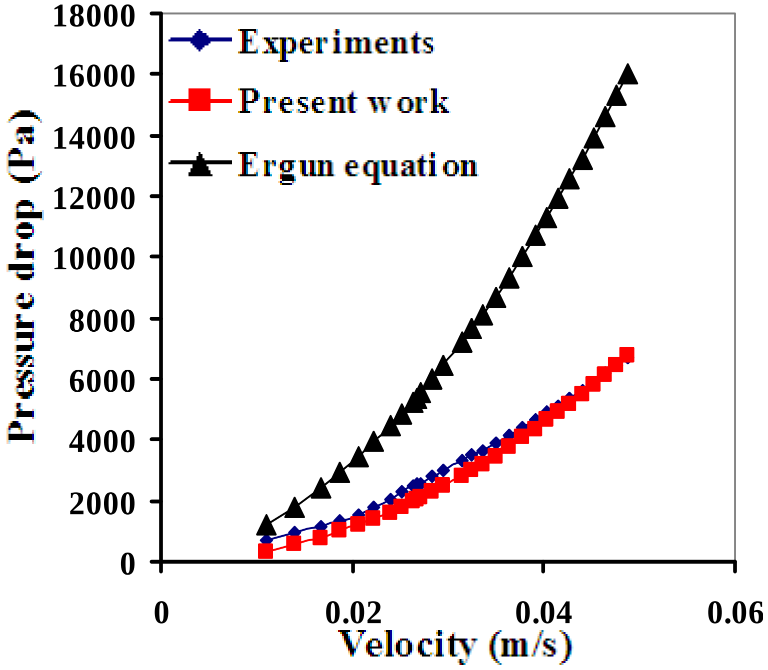


Figure 4.19 Pressure drop versus velocity for Acrylic balls of particles diameter (dp_1=0.655, dp_2=1.27 with dp_{eff}=0.7056 cm), fractions of (x_1=0.8,x_2=0.2), pore diameter of 0.21cm, bed porosity of 0.379, packing

height of 48.26 cm, bed diameter of 8 cm, tortousity of 1.31 and permeability of 2.65E-10 m^2 [3], (Appendix A.17)

Figures 4.15 - 4.19 show that the suggested equation results of pressure drop-velocity curves are very close to the experimental results curves, while the results obtained from Ergun equation lie above them. This is may be due to the differences in beds dimensions, packing shapes and sizes used by Ergun [4,8].

In binary system, mixing of two sizes of particles is the decrease in porosity with respect to mono sized particles. This is because in binary system the particles with smaller sizes tend to fill the voids between the larger sizes particles [116].

As the velocity of fluid increases the pressure drop across the bed increases.

The wall effect correction factor semi-empirical equation (3.50a) that can be used for this case can be represented as follows:

$$f_w = 1.00025 - 3.572\frac{S_C}{S} \quad \text{... (4.17)}$$

The results of the wall correction factor equation (equation 4.17) are shown in tables 4.24 and A.19 (appendix A), the tables also show the experimental values of particle diameter, bed diameter and bed length.

4.3.3.3 Ternary Sized Spherical Particles System

In this system of packing the mixture contains three sizes of glass spherical particles. The percentage of each size is equal 1/3 of the total packing.

The pressure drop equation (3.45a) was fitted for water flow through packed beds of ternary-sizes spherical particles. In this fitting 20 sets of data from literatures [116, 117] were used. In these sets 190 values of pressure drop versus velocity were taken. Equation (3.45a) for this case can be written as follows:

$$\frac{\Delta \mathrm{P}}{L} = -0.8265 \frac{\mu u}{k^{-3.8787}} + 0.0025 \frac{\rho u^2}{k^{0.6296}} \qquad \ldots (4.18)$$

The permeability used in equation 4.18 was taken from equation 3.46 which include the pore size and porosity modified equations. The permeability equation (3.46) was fitting for this case using experimental data from literatures [116, 117], and can be written as follows:

$$k = 0.0005 \frac{d_{pore}^{2.3369} \; \varepsilon^{0.9209}}{\tau^{-0.2841} (1-\varepsilon)^{1.3469}} \qquad \ldots (4.19)$$

The tortousity used in equation 4.19 was suggested from best fitting of experimental data. This tortousity can be represented in the following equation:

$$\tau = 1.3906 - 0.3107 \; \varepsilon^{1.3818} \qquad \ldots (4.20)$$

In tables A.20 comparisons between the permeability equation, Carman equation (2.16) and experimental data results [116,117] have been shown. From these comparisons, it can be seen that the permeability equation results gave a good fitting to the experimental data rather than Carman equation.

Comparisons between the pressure drop proposed equation results (equation 4.18), experimental data results and Ergun equation, are shown in figures 4.20 to 4.24.

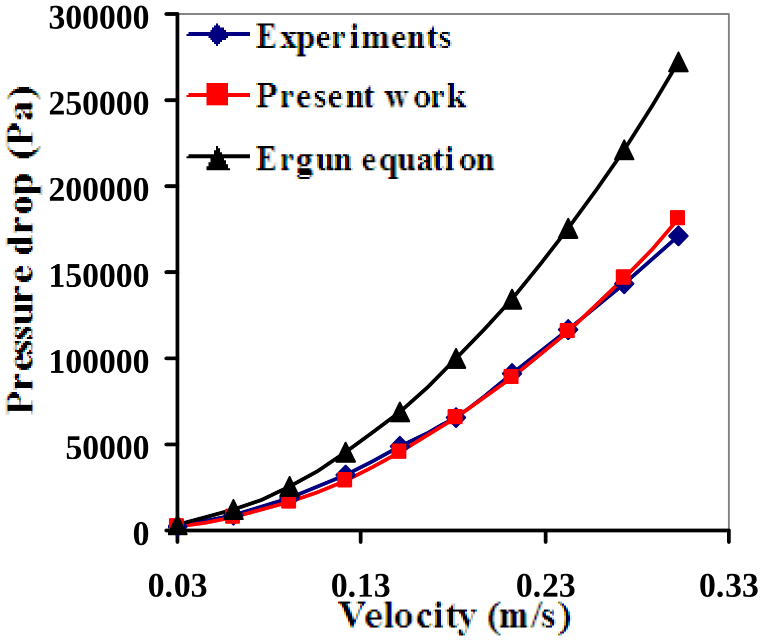


Figure 4.20 Pressure drop versus velocity for particles diameter (0.51, 0.61, 0.79 and dp_{eff}=0.617cm), pore diameter of 0.258 cm, bed porosity of 0.389, packing height of 15.15 cm, bed diameter of 7.64 cm, tortousity of 1.306 and permeability of 4.42E-10 m^2 [116], (Table 4.26)

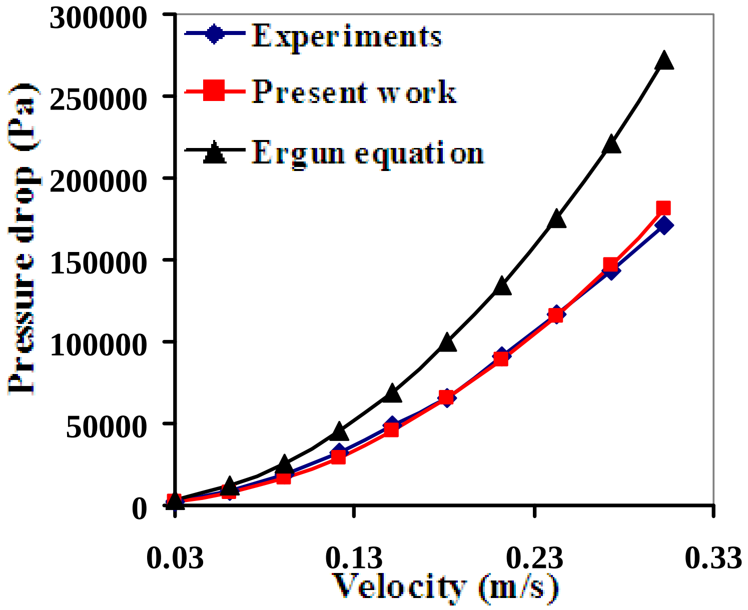


Figure 4.21 Pressure drop versus velocity for particles diameter (0.42, 0.51, 0.61 and dp_{eff}=0.502 cm), pore diameter of 0.1881 cm, bed porosity of 0.372, packing height of 20 cm, bed diameter of 7.62 cm, tortousity of 1.31 and permeability of 1.73E-10 m^2 [117], (Appendix A.21)

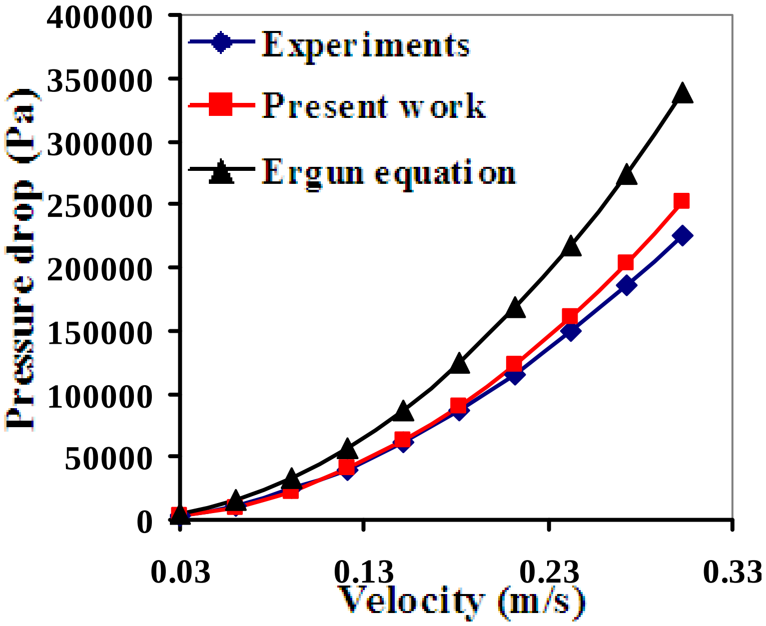


Figure 4.22 Pressure drop versus velocity for particles diameter (0.421, 0.5099, 0.7955 and dp_{eff}=0.536 cm), pore diameter of 0.22 cm, bed porosity of 0.376, packing height of 15.15 cm, bed diameter of 7.64 cm, tortousity of 1.31 and permeability of 2.6E-10 m^2 [116] (Appendix A.22)

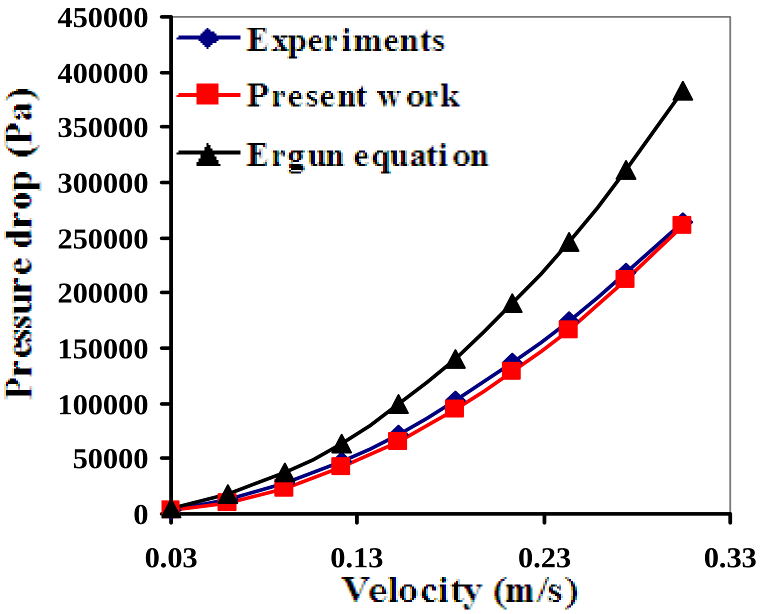


Figure 4.23 Pressure drop versus velocity for particles diameter (0.42, 0.61, 1.01 and dp_{eff}=0.5627 cm), pore diameter of 0.217 cm, bed porosity of 0.379, packing height of 20 cm, bed diameter of 7.62 cm, tortousity of 1.309 and permeability of 2.5E-10 m^2 [117], (Appendix A.23)

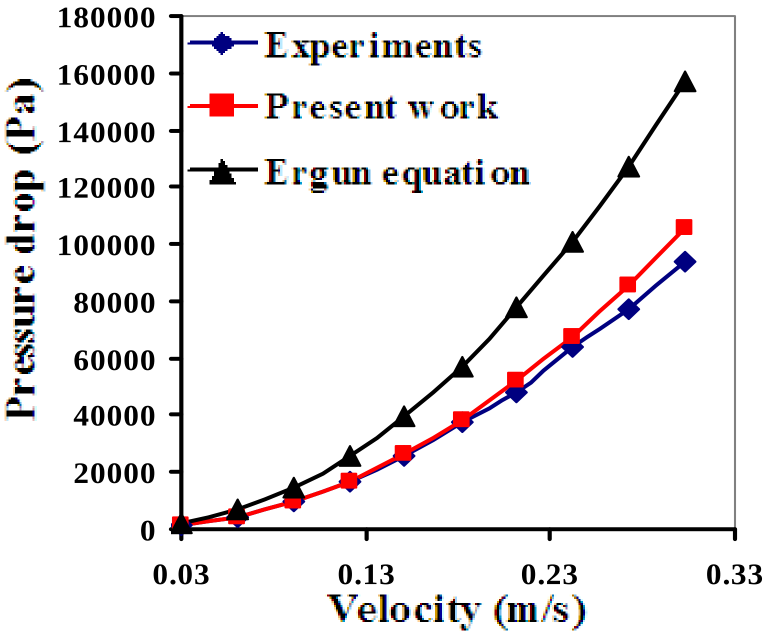


Figure 4.24 Pressure drop versus velocity for particles diameter (0.6015, 0.7955, 0.998 and dp_{eff}=0.7651cm), pore diameter of 0.381cm, bed porosity of 0.4024, packing height of 15.15 cm, bed diameter of 7.64 cm, tortousity of 1.302 and permeability of 1.04E-09m^2 [116], (Appendix A.24)

It can be noticed from figures 4.20 to 4.24 that the suggested equation results gave a good fitting to the experiment rather than Ergun equation. This is due to the fact that in Ergun equation non-spherical, rough and small packing was used [2].

The above figures show that when the tortuosity increases , the bed porosity decreases this leads to an increase in the pressure drop. This is because when the tortuosity of the porous media increased the void fraction decreased, which leads to an increase in pressure drop for the same bed diameter and packing height [69].

The wall effect correction factor semi-empirical equation (3.50a) for this case can be represented in the following equation:

$$f_w = 1 + 0.2495 \frac{S_C}{S} \qquad \text{... (4.21)}$$

The results of the wall correction factor equation (equation 4.21) are shown in and A.25. The table also show the experimental values of particle diameter, bed diameter and bed length. From thes tables it can be seen that the value of the wall correction factor is one this is due to the small ratio of particle to bed diameters used in this experiments.

4.3.3.4 General Equation Results

The pressure drop equation (3.45a) was fitted for water flow through packed beds. In this fitting 74 sets of data from literatures [1,3,112,113,114, 115,116,117] were used. In these sets 788 values of pressure drop versus velocity were taken. The general form of equation (3.45a) will be as follows:

$$\frac{\Delta \mathrm{P}}{L} = -13.0582 \frac{\mu u}{k^{-29.2316}} + 0.1493 \frac{\rho u^2}{k^{0.4243}} \quad \ldots (4.22)$$

The permeability used in equation 4.22 was taken from equation 3.46 which include the pore size and porosity modified equations. The permeability equation (3.46) was fitted for this case using experimental data from literatures [1,3,112,113,114,115,116,117], and can be written as follows:

$$k = 0.3109 \frac{d_{pore}^{3.2354} \ \varepsilon^{0.6162}}{\tau^{0.7682} (1-\varepsilon)^{1.6322}} \quad \ldots (4.23)$$

The tortousity used in equation 4.23 was suggested from best fitting of experimental data. This tortousity can be represented in the following equation:

$$\tau = 0.5787 + 0.5264 \ \varepsilon^{-0.3384} \quad \ldots (4.24)$$

Equation 4.22, 4.23 and 4.24 shown above can be used for all types of packing systems.

The permeability suggested equation results are shown in tables A.26. These tables show comparisons between the permeability equation results, experimental data results and theoretical results. The tables also show that, the equation results were closer to the experimental results rather than Carmen -Kozeny equation results.

The values of pressure drop versus velocity for water flow through packed beds were plotted in Figs. 4.25 and 4.26 for mono spherical particles, Fig. 4.27 and 4.28 for binary spherical particles and Figs. 4.29 and 4.30 for ternary spherical particles. The figures also show the comparisons between the pressure drop of the general equation results and Ergun equation results.

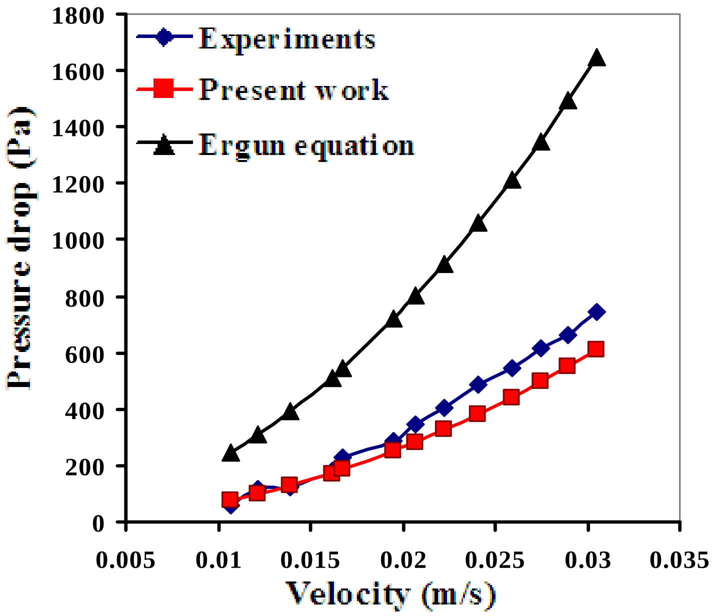


Figure 4.25 Pressure drop versus velocity for pea gravel spherical particles diameter of 1.27 cm, pore diameter of 0.5189 cm, bed porosity of 0.3902, packing height of 52.07 cm, bed diameter of 8.89 cm, tortousity of 1.3026, permeability of 3.82E-09 m^2 [112] (Appendix A.27)

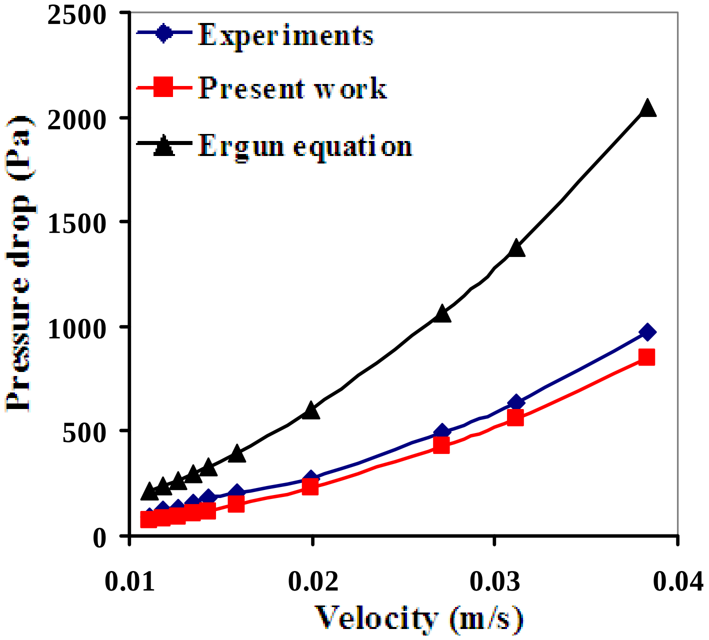


Figure 4.26 Pressure drop versus velocity for acrylic balls spherical particles diameter of 1.27 cm, pore diameter of 0.571cm, bed porosity of 0.3915, packing

height of 48.26cm, bed diameter of 8.001cm, tortousity of 1.3017, permeability of 3.49E-09 m^2[114] (Appendix A.28)

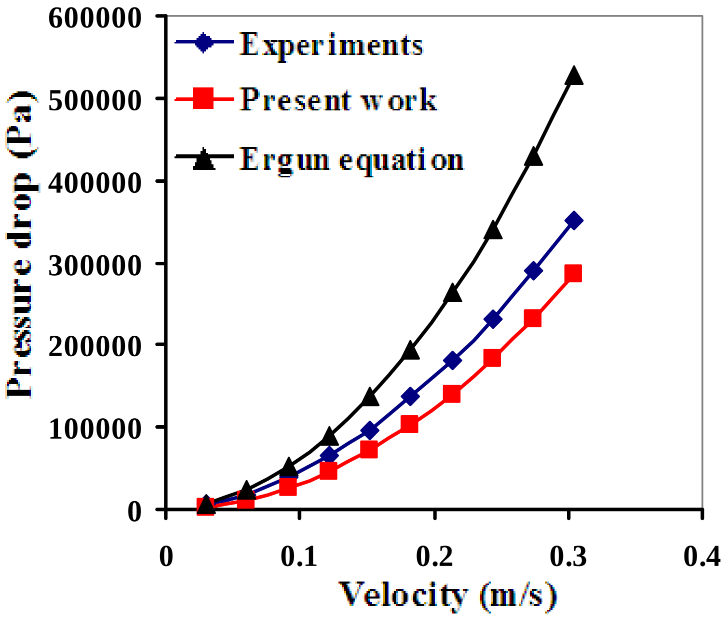


Figure 4.27 Pressure drop versus velocity for glass spheres of particles diameter (dp_1=0.42 and dp_2=0.51 with dp_{eff}=0.46cm),fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.1689 cm, bed porosity of 0.382, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.3075,permeability of 6.83E-11 m^2 [117],(Appendix A.33)

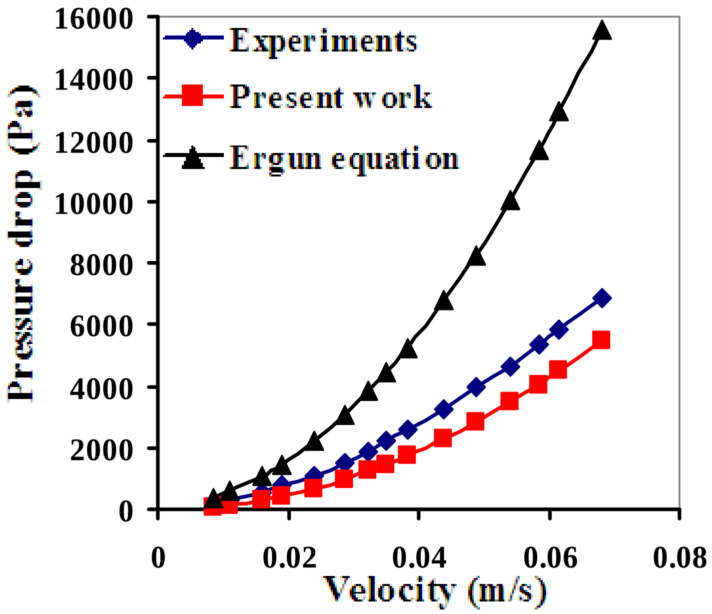


Figure 4.28 Pressure drop versus velocity for Acrylic balls of diameters(dp_1=0.655, dp_2=1.27 with dp_{eff}=1.016cm), fractions of (x_1=0.25,x_2=0.75), pore diameter of

0.338cm, bed porosity of 0.388, packing height of 49.53 cm, bed diameter of 8cm, tortousity of 1.303, permeability of 6.4E-10 m^2[114], (Appendix A.34)

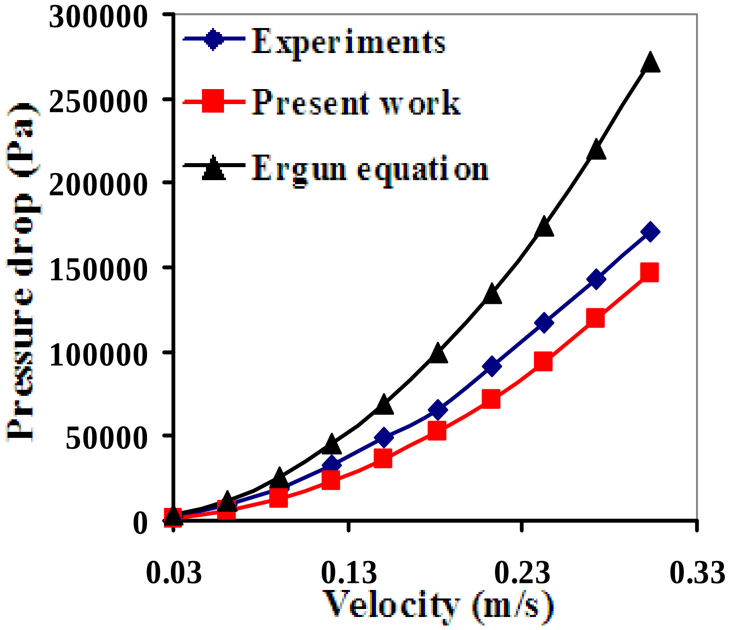


Figure 4.29 Pressure drop versus velocity for particles diameter (0.51, 0.61, 0.79 and dp_{eff}=0.617 cm), pore diameter of 0.258 cm, bed porosity of 0.385, packing height of 15.15 cm, bed diameter of 7.64cm, tortousity of 1.305 and permeability of 1.36E-10 m^2 [116], (Appendix A.39)

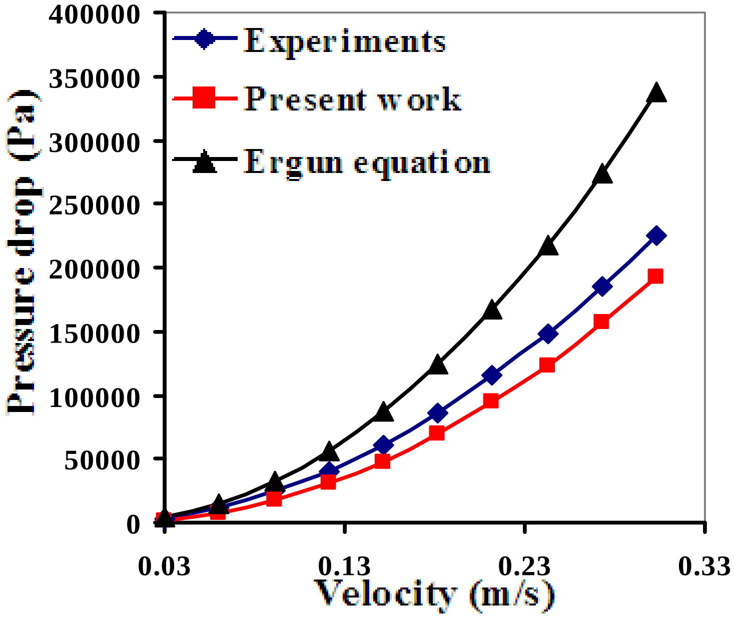


Figure 4.30 Pressure drop versus velocity for particles diameter (0.421, 0.5099,

0.7955and dp_{eff}=0.536 cm), pore diameter of 0.22cm, bed porosity of 0.383, packing height of 15.15cm, bed diameter of 7.64cm, tortousity of 1.307 and permeability of 1.7E-10 m^2 [116], (Appendix A.40)

Similar comparisons between the pressure drop of the general equation results, experimental data and Ergun equation can be seen in appendix A (tables A.29 to A.32 for mono size spherical particles, tables A.35 to A.38 for binary sized spherical particles and in tables A.41 to A.43 for ternary sized spherical particles).

It can be noticed from figures 4.25 to 4.30 that the suggested equation results gave a good fitting to the experimental data results better than Ergun equation results. The suggested equation results of pressure drop-velocity curves were closer to experimental results curves, while the results from Ergun equation were far from the experimental results; this may be due to:

1. The difference of bed dimensions (diameter and height of bed) [121].
2. The difference of void fraction (difference of packing shape and size) [17].
3. Ergun designed his equation using completely different procedures than experimental data work, so it was clear that the failure was confirmed. Ergun used pea gravel for the packed bed and air for the fluid; unlike experiments were pea gravel was used for the packed bed and water for the fluid [120].
4. Other reasons of this large deviation from Ergun equation, that Ergun's equation does not take in to consideration wall effects, because Ergun considered that,it is generally assumed that the diameter of the packing is close to that of the column; therefore, there is no wall effects [115].

The general equation could be used for any packing system, while the equations written for a certain type can be used for only one type of packing (which is written for it), and can not be used for another type. Figures 4.25 to

4.30 show the results of the general equation for multi sized particles (equation 4.22), which can be compared with the results of singular equations for different types of packing systems which have been shown before in figures 4.10, 4.11, 4.15, 4.18, 4.20 and 4.22 respectively. The comparisons between general and singular equations results show quite good agreement.

Figure 4.27 and 4.28 show that when the pore diameter decreases , the pressure drop increase (fig.4.27), this is because when the porosity decreases the pressure drop increases for the same bed diameter and packing height .

The general form of the wall effect correction factor semi-empirical equation (3.50a) can be represented in the following equation:

$$f_w = 1 - 0.2861 \frac{S_C}{S} \qquad \text{... (4.25)}$$

The results of the general equation of the wall correction factor that can be used for any type of packing systems are shown in tables A.44 (appendix A), the table also shows the experimental values of particle diameter, bed diameter and bed length.

Chapter Five

Conclusions and Recommendations for Future Work

5.1 Conclusions

1. The pore size and porosity modified equations had successfully described the effects of particles diameter, bed diameter and permeability on the fluid flow through packed beds, compared with the literature results.
2. The porosity modified equations results from literature with a very small average percentage error, while Furnas equation of porosity was far from the experimental data.
3. The porosity has a great effect on the properties of packed beds. It highly affects the pressure drop of the packed and inversely proportional to it.
4. The particle size and size distribution highly affect the bed porosity. The porosity value of the multi- size systems are generally less than those of mono size systems, because the particles of smaller sizes tend to fill the void spaces between the larger sizes particles.
5. Comparing the results of the suggested equations of pressure drop-velocity curves with those of literature data from literature and Ergun equation results; it indicates that the suggested equations results coincide with literature results, while the results from Ergun equation was far away from them.
6. The suggested general equations for the pore size, porosity, pressure drop, permeability and tortousity can be used for any system of packing, while

the equation that written for a certain type of packing can be used for only one type of packing.

7. It was found that the wall effect correction factor were small, this is may be due to the fact that the ratio of bed diameter to particle diameter of most of the used experimental data was less than ten.

5.2 Recommendations for Future Work

The following suggestions could be considered for future work:

The suggested equations can be extended to:

1. Include fluidization conditions.
2. Used for non spherical particles systems.
3. Used for air flow through packed bed of different sizes of packing systems.
4. Studying two phase flow through packed bed .

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Appendix A

Water Flow through Packed Bed

A.1 Pore size equation

Table A.1.1 The pore size results for water flow through packed bed for mono sphere particles

Type of packing	ε	k (m^2)	d_p (m) (Experiment)	d_{pore} (m) (Experiment)	d_{pore} (m) (Present work)
Pea gravel[112]	0.38	4E-06	0.0127	0.00519	0.00559
Marbles[113]	0.3	2E-06	0.0127	0.00363	0.00384
Marbles[113]	0.35	4E-06	0.0127	0.00456	0.00489
Marbles[113]	0.4	5E-06	0.0127	0.00564	0.00608
Marbles[3]	0.4	5E-06	0.0127	0.00564	0.00608
Acrylic ball [114]	0.41	5E-06	0.0127	0.00577	0.00621
Glass marbles[114]	0.41	5E-06	0.0127	0.00579	0.00623
Glass marbles[114]	0.38	4E-06	0.0127	0.0051	0.00549
Black marbles[115]	0.41	6E-06	0.019	0.0088	0.0068
Celite spheres[105]	0.37	3E-06	0.0064	0.00251	0.00474
Celite spheres[105]	0.38	3E-06	0.0064	0.00262	0.00495

Table A.1.2 The pore size results for water flow through packed bed for binary sphere particles

Type of packing	ε	k (m^2)	dp_{eff} (m) (Experiment)	d_{pore} (m) (Experiment)	d_{pore} (m) (Present work)
Acrylic ball [114]	0.329	1.26E-09	0.0115	0.00377	0.00345
Acrylic ball [114]	0.31	2.8E-10	0.0071	0.00211	0.00218
Acrylic ball [114]	0.33	3.76E-10	0.0073	0.00238	0.00243
Acrylic ball [114]	0.33	3.76E-10	0.0073	0.00238	0.00243
Glass [117]	0.348	2.7E-10	0.0059	0.00211	0.00225
Glass [117]	0.355	1.48E-10	0.0046	0.00169	0.0019
Glass [117]	0.359	2.46E-10	0.0055	0.00205	0.00221
Glass [117]	0.365	2.03E-10	0.005	0.00191	0.0021

Glass [117]	0.367	1.01E-09	0.0091	0.00351	0.00337
Glass [117]	0.373	4.99E-10	0.0068	0.00269	0.00276
Glass [116]	0.382	3.86E-10	0.0059	0.00244	0.00258

Table A.1.3 The pore size results for water flow through packed bed for ternary sphere particles

Type of packing	ε	k (m^2)	dp_{eff} (m) (Experiment)	d_{pore} (m) (Experiment)	d_{pore} (m) (Present work)
Glass [117]	0.388	4.64E-10	0.0062	0.00262	0.00274
Glass [117]	0.39	8.16E-10	0.0076	0.00325	0.00324
Glass [116]	0.393	2.25E-10	0.0046	0.00199	0.00223
Glass [116]	0.397	3.72E-10	0.0055	0.00241	0.00259
Glass [116]	0.409	3.21E-09	0.0119	0.00547	0.00492
Glass [117]	0.409	7.66E-10	0.0069	0.00318	0.00324
Glass [116]	0.412	4.42E-10	0.0055	0.00258	0.00276
Glass [116]	0.417	6.34E-10	0.0062	0.00296	0.00308
Glass [116]	0.417	7.92E-10	0.0067	0.00322	0.00329
Glass [116]	0.417	8.28E-10	0.0069	0.00327	0.00333
Glass [116]	0.419	1.07E-09	0.0075	0.0036	0.0036
Glass [117]	0.422	1.74E-09	0.0089	0.00434	0.00416
Glass [116]	0.423	1.74E-09	0.0089	0.00433	0.00416
Glass [116]	0.4276	1.16E-09	0.0077	0.0038	0.0037
Glass [116]	0.4019	6.78E-10	0.0071	0.0032	0.0031
Glass [116]	0.3861	4.36E-10	0.0065	0.0027	0.0027
Glass [116]	0.3998	5.32E-10	0.0065	0.0029	0.0029
Glass [116]	0.3906	3.83E-10	0.0060	0.0025	0.0026
Glass [116]	0.3838	3.03E-10	0.0056	0.0023	0.0024
Glass [116]	0.3985	4.6E-10	0.0061	0.0027	0.0028
Glass [116]	0.3984	3.81E-10	0.0057	0.0025	0.0026
Glass [116]	0.3837	2.71E-10	0.0054	0.0022	0.0023
Glass [116]	0.3766	2.08E-10	0.0050	0.0020	0.0021
Glass [117]	0.36	1.65E-10	0.005	0.0019	0.0020
Glass [117]	0.3665	2.37E-10	0.0056	0.0022	0.0022
Glass [117]	0.3965	7.59E-10	0.0077	0.0034	0.0032
Glass [117]	0.3648	2.05E-10	0.0054	0.0020	0.0021
Glass [117]	0.3562	2.08E-10	0.0057	0.0021	0.0021

Glass [117]	0.3712	2.93E-10	0.006	0.0024	0.0023
Glass [117]	0.3854	4.31E-10	0.0065	0.0027	0.0027
Glass [117]	0.3771	3.42E-10	0.0062	0.0025	0.0025
Glass [117]	0.3895	4.68E-10	0.0065	0.0028	0.0027

Table A.1.4 The pore size results for water flow through packed bed for general equation

Type of packing	ε	k (m^2)	d_p (m) (Experiment)	d_{pore} (m) (Experiment)	d_{pore} (m) (Present work)
Pea gravel [112]	0.388	2.83E-11	0.00305	0.00129	0.00137
Pea gravel [112]	0.3	8.36E-10	0.0127	0.00363	0.00351
Marbles [113]	0.35	1.73E-09	0.0127	0.00456	0.00449
Marbles [113]	0.4	3.34E-09	0.0127	0.00564	0.0056
Acrylic ball[114]	0.357	2.02E-10	0.00635	0.00235	0.0024
Celite spheres [105]	0.38	2.81E-10	0.0064	0.00262	0.00268
Celite spheres [105]	0.46	7.51E-10	0.0064	0.00363	0.00372
Celite spheres [105]	0.47	8.43E-10	0.0064	0.00378	0.00387
Acrylic ball [114]	0.33	2.14E-10	0.00726	0.00238	0.0024
Acrylic ball [114]	0.33	2.14E-10	0.00726	0.00238	0.0024
Glass [116]	0.348	1.44E-10	0.00593	0.00211	0.00216
Glass [116]	0.355	6.94E-11	0.0046	0.00169	0.00175
Glass [116]	0.359	1.29E-10	0.00548	0.00205	0.00211
Glass [116]	0.365	1.02E-10	0.00498	0.00191	0.00197
Glass [116]	0.367	7.33E-10	0.00907	0.00351	0.00353
Glass [116]	0.373	3.09E-10	0.00678	0.00269	0.00274
Glass [116]	0.382	2.25E-10	0.00592	0.00244	0.00251
Glass [116]	0.386	1.93E-10	0.00556	0.00233	0.0024
Glass [116]	0.388	2.82E-10	0.0062	0.00262	0.00269
Glass [117]	0.409	5.15E-10	0.00688	0.00318	0.00325
Glass [117]	0.412	2.61E-10	0.00551	0.00258	0.00267
Glass [117]	0.417	4.05E-10	0.00621	0.00296	0.00304
Glass [117]	0.417	5.32E-10	0.00674	0.00322	0.0033
Glass [117]	0.417	5.62E-10	0.00685	0.00327	0.00335
Glass [117]	0.419	7.68E-10	0.00751	0.0036	0.00367
Glass [117]	0.422	1.39E-09	0.0089	0.00434	0.00438
Glass [117]	0.423	1.39E-09	0.00886	0.00433	0.00438

Glass [116]	0.428	9.11E-10	0.00765	0.00381	0.00388
Glass [116]	0.402	5.22E-10	0.0071	0.00318	0.00325
Glass [116]	0.386	3.16E-10	0.00647	0.00271	0.00278
Glass [116]	0.4	3.78E-10	0.00648	0.00288	0.00295
Glass [116]	0.391	2.55E-10	0.00595	0.00254	0.00262
Glass [116]	0.384	1.94E-10	0.00562	0.00233	0.00241
Glass [116]	0.399	3.12E-10	0.00614	0.00271	0.00279
Glass [117]	0.398	2.4E-10	0.00567	0.0025	0.00258
Glass [117]	0.384	1.66E-10	0.00536	0.00222	0.0023
Glass [117]	0.377	1.21E-10	0.005	0.00201	0.00209
Glass [117]	0.36	9.81E-11	0.00502	0.00188	0.00194
Glass [117]	0.367	1.55E-10	0.00563	0.00217	0.00223
Glass [117]	0.397	6.33E-10	0.0077	0.00337	0.00343
Glass [117]	0.365	1.29E-10	0.00535	0.00205	0.00211
Glass [116]	0.393	1.15E-10	0.00461	0.00199	0.00208
Glass [117]	0.397	2.14E-10	0.00551	0.00241	0.00249
Glass [117]	0.409	2.99E-09	0.01186	0.00547	0.00545
Glass [117]	0.356	1.39E-10	0.00568	0.00209	0.00215
Glass [117]	0.371	2.02E-10	0.00599	0.00236	0.00242
Glass [117]	0.385	3.13E-10	0.00647	0.0027	0.00277
Glass [116]	0.393	1.15E-10	0.00461	0.00199	0.00208
Glass [117]	0.397	2.14E-10	0.00551	0.00241	0.00249
Glass [117]	0.409	2.99E-09	0.01186	0.00547	0.00545
Glass [117]	0.356	1.39E-10	0.00568	0.00209	0.00215
Glass [117]	0.371	2.02E-10	0.00599	0.00236	0.00242
Glass [117]	0.385	3.13E-10	0.00647	0.0027	0.00277

Table A.1.5 The experiment used to evaluate the pore size equation constants

Type of packing	ε	d_k	d_m	d_g	dp_{eff} (m) (Experiment)	k (m^2)
Glass[116]	0.3984	0.00421	0.00602	0.00796	0.00567	3.8E-10
Glass[116]	0.3836	0.00421	0.00509	0.00796	0.00536	2.7E-10
Glass[116]	0.3984	0.00421	0.00602	0.00796	0.00567	3.8E-10
Glass[116]	0.3836	0.00421	0.00509	0.00796	0.00536	2.7E-10
Glass[117]	0.36	0.00421	0.0051	0.0061	0.00502	1.7E-10
Glass[117]	0.3648	0.0042	0.0051	0.0079	0.00535	2.1E-10
Glass[117]	0.3562	0.0042	0.0051	0.0101	0.00563	2E-10

Glass[117]	0.3695	0.0042	0.0061	0.0079	0.00568	2.5E-10
Glass[117]	0.3665	0.0042	0.0061	0.0101	0.00599	2.7E-10
Glass[117]	0.3712	0.0042	0.0079	0.0101	0.00647	3.5E-10
Glass[117]	0.3854	0.0051	0.0061	0.0079	0.00617	3.9E-10
Glass[117]	0.3771	0.0051	0.0061	0.0101	0.00654	3.9E-10
Glass[117]	0.3895	0.0051	0.0079	0.0101	0.00712	5.7E-10
Glass[117]	0.3962	0.0061	0.0079	0.0101	0.00771	7.6E-10

Table A.1.6 The pore size results for water flow through packed bed

Type of packing	ε	d_{pore} (m) (Experiment)	d_{pore} (m) Equation (4.4)	d_{pore} (m) Equation (4.3)
Glass[116]	0.3984	0.0025	0.00243	0.00261
Glass[116]	0.3836	0.00222	0.00233	0.00233
Glass[116]	0.3984	0.0025	0.00243	0.00261
Glass[116]	0.3836	0.00222	0.0023	0.00233
Glass[117]	0.36	0.00188	0.00253	0.00196
Glass[117]	0.3648	0.00205	0.0023	0.0021
Glass[117]	0.3562	0.00208	0.00212	0.00208
Glass[117]	0.3695	0.00222	0.00245	0.00224
Glass[117]	0.3665	0.00231	0.00225	0.00229
Glass[117]	0.3712	0.00255	0.00245	0.00248
Glass[117]	0.3854	0.00258	0.00298	0.00258
Glass[117]	0.3771	0.00264	0.00273	0.00257
Glass[117]	0.3895	0.00303	0.00298	0.00291
Glass[117]	0.3962	0.00337	0.00357	0.00318

Table A.2 The pore size results for water flow through packed bed

For general equation

Type of packing	ε	k (m^2)	d_p (m) (Experiment)	d_{pore} (m) (Experiment)	d_{pore} (m) (Present work)
Pea gravel [112]	0.38	2.58E-09	0.0127	0.00519	0.00514
Marbles [113]	0.35	1.73E-09	0.0127	0.00456	0.00449
Marbles [113]	0.386	1.63E-11	0.00259	0.00109	0.00116
Acrylic ball[114]	0.405	3.57E-09	0.0127	0.00577	0.00573
Glass marbles[113]	0.406	3.6E-09	0.0127	0.00579	0.00575

Glass [1]	0.376	2.45E-09	0.0127	0.0051	0.00505
Black marbles[115]	0.47	2.87E-08	0.01905	0.01126	0.01091
Black marbles[115]	0.41	1.39E-08	0.019	0.0088	0.00857
Celite spheres [105]	0.37	2.47E-10	0.0064	0.00251	0.00256
Acrylic ball [114]	0.329	9.4E-10	0.01155	0.00377	0.00371
Acrylic ball [114]	0.31	1.45E-10	0.00706	0.00211	0.00212
Glass [116]	0.39	5.63E-10	0.00761	0.00325	0.0033
Glass [117]	0.377	2.4E-10	0.00617	0.00249	0.00255

A.2 Porosity Equation

Table A.3 The porosity results for mono sphere particles

Type of packing	Dr (m)	d_p (m)	ε (Experiment)	ε (Present work)	ε Furnas
Pea gravel[112]	0.0889	0.0127	0.38	0.3953	0.4236
Pea gravel[112]	0.0889	0.0127	0.38	0.3953	0.4236
Pea gravel[112]	0.1524	0.0127	0.38	0.388	0.4033
Marbles[113]	0.0889	0.0127	0.38	0.3953	0.4236
Pea gravel[113]	0.0826	0.0127	0.3	0.3964	0.4273
Pea gravel[113]	0.0826	0.0127	0.35	0.3964	0.4273
Acrylic ball [114]	0.08	0.0064	0.3571	0.3875	0.402
Acrylic ball [114]	0.08	0.0127	0.4028	0.3969	0.429
Acrylic ball [114]	0.08	0.0127	0.4028	0.3969	0.429
Glass marbles[114]	0.08	0.0127	0.406	0.3969	0.429
Glass marbles[114]	0.08	0.0127	0.406	0.3969	0.429
Marbles[112]	0.0889	0.0127	0.38	0.3953	0.4236
Marbles[112]	0.1524	0.0127	0.38	0.388	0.4033
Marbles[114]	0.1524	0.0127	0.406	0.388	0.4033
Black marbles[115]	0.1461	0.019	0.41	0.3939	0.4192
Black marbles[115]	0.0889	0.019	0.4	0.4022	0.4477
Glass[117]	0.0762	0.0101	0.4321	0.3942	0.4201
Glass[117]	0.0762	0.0051	0.4051	0.3856	0.3978
Glass[117]	0.0762	0.0061	0.4156	0.3876	0.4022
Glass[117]	0.0762	0.0079	0.4265	0.3907	0.4102
Pea gravel[113]	0.0825	0.0026	0.386	0.379	0.3857
Acrylic ball [114]	0.08	0.0064	0.3571	0.3875	0.402

Marbles[113]	0.0826	0.0127	0.38	0.3964	0.4273
Marbles[113]	0.0826	0.0021	0.38	0.3777	0.3838
Pea gravel[113]	0.0825	0.003	0.388	0.3802	0.3876
Marbles[3]	0.1524	0.0127	0.4	0.388	0.4033
Acrylic ball [114]	0.08	0.0127	0.4054	0.3969	0.429
Glass marbles[114]	0.08	0.0127	0.406	0.3969	0.429
Glass[116]	0.0762	0.0042	0.3793	0.3837	0.3937
Pea gravel[112]	0.1524	0.0127	0.38	0.388	0.4033
Marbles[114]	0.1524	0.0889	0.4207	0.4269	0.5733
Marbles[114]	0.08	0.0127	0.406	0.3969	0.429
Marbles[113]	0.0826	0.0127	0.4	0.3964	0.4273
Marbles[3]	0.0889	0.0127	0.4	0.3953	0.4236
Marbles[112]	0.1524	0.0127	0.38	0.388	0.4033
Marbles[114]	0.08	0.0127	0.406	0.3969	0.429
Marbles[114]	0.1524	0.0127	0.376	0.388	0.4033
Black marbles[1]	0.0889	0.0191	0.47	0.4022	0.4479
Black marbles[115]	0.1461	0.019	0.41	0.3939	0.4192
Black marbles[115]	0.0889	0.019	0.4	0.4022	0.4477

Table A.4 The porosity results for binary sphere particles

Type of packing	Dr (m)	d_p (m)	ε (Experiment)	ε (Present work)	ε Furnas
Glass [117]	0.0762	0.00498	0.365	0.38127	0.3972
Glass [117]	0.0762	0.00548	0.359	0.38097	0.3995
Glass [117]	0.0762	0.00593	0.3484	0.38072	0.4015
Glass [117]	0.0762	0.00556	0.3861	0.38093	0.3998
Glass [117]	0.0762	0.0062	0.3882	0.38057	0.4027
Glass [117]	0.0762	0.00678	0.373	0.38025	0.4052
Glass [117]	0.0762	0.00761	0.3904	0.3798	0.4089
Glass [116]	0.0764	0.00886	0.3832	0.37915	0.4144
Glass [116]	0.0764	0.00751	0.3786	0.37987	0.4084
Glass [116]	0.0764	0.00674	0.3871	0.38028	0.405
Acrylic ball [114]	0.08001	0.00706	0.31	0.38028	0.405
Acrylic ball [114]	0.08001	0.00726	0.33	0.38018	0.4058
Glass [116]	0.0764	0.00592	0.3822	0.38074	0.4014

Glass [116]	0.0764	0.00685	0.3873	0.38022	0.4055
Acrylic ball [114]	0.08001	0.01155	0.3286	0.37803	0.4241
Acrylic ball [114]	0.08001	0.01016	0.333	0.37871	0.4182
Acrylic ball [114]	0.08001	0.00907	0.367	0.37925	0.4135
Acrylic ball [114]	0.08001	0.00726	0.33	0.38018	0.4058
Glass [116]	0.0764	0.00621	0.3966	0.38058	0.4026
Glass [116]	0.0764	0.00551	0.3865	0.38097	0.3995
Glass [116]	0.0764	0.00551	0.3921	0.38097	0.3995
Glass [116]	0.0764	0.01186	0.409	0.37761	0.4278
Glass [116]	0.0764	0.00461	0.3833	0.38149	0.3955
Glass [117]	0.0762	0.0046	0.3561	0.38148	0.3955
Glass [117]	0.0762	0.00688	0.3891	0.3802	0.4057
Glass [117]	0.0762	0.0089	0.3723	0.3791	0.4147

Table A.5 The porosity results for ternary sphere packing

Type of packing	Dr (m)	dp_{eff} (m)	ε (Experiment)	ε (Present work)	ε Furnas
Glass [116]	0.0764	0.00647	0.3861	0.3891	0.4038
Glass [116]	0.0764	0.00614	0.3985	0.3853	0.4023
Glass [116]	0.0764	0.00567	0.3984	0.3798	0.4002
Glass [116]	0.0764	0.00536	0.3837	0.3762	0.3989
Glass [116]	0.0764	0.005	0.3766	0.3719	0.3972
Glass [117]	0.0762	0.00502	0.36	0.3723	0.3974
Glass [117]	0.0762	0.00563	0.3665	0.3795	0.4001
Glass [117]	0.0762	0.0077	0.3965	0.4033	0.4094
Glass [116]	0.0764	0.00647	0.3861	0.3891	0.4038
Glass [116]	0.0764	0.00648	0.3998	0.3892	0.4038
Glass [116]	0.0764	0.00595	0.3906	0.3831	0.4015
Glass [116]	0.0764	0.00562	0.3838	0.3792	0.4
Glass [117]	0.0762	0.00535	0.3648	0.3762	0.3989
Glass [117]	0.0762	0.00568	0.3562	0.38	0.4003
Glass [117]	0.0762	0.00599	0.3712	0.3837	0.4017
Glass [117]	0.0762	0.00647	0.3854	0.3892	0.4039

Glass [117]	0.0762	0.00617	0.3771	0.3857	0.4025
Glass [117]	0.0762	0.00654	0.3895	0.39	0.4042

Table A.6 The porosity results for the general equation

Type of packing	Dr (m)	d_{peff} (m)	ε (Experiment)	ε (Present work)	ε Furnas
Acrylic ball[118]	0.08	0.0064	0.3571	0.3843	0.402
Acrylic ball[118]	0.08	0.0064	0.3571	0.3843	0.402
Acrylic ball[118]	0.08	0.0127	0.406	0.3915	0.429
Marbles [113]	0.0826	0.0127	0.4	0.3911	0.4273
Marbles [113]	0.0889	0.0127	0.4	0.3901	0.4236
Acrylic ball[118]	0.08	0.0127	0.406	0.3915	0.429
Marbles [113]	0.1524	0.0127	0.376	0.3847	0.4033
Marbles [113]	0.1524	0.0127	0.406	0.3847	0.4033
Black marbles [115]	0.0889	0.0191	0.407	0.3959	0.4479
Black marbles [115]	0.1461	0.019	0.4001	0.389	0.4192
Black marbles [115]	0.1461	0.019	0.4021	0.389	0.4192
Black marbles [115]	0.0889	0.019	0.4	0.3958	0.4477
Marbles [117]	0.0762	0.0101	0.4321	0.3892	0.4201
Marbles [117]	0.0762	0.0051	0.4051	0.383	0.3978
Marbles [117]	0.0762	0.0061	0.4056	0.3844	0.4022
Marbles [117]	0.0762	0.0079	0.3865	0.3866	0.4102
Marbles [117]	0.0762	0.0042	0.3793	0.3818	0.3937
Marbles [118]	0.1524	0.0889	0.4207	0.4199	0.5733
Glass [117]	0.0764	0.0075	0.4186	0.3861	0.4084
Glass [117]	0.0764	0.0067	0.4171	0.3852	0.405
Glass [117]	0.0764	0.0059	0.3822	0.3841	0.4014
Glass [117]	0.0764	0.0069	0.4173	0.3853	0.4055
Glass [117]	0.0764	0.0062	0.4166	0.3845	0.4026
Glass [117]	0.0764	0.0055	0.3965	0.3836	0.3995
Glass [117]	0.0764	0.0055	0.4121	0.3836	0.3995
Glass [117]	0.0764	0.0119	0.409	0.3912	0.4278

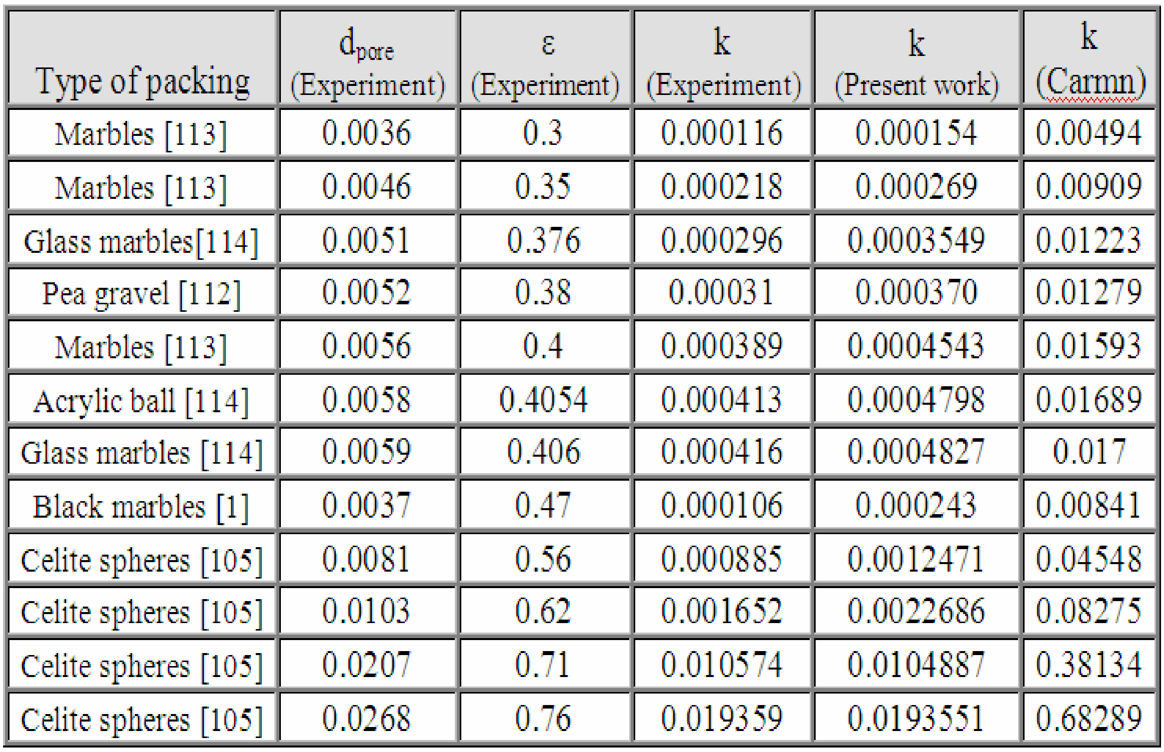
Glass [116]	0.0762	0.0068	0.373	0.3852	0.4052
Glass [116]	0.0762	0.0076	0.3904	0.3863	0.4089
Glass [116]	0.0764	0.0077	0.4276	0.3863	0.4091
Glass [116]	0.0764	0.0071	0.4019	0.3856	0.4066
Glass [116]	0.0764	0.0065	0.3861	0.3848	0.4038
Glass [116]	0.0764	0.0065	0.3998	0.3848	0.4038
Glass [116]	0.0764	0.006	0.3906	0.3842	0.4015
Glass [116]	0.0764	0.0056	0.3838	0.3837	0.4
Glass [116]	0.0764	0.0061	0.3985	0.3844	0.4023
Glass [116]	0.0764	0.0057	0.3984	0.3838	0.4002
Glass [116]	0.0764	0.005	0.3766	0.3829	0.3972
Glass [117]	0.0762	0.005	0.36	0.3829	0.3974
Glass [117]	0.0762	0.0056	0.3665	0.3837	0.4001
Glass [117]	0.0762	0.0077	0.3965	0.3864	0.4094
Glass [117]	0.0762	0.0077	0.3965	0.3864	0.4094
Glass [117]	0.0762	0.0054	0.3648	0.3834	0.3989
Glass [117]	0.0762	0.0057	0.3562	0.3838	0.4003
Glass [117]	0.0762	0.006	0.3712	0.3842	0.4017
Glass [117]	0.0762	0.0062	0.3771	0.3845	0.4025
Pea gravel[112]	0.0826	0.0127	0.35	0.3911	0.4273
Marbles [113]	0.0826	0.0127	0.38	0.3911	0.4273
Acrylic ball[118]	0.08	0.0073	0.383	0.3854	0.4058
Acrylic ball[118]	0.08	0.0071	0.3851	0.3852	0.405
Acrylic ball[118]	0.08	0.0073	0.33	0.3854	0.4058
Glass [116]	0.0762	0.0089	0.4223	0.3878	0.4147
Glass [116]	0.0762	0.005	0.365	0.3829	0.3972
Glass [116]	0.0762	0.0055	0.359	0.3836	0.3995
Glass [116]	0.0762	0.0059	0.3484	0.3842	0.4015
Glass [117]	0.0764	0.0046	0.3933	0.3823	0.3955
Glass [116]	0.0762	0.0046	0.3551	0.3823	0.3955
Glass [116]	0.0762	0.0069	0.4091	0.3854	0.4057
Glass [116]	0.0764	0.0054	0.3837	0.3834	0.3989
Pea gravel[112]	0.1524	0.0127	0.3837	0.3847	0.4033
Pea gravel[112]	0.0889	0.0127	0.38	0.3901	0.4236
Pea gravel[112]	0.1524	0.0127	0.38	0.3847	0.4033

Marbles [113]	0.1524	0.0127	0.38	0.3847	0.4033
Marbles [118]	0.1524	0.0889	0.41	0.4199	0.5733
Acrylic ball[118]	0.08	0.0115	0.3286	0.3902	0.4241
Acrylic ball[118]	0.08	0.0102	0.3883	0.3888	0.4182
Acrylic ball[118]	0.08	0.0091	0.367	0.3875	0.4135
Glass [117]	0.0762	0.0065	0.3895	0.3849	0.4042
Glass [117]	0.0762	0.0065	0.3854	0.3849	0.4039
Glass [116]	0.0762	0.0056	0.3861	0.3837	0.3998
Glass [116]	0.0762	0.0062	0.3882	0.3845	0.4027

A.3 Pressure Drop Equation

A.3.1 Mono Size Spherical Particle System

Table A.7 The permeability results for water flow through packed bed



Type of packing	d_{pore} (Experiment)	ε (Experiment)	k (Experiment)	k (Present work)	k (Carmn)
Marbles [113]	0.0036	0.3	0.000116	0.000154	0.00494
Marbles [113]	0.0046	0.35	0.000218	0.000269	0.00909
Glass marbles[114]	0.0051	0.376	0.000296	0.0003549	0.01223
Pea gravel [112]	0.0052	0.38	0.00031	0.000370	0.01279
Marbles [113]	0.0056	0.4	0.000389	0.0004543	0.01593
Acrylic ball [114]	0.0058	0.4054	0.000413	0.0004798	0.01689
Glass marbles [114]	0.0059	0.406	0.000416	0.0004827	0.017
Black marbles [1]	0.0037	0.47	0.000106	0.000243	0.00841
Celite spheres [105]	0.0081	0.56	0.000885	0.0012471	0.04548
Celite spheres [105]	0.0103	0.62	0.001652	0.0022686	0.08275
Celite spheres [105]	0.0207	0.71	0.010574	0.0104887	0.38134
Celite spheres [105]	0.0268	0.76	0.019359	0.0193551	0.68289

Table A.8 For pea gravel spherical particles diameter of 1.27 cm, pore diameter of 0.5189 cm, bed porosity of 0.395, packing height of 52.07 cm, bed diameter of 8.89 cm, tortousity of 1.3024 , permeability of 3.82E-09 m^2 [112]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.010668	57.42439	90.67393	246.6932
0.012192	114.8488	118.43126	310.8642
0.013899	124.5464	153.91326	391.3218
0.016154	181.9708	207.92087	511.5558
0.016764	229.6976	223.90909	546.7708
0.019507	287.122	303.18401	719.5583
0.020726	344.5463	342.26633	803.8743
0.02225	401.9707	394.4501	915.7783
0.024079	488.2977	461.95592	1059.61
0.025908	545.7221	534.79114	1213.856
0.027432	612.8441	599.55823	1350.349
0.028956	660.7612	668.0263	1494.076
0.03048	747.0882	740.19535	1645.034

Table A.9 For acrylic balls of particles diameter of 1.27 cm, pore diameter of 0.571cm, bed porosity of 0.3969, packing height of 48.26cm, bed diameter of 8.001 cm, tortousity of 1.3016 , permeability of 4.61E-09 m^2 [114]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.011078	93.24492	86.61931	210.7837
0.01188	120.438	99.627616	238.0685
0.012683	134.6457	113.54559	266.9763
0.013486	157.0431	128.37323	297.5072
0.014288	178.4271	144.11054	329.6612
0.015894	209.3086	178.31414	398.8383
0.019908	275.1347	279.74236	600.1845
0.027132	498.7567	519.62226	1064.86
0.031146	633.4438	684.72722	1379.821

0.03837	973.6635	1039.2253	2049.002

Table A.10 Pressure drop versus velocity for black marbles of diameters 1.905cm, pore diameter of 1.12cm, bed porosity of 0.402, packing height of 45.72cm, bed diameter of 8.89 cm, tortousity of 1.2988,permeability of 1.73E-08m^2[1]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0153	108.9239	70.560804	126.8862
0.0269	326.7717	218.11484	369.4183
0.0308	381.2336	285.94473	479.3073
0.0424	708.0052	541.89154	890.4733
0.0502	871.3911	759.60549	1237.908
0.058	1143.701	1013.9969	1642.405
0.0657	1416.01	1301.1022	2097.685
0.0735	1688.32	1628.3784	2615.574
0.089	2505.249	2387.5951	3814.072
0.0983	3104.331	2912.6459	4641.329
0.1061	3431.102	3393.2154	5397.706
0.1154	4084.646	4014.1378	6374.117
0.1294	4886.264	5047.1848	7996.955

Table A.11 Pressure drop versus velocity for black marbles of particles diameter of 1.9 cm, pore diameter of 0.84 cm,bed porosity of 0.4, packing height of 67.2cm, bed diameter of 8.89cm, tortousity of 1.29, permeability of 9.92E-09m^2 [115]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.014	73.37296	84.487591	201.965
0.02	220.1189	172.42365	392.1399
0.027	366.8499	314.24211	692.5879
0.033	513.5958	469.4234	1017.467
0.039	660.3418	655.64094	1404.517
0.046	953.8187	912.12113	1934.652

0.052	1100.565	1165.5839	2456.406
0.059	1467.415	1500.5168	3143.696

Table A.12 Pressure drop versus velocity for glass sphere of particles diameter of 0.42 cm, pore diameter of 0.171 cm, bed porosity of 0.3837, packing height of 20 cm, bed diameter of 7.62 cm, tortousity of 1.308 and permeability of 4.4E-10m^2 [117]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	ΔP (pa) (Ergun equation)
0.0305	4700	2988.3971	6239.893
0.0609	15455	11914.428	21233.64
0.0914	32145	26836.818	45079.87
0.1218	53165	47657.714	77622.23
0.1523	82835	74514.095	119074.8
0.1827	114360	107229.86	169165.8
0.2132	151455	146020.23	228224.7
0.2436	193490	190630.85	295864.3
0.2741	242325	241355.22	372529.5
0.3046	295485	298056.38	458012.4

Table A.13 Pressure drop versus velocity for glass sphere of particles diameter of 1.01 cm, pore diameter of 0.51 cm, bed porosity of 0.394, packing height of 20 cm, bed diameter of 7.62cm, tortousity of 1.3 and permeability of 3.72E-09 m^2[117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	1115	754.23264	1313.864
0.0609	3770	3007.0471	4881.434
0.0914	8035	6773.2645	10726.18
0.1218	13910	12028.188	18809.78
0.1523	21020	18806.391	29185.41
0.1827	29055	27063.424	41785.04
0.2132	39565	36853.611	56691.55

0.2436	51310	48112.753	73807.2
0.2741	64910	60914.925	93244.6
0.3046	78510	75225.562	114951.2

Table A.14 Pressure drop versus velocity for glass sphere of particles diameter of 0.51cm, pore diameter of 0.23 cm, bed porosity of 0.385, packing height of 20 cm, bed diameter of 7.62 cm, tortousity of 1.307 and permeability of 7.9E-10m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	3030	2047.05	3792.818
0.0609	9890	8161.3752	13258.01
0.0914	21020	18383.202	28457.85
0.1218	35235	32645.501	49292.66
0.1523	53165	51042.104	75899.52
0.1827	74180	73452.377	108104
0.2132	99525	100023.76	146117.8
0.2436	127345	130582	189691.9
0.2741	158870	165328.16	239112.8
0.3046	194725	204168.42	294246.6

Table A.15 Wall correction factor results for mono size spherical particles

Type of packing	Dr (m)	d_p (m)	L (m)	Sc/S	f_w (Present model)
Marbles [113]	0.0889	0.0127	0.413	0.00014	1.000034
Marbles [113]	0.0889	0.0127	0.533	0.00017	1.000043
Glass marbles [114]	0.0889	0.0127	0.26	9E-05	1.000022
Pea gravel [112]	0.0889	0.0127	0.521	0.00017	1.000042
Marbles [113]	0.0889	0.019	0.673	0.00032	1.000079
Acrylic ball [114]	0.1461	0.019	0.673	0.00054	1.000135
Glass marbles [114]	0.1524	0.0127	0.394	0.00024	1.000059
Glass marbles [114]	0.1524	0.0127	0.508	0.0003	1.000074
Black marbles [1]	0.1524	0.0127	0.279	0.00018	1.000045

Pea gravel [112]	0.1524	0.0889	0.343	0.00149	1.000371
Pea gravel [112]	0.1524	0.0127	0.47	0.00028	1.000069
Pea gravel [112]	0.1524	0.0127	0.559	0.00032	1.00008
Pea gravel [112]	0.1524	0.0127	0.33	0.00021	1.000051
Pea gravel [112]	0.0762	0.0042	0.2	2E-05	1.000005
Pea gravel [112]	0.0889	0.0191	0.457	0.00022	1.000055
Glass marbles [116]	0.08	0.0064	0.457	6.6E-05	1.000016
Glass marbles [116]	0.08	0.0127	0.478	0.00014	1.000034
Pea gravel [112]	0.08	0.0127	0.483	0.00014	1.000035
Pea gravel [112]	0.0889	0.0127	0.483	0.00016	1.000039
Acrylic ball [114]	0.0889	0.0127	0.47	0.00015	1.000038
Acrylic ball [114]	0.0826	0.0127	0.413	0.00012	1.000031
Acrylic ball [114]	0.0825	0.003	0.413	3E-05	1.000007

A.3.2 Binary Sized Spherical Particle System

Table A.16 The permeability results for water flow through packed bed

Type of packing	d_{pore}(m) (Experiment)	ε (Experiment)	k(m^2) (Experiment)	k (m^2) (Present work)	k (m^2) (Carmen)
Acrylic ball [114]	0.0021	0.31	2.27E-05	2.8E-05	0.0017
Acrylic ball [114]	0.0024	0.33	3.18E-05	3.76E-05	0.0023
Acrylic ball [114]	0.0024	0.33	3.18E-05	3.76E-05	0.0023
Glass [117]	0.0021	0.3484	2.18E-05	2.7E-05	0.0019
Glass [117]	0.0017	0.3551	1.1E-05	1.48E-05	0.0013
Glass [117]	0.0035	0.367	9.72E-05	0.000101	0.0056
Glass [117]	0.0027	0.373	4.35E-05	4.99E-05	0.0034
Glass [116]	0.0024	0.3822	3.23E-05	3.86E-05	0.0029
Glass [117]	0.0023	0.3861	2.78E-05	3.4E-05	0.0026
Glass [117]	0.0026	0.3882	3.96E-05	4.64E-05	0.0033
Glass [117]	0.0032	0.3904	7.5E-05	8.16E-05	0.0051
Glass [116]	0.002	0.3933	1.72E-05	2.25E-05	0.002
Glass [116]	0.0024	0.3965	3.05E-05	3.72E-05	0.0029
Glass [116]	0.0055	0.409	0.00035	0.000321	0.0153
Glass [117]	0.0032	0.4091	6.85E-05	7.66E-05	0.0052

Glass [116]	0.0026	0.41208	3.64E-05	4.42E-05	0.0034
Glass [116]	0.003	0.4166	5.46E-05	6.34E-05	0.0045
Glass [116]	0.0032	0.4171	7.04E-05	7.92E-05	0.0054
Glass [116]	0.0033	0.4173	7.4E-05	8.28E-05	0.0056
Glass [116]	0.0036	0.4186	9.88E-05	0.000107	0.0068
Glass [117]	0.0043	0.4223	0.000171	0.000174	0.0099
Glass [117]	0.002	0.359	1.95E-05	2.46E-05	0.0019
Glass [117]	0.0019	0.365	1.57E-05	2.03E-05	0.0017

Table A.17 For Glass spheres of particles diameter (dp_1=0.42 and dp_2=0.51with dp_{eff}=0.46cm),fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.1689cm, bed porosity of 0.3809, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.3094 and permeability of 1.46E-10 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	5685	3687.9865	7105.2716
0.0609	18545	14703.619	24331.299
0.0914	38945	33119.368	51791.411
0.1218	64910	58814.474	89305.542
0.1523	97055	91957.986	137120.49
0.1827	137235	132332.57	194922.73
0.2132	181745	180203.84	263092.52
0.2436	231815	235257.9	341182.86
0.2741	289305	297856.93	429707.49
0.3046	351125	367831.94	528426.26

Table A.18 Pressure drop results for glass spheres of particles diameter (dp_1=0.42 and dp_2=0.61with dp_{eff}=0.4975cm),fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.1906cm, bed porosity of 0.3807, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.3096 and permeability of 2.02E-10 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	4820	3080.5492	5805.8264
0.0609	16380	12281.829	20096.768
0.0914	34620	27664.375	42966.846
0.1218	58110	49127.316	74266.09
0.1523	88400	76811.858	114200.42
0.1827	124255	110536.46	162507.96
0.2132	165050	150523	219506.54
0.2436	210180	196509.27	284822.39
0.2741	266435	248797.8	358885.22
0.3046	323925	307247.43	441494.19

Table A.19 Pressure drop results for glass spheres of particles diameter (dp_1=0.51 and dp_2=0.79 with dp_{eff}=0.6198cm),fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.26cm, bed porosity of 0.379, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.3098 and permeability of 4.67E-10 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	2905	1920.2325	3508.7869
0.0609	9890	7655.7673	12472.707
0.0914	21020	17244.338	26950.732
0.1218	35235	30623.069	46847.926
0.1523	54400	47879.978	72295.19
0.1827	76035	68901.905	103125.66
0.2132	102615	93827.153	139542.16
0.2436	131670	122492.28	181305.9
0.2741	165670	155085.86	228691.64
0.3046	202145	191519.91	281571.01

Table A.20 Pressure drop results for Acrylic balls of particles diameter (dp_1=0.655,dp_2=1.27 with dp_{eff}=1.016 cm), fractions of (x_1=0.25,x_2=0.75), pore diameter of 0.338 cm, bed porosity of 0.3778, packing height of 49.53 cm, bed diameter of 8 cm, tortousity of 1.31 and permeability of 9.1E-10 m^2 [114]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.00866944	200.9771855	106.32716	385.62045
0.01107761	301.4657783	173.60162	575.73503
0.01589394	552.6872602	357.37556	1064.2226
0.01910483	753.6644458	516.35451	1470.0725
0.02392116	1055.130224	809.51732	2199.1342
0.02873749	1507.328892	1168.3134	3072.5403
0.03234974	1859.038966	1480.4824	3822.3209
0.03515927	2210.749041	1748.8046	4461.6178
0.03837015	2612.703412	2082.8057	5252.386
0.04398921	3265.879265	2737.4988	6790.599
0.04880554	3969.299414	3369.7671	8265.4404
0.05402323	4622.475268	4128.7898	10026.074
0.0584382	5376.139713	4831.2036	11648.157
0.06164909	5828.338381	5376.6898	12904.035
0.06807086	6883.468605	6555.1734	15608.251

Table A.21 Pressure drop results for Acrylic balls of particles diameter (dp_1=0.655,dp_2=1.27 with dp_{eff}=0.7056 cm), fractions of (x_1=0.8,x_2=0.2), pore diameter of 0.21 cm, bed porosity of 0.379, packing height of 48.26 cm, bed diameter of 8 cm, tortousity of 1.31 and permeability of 2.65E-10 m^2 [3]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.01107761	721.931206	348.72232	1238.4003
0.01388714	928.1972648	548.04097	1776.6245
0.01669666	1134.463324	792.22217	2405.5404
0.01870347	1340.729383	994.10416	2910.2914
0.02071027	1546.995441	1218.8752	3461.3137

0.02231572	1805.373809	1415.1722	3935.4469
0.02392116	2063.752176	1626.1182	4439.1937
0.02512524	2269.944882	1793.9411	4836.4376
0.02632932	2476.137588	1970.004	5250.3391
0.02673069	2527.704103	2030.5227	5392.008
0.02713205	2579.270617	2091.9571	5535.5278
0.02833613	2785.536676	2281.7535	5977.1922
0.02954021	2991.802735	2479.7899	6435.5143
0.03154702	3326.985081	2828.1619	7236.4011
0.03255042	3494.576254	3010.9313	7654.1963
0.03355382	3662.167426	3199.4229	8083.5592
0.03495858	3907.108371	3472.9246	8704.1013
0.03636335	4152.049316	3757.6419	9347.3163
0.03776811	4396.990261	4053.5749	10013.204
0.03917287	4641.931206	4360.7235	10701.765
0.04037696	4886.872151	4632.9205	11310.006
0.04158104	5131.813096	4913.3575	11934.904
0.04278512	5376.754041	5202.0346	12576.46
0.04398921	5621.694985	5498.9517	13234.674
0.04519329	5892.655408	5804.1089	13909.545
0.04639737	6163.615831	6117.5062	14601.074
0.04760145	6434.576254	6439.1435	15309.261
0.04880554	6705.536676	6769.0209	16034.105

Table A.22 Pressure drop results for glass of particles diameter (dp_1=0.79, dp_2=1.01 with dp_{eff}=0.8856cm), fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.43cm, bed porosity of 0.378, packing height of 15.15cm, bed diameter of 7.64cm, tortousity of 1.31 and permeability of 1.75E-09 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0303	1029.887789	899.78693	1635.0072
0.0606	3510.990099	3599.1477	6031.2737
0.0909	7490.112211	8098.0824	13188.8
0.1211	12171.43234	14372.844	23070.308

0.1511	19661.53795	22376.046	35602.251
0.1817	29960.44224	32356.704	51173.431
0.2121	37450.55446	44089.56	69431.497
0.2424	48217.58416	57586.364	90395.32
0.2726	60857.14851	72829.299	114037.56
0.303	72560.46205	89978.693	140606.74

Table A.23 Wall correction factor results

Type of packing	Dr (m)	d_{peff} (m)	L (m)	Sc/S	f_w (Present model)
Acrylic ball[114]	0.08001	0.01016	0.4953	0.00011	0.99984
Acrylic ball[114]	0.08001	0.00907	0.4826	9.9E-05	0.9999
Acrylic ball[114]	0.08001	0.00726	0.508	8.3E-05	0.99995
Acrylic ball[114]	0.08001	0.00706	0.4826	7.7E-05	0.99997
Acrylic ball[3]	0.08001	0.00726	0.4953	8.1E-05	0.99996
Acrylic ball[3]	0.0764	0.00886	0.1515	3.4E-05	1.00013
Glass[117]	0.0764	0.00751	0.1515	2.8E-05	1.00015
Glass[117]	0.0764	0.00674	0.1515	2.6E-05	1.00016
Glass[116]	0.0762	0.00678	0.2	3.2E-05	1.00014
Glass[116]	0.0762	0.00761	0.2	3.6E-05	1.00012
Acrylic ball[114]	0.08001	0.01155	0.4064	0.00011	0.99986
Glass[117]	0.0764	0.00592	0.1515	2.2E-05	1.00017
Glass[117]	0.0764	0.00685	0.1515	2.6E-05	1.00016
Glass[117]	0.0764	0.00621	0.1515	2.4E-05	1.00017
Glass[117]	0.0764	0.00551	0.1515	2.1E-05	1.00018
Glass[117]	0.0764	0.00551	0.1515	2.1E-05	1.00018
Glass[117]	0.0764	0.00461	0.1515	1.7E-05	1.00019
Glass[116]	0.0762	0.0046	0.2	2.2E-05	1.00017
Glass[116]	0.0762	0.00688	0.2	3.3E-05	1.00013
Glass[116]	0.0762	0.0089	0.2	4.2E-05	1.0001
Glass[116]	0.0762	0.00498	0.2	2.4E-05	1.00017
Glass[116]	0.0762	0.00548	0.2	2.6E-05	1.00016
Glass[116]	0.0762	0.00593	0.2	2.8E-05	1.00015
Glass[116]	0.0762	0.0062	0.2	2.9E-05	1.00014

Acrylic ball[114]	0.08001	0.00706	0.4826	7.7E-05	0.99997

A.3.3 Ternary Sized Spherical Particles System

Table A.24 The permeability results for water flow through packed bed

Type of packing	d_{pore}(m) (Experiment)	ε (Experiment)	k(m^2) (Experiment)	k (m^2) (Present work)	k (m^2) (Carmn)
Glass[116]	0.0038	0.428	0.00012	0.00012	0.0078
Glass[116]	0.0032	0.402	7E-05	6.8E-05	0.0051
Glass[116]	0.0027	0.386	4.4E-05	4.4E-05	0.0036
Glass[117]	0.002	0.365	1.9E-05	2.1E-05	0.0019
Glass[117]	0.0024	0.371	2.9E-05	2.9E-05	0.0026
Glass[117]	0.0027	0.385	4.4E-05	4.3E-05	0.0035
Glass[117]	0.0025	0.377	3.4E-05	3.4E-05	0.0029
Glass[117]	0.0028	0.39	4.7E-05	4.7E-05	0.0038
Glass[117]	0.0022	0.367	2.3E-05	2.4E-05	0.0022
Glass[117]	0.0034	0.397	8.3E-05	7.6E-05	0.0056
Glass[117]	0.0021	0.356	2.1E-05	2.1E-05	0.002
Glass[116]	0.0029	0.4	5.2E-05	5.3E-05	0.0041
Glass[116]	0.0025	0.391	3.6E-05	3.8E-05	0.0032
Glass[116]	0.0023	0.384	2.8E-05	3E-05	0.0026
Glass[116]	0.0027	0.399	4.3E-05	4.6E-05	0.0037
Glass[116]	0.0025	0.398	3.4E-05	3.8E-05	0.0031
Glass[116]	0.002	0.377	1.8E-05	2.1E-05	0.0019
Glass[117]	0.0019	0.36	1.5E-05	1.6E-05	0.0016

Table A.25 Pressure drop results for glass of particles diameter (0.51, 0.61, 0.79 and dp_{eff}=0.617 cm), pore diameter of 0.258 cm, bed porosity of 0.389, packing height of 15.15 cm, bed diameter of 7.64 cm, tortousity of 1.306 and permeability of 4.42E-10 m^2 [116]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0303	2574.7254	1810.1448	3358.1001
0.0606	9362.638	7240.5793	12011.843
0.0909	18725.276	16291.303	25961.227
0.1211	32769.233	28914.544	45134.029
0.1511	49153.849	45014.973	69388.461
0.1817	65538.466	65093.544	99476.055
0.2121	91285.72	88697.096	134715.19
0.2424	117032.97	115849.27	175142.78
0.2726	142780.23	146514.22	220706.41
0.303	170868.14	181014.48	271884.9

Table A.26 Pressure drop results for particles diameter (0.42,0.51,0.61 and dp_{eff}=0.502cm), pore diameter of 0.1881cm, bed porosity of 0.372, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.31 and permeability of 1.73E-10 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	5070	3300.3956	6048.3259
0.0609	16690	13158.334	20937.028
0.0914	35235	29638.67	44764.059
0.1218	58725	52633.336	77373.175
0.1523	88400	82293.612	118978.91
0.1827	122400	118425.01	169308.44
0.2132	161345	161265.22	228692.88
0.2436	208325	210533.34	296742.83

0.2741	260250	266553.5	373905.97
0.3046	314035	329174.45	459973.07

Table A.27 Pressure drop results for particles diameter (0.421,0.5099,0.7955 and dp_{eff}=0.536cm), pore diameter of 0.22cm, bed porosity of 0.376, packing height of 15.15cm, bed diameter of 7.64cm, tortousity of 1.31 and permeability of 2.6E-10 m^2 [116]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0303	3230.1122	2517.0951	4336.0155
0.0606	11703.3	10068.381	15215.522
0.0909	24810.99	22653.856	32638.519
0.1211	39791.215	40207.092	56515.147
0.1511	60857.149	62595.527	86669.592
0.1817	86604.422	90515.766	124035.4
0.2121	115628.58	123337.66	167765.41
0.2424	148865.94	161094.09	217905.87
0.2726	185380.26	203735.2	274391.97
0.303	224703.3	251709.51	337817.24

Table A.28 Pressure drop results for particles diameter (0.42,0.61,1.01 and dp_{eff}=0.5627cm), pore diameter of 0.217cm, bed porosity of 0.379, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.309 and permeability of 2.5E-10 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	3835	2619.5279	4900.8358
0.0609	12980	10443.785	17194.895
0.0914	27820	23524.247	36963.059
0.1218	47600	41775.14	64075.702
0.1523	72325	65316.537	98711.195
0.1827	102000	93994.066	140642.42

0.2132	136615	127996.4	190145.24
0.2436	174945	167100.56	246895.05
0.2741	218835	211563.83	311265.2
0.3046	263960	261266.15	383081.22

Table A.29 Pressure drop results for particles diameter (0.6015,0.7955,0.998 and dp_{eff}=0.7651cm), pore diameter of 0.381cm, bed porosity of 0.4024, packing height of 15.15cm, bed diameter of 7.64cm, tortousity of 1.302 and permeability of 1.04E-09m^2 [116]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0303	1357.5825	1057.8523	1862.1881
0.0606	4213.1871	4231.409	6798.266
0.0909	9830.7698	9520.6704	14808.234
0.1211	16384.616	16897.718	25850.455
0.1511	25747.254	26306.84	39842.915
0.1817	37450.552	38040.798	57219.549
0.2121	47749.453	51834.761	77587.001
0.2424	64134.07	67702.545	100966.42
0.2726	77241.763	85623.202	127327.36
0.303	93626.38	105785.23	156946.92

Table A.30 Wall correction factor results

Type of packing	Dr (m)	d_{peff}(m)	L (m)	Sc/S	f_w (Present model)
Glass [116]	0.0764	0.0071	0.1515	2.6941E-05	1.00000672
Glass [116]	0.0764	0.0065	0.1515	2.4581E-05	1.00000613
Glass [116]	0.0764	0.006	0.1515	2.2578E-05	1.00000563
Glass [116]	0.0764	0.0057	0.1515	2.1487E-05	1.00000536
Glass [116]	0.0764	0.0054	0.1515	2.0327E-05	1.00000507
Glass [117]	0.0762	0.0054	0.2	2.5399E-05	1.00000634
Glass [117]	0.0762	0.0065	0.2	3.0716E-05	1.00000766
Glass [116]	0.0764	0.0077	0.1515	2.9017E-05	1.00000724

Glass [116]	0.0764	0.0065	0.1515	2.4553E-05	1.00000613
Glass [116]	0.0764	0.0056	0.1515	2.13E-05	1.00000531
Glass [116]	0.0764	0.0061	0.1515	2.3294E-05	1.00000581
Glass [117]	0.0762	0.005	0.2	2.3813E-05	1.00000594
Glass [117]	0.0762	0.0056	0.2	2.6714E-05	1.00000667
Glass [117]	0.0762	0.0057	0.2	2.6942E-05	1.00000672
Glass [117]	0.0762	0.006	0.2	2.8428E-05	1.00000709
Glass [117]	0.0762	0.0062	0.2	2.9268E-05	1.0000073
Glass [117]	0.0762	0.0065	0.2	3.103E-05	1.00000774
Glass [116]	0.0764	0.005	0.1515	1.8954E-05	1.00000473
Glass [117]	0.0762	0.0077	0.2	3.6565E-05	1.00000912

A.3.4 General Equation Results

Table A.31 The permeability results for the general equation

Type of packing	d_{pore} (m) (Experiment)	ε (Experiment)	k(m^2) (Experiment)	k (m^2) (present work)	k (m^2) (Carmn)
Pea gravel[112]	0.0056	0.4	0.000389	0.000334	0.01593
Pea gravel[112]	0.0052	0.38	0.00027	0.000258	0.01279
Marbles [113]	0.0081	0.56	0.000885	0.000823	0.04548
Glass [116]	0.0038	0.428	0.000115	9.11E-05	0.00776
Glass [116]	0.0032	0.402	6.96E-05	5.22E-05	0.00509
Glass [116]	0.0027	0.386	4.4E-05	3.16E-05	0.00355
Glass [116]	0.0027	0.399	4.33E-05	3.12E-05	0.00367
Glass [116]	0.0025	0.398	3.39E-05	2.4E-05	0.00312
Glass [116]	0.0022	0.384	2.43E-05	1.66E-05	0.00237
Glass [116]	0.002	0.377	1.82E-05	1.21E-05	0.00191
Glass [116]	0.0043	0.423	0.000171	0.000139	0.00993
Pea gravel[112]	0.0052	0.38	0.00031	0.000258	0.01279
Pea gravel[112]	0.0056	0.4	0.000389	0.000334	0.01593
Acrylic ball[118]	0.0058	0.405	0.000413	0.000357	0.01689
Marbles [113]	0.0026	0.38	3.97E-05	2.81E-05	0.00325
Acrylic ball[118]	0.0036	0.46	9.57E-05	7.51E-05	0.0076

Marbles [117]	0.0021	0.31	2.27E-05	1.45E-05	0.00173
Marbles [117]	0.0024	0.33	3.18E-05	2.14E-05	0.00234
Marbles [117]	0.0024	0.33	3.18E-05	2.14E-05	0.00234
Marbles [117]	0.0021	0.348	2.18E-05	1.44E-05	0.00195
Marbles [118]	0.002	0.359	1.95E-05	1.29E-05	0.00188
Marbles [118]	0.0019	0.365	1.57E-05	1.02E-05	0.00166
Acrylic ball[118]	0.0035	0.367	9.72E-05	7.33E-05	0.00564
Acrylic ball[118]	0.0027	0.373	4.35E-05	3.09E-05	0.00337
Acrylic ball[118]	0.0024	0.382	3.23E-05	2.25E-05	0.00285
Acrylic ball[118]	0.0023	0.386	2.78E-05	1.93E-05	0.00262
Acrylic ball[118]	0.0026	0.388	3.96E-05	2.82E-05	0.00334
Acrylic ball[118]	0.0032	0.39	7.5E-05	5.63E-05	0.00515
Glass [117]	0.002	0.393	1.72E-05	1.15E-05	0.00195
Glass [117]	0.0024	0.397	3.05E-05	2.14E-05	0.00288
Glass [117]	0.0055	0.409	0.00035	0.000299	0.01531
Black marble [115]	0.0103	0.62	0.001652	0.001571	0.08275
Black marbles [115]	0.0207	0.71	0.010574	0.010759	0.38134
Black marbles [115]	0.0268	0.76	0.019359	0.019263	0.68289
Glass [117]	0.0032	0.409	6.85E-05	5.15E-05	0.00516
Glass [117]	0.0026	0.412	3.64E-05	2.61E-05	0.00342
Glass [117]	0.003	0.417	5.46E-05	4.05E-05	0.00455
Glass [117]	0.0032	0.417	7.04E-05	5.32E-05	0.0054
Glass [117]	0.0033	0.417	7.4E-05	5.62E-05	0.00558
Glass [117]	0.0036	0.419	9.88E-05	7.68E-05	0.0068
Glass [117]	0.0043	0.422	0.000171	0.000139	0.00993
Glass [116]	0.0022	0.367	2.31E-05	1.55E-05	0.00216
Glass [116]	0.0034	0.397	8.34E-05	6.33E-05	0.00564
Glass [116]	0.002	0.365	1.94E-05	1.29E-05	0.00191
Glass [116]	0.0021	0.356	2.09E-05	1.39E-05	0.00195
Glass [116]	0.0024	0.371	2.93E-05	2.02E-05	0.00258
Glass [116]	0.0027	0.385	4.36E-05	3.13E-05	0.00352
Glass [116]	0.0025	0.377	3.43E-05	2.4E-05	0.00292

Glass [116]	0.0028	0.39	4.71E-05	3.4E-05	0.00376
Pea gravel[112]	0.0011	0.386	2.82E-06	1.63E-06	0.00057
Pea gravel[113]	0.0013	0.388	4.7E-06	2.83E-06	0.0008
Acrylic ball[118]	0.0058	0.406	0.000416	0.00036	0.017
Pea gravel[113]	0.0046	0.35	0.000218	0.000173	0.00909
Marbles [113]	0.0113	0.47	0.002805	0.002873	0.07452
Marbles [113]	0.0088	0.41	0.001456	0.001392	0.03971
Pea gravel[112]	0.0024	0.357	2.97E-05	2.02E-05	0.00247
Acrylic ball[118]	0.0051	0.376	0.000296	0.000245	0.01223
Marbles [113]	0.0025	0.37	3.53E-05	2.47E-05	0.0029
Glass [116]	0.0029	0.4	5.16E-05	3.78E-05	0.00414
Glass [116]	0.0025	0.391	3.61E-05	2.55E-05	0.00316
Glass [116]	0.0023	0.384	2.8E-05	1.94E-05	0.00261

Table A.32 Pressure drop results for pea gravel of particles diameter 1.27cm, pore diameter of 0.5189 cm, bed porosity of 0.3902, packing height of 52.07cm, bed diameter of 8.89 cm, tortousity of 1.3026, permeability of 3.82E-09 m^2 [112]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.01067	57.42439	75.004694	246.6932
0.01219	114.8488	97.965314	310.8642
0.0139	124.5464	127.31572	391.3218
0.01615	181.9708	171.99035	511.5558
0.01676	229.6976	185.21567	546.7708
0.01951	287.122	250.7912	719.5583
0.02073	344.5463	283.11976	803.8743
0.02225	401.9707	326.28572	915.7783
0.02408	488.2977	382.12595	1059.61
0.02591	545.7221	442.37462	1213.856
0.02743	612.8441	495.9494	1350.349
0.02896	660.7612	552.5856	1494.076
0.03048	747.0882	612.28321	1645.034

Table A.33 For acrylic balls spherical particles diameter of 1.27 cm, pore diameter of 0.571cm, bed porosity of 0.3915, packing height of 48.26cm, bed diameter of 8.001 cm, tortousity of 1.3017, permeability of 3.49E-09 m^2 [114]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.01108	93.24492	70.953588	210.7837
0.01188	120.438	81.609248	238.0685
0.01268	134.6457	93.010057	266.9763
0.01349	157.0431	105.15601	297.5072
0.01429	178.4271	118.04712	329.6612
0.01589	209.3086	146.06475	398.8383
0.01991	275.1347	229.14896	600.1845
0.02713	498.7567	425.64485	1064.86
0.03115	633.4438	560.8894	1379.821
0.03837	973.6635	851.27393	2049.002

Table A.34 For black marbles of particles diameter of 1.905 cm, pore diameter of 1.1262cm, bed porosity of 0.3959, packing height of 45.72cm, bed diameter of 8.89cm, tortousity of 1.2989, permeability of 3.13E-08m^2 [1]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0153	108.9239	53.351649	126.8862
0.0269	326.7717	164.91856	369.4183
0.0308	381.2336	216.20534	479.3073
0.0424	708.0052	409.72899	890.4733
0.0502	871.3911	574.34443	1237.908
0.058	1143.701	766.69207	1642.405
0.0657	1416.01	983.77486	2097.685
0.0735	1688.32	1231.2313	2615.574
0.089	2505.249	1805.2818	3814.072
0.0983	3104.331	2202.2774	4641.329
0.1061	3431.102	2565.6402	5397.706
0.1154	4084.646	3035.1251	6374.117
0.1294	4886.264	3816.2212	7996.955

Table A.35 Pressure drop versus velocity for black marbles of particles diameter of 1.9 cm, pore diameter of 0.84 cm, bed porosity of 0.395, packing height of 67.2 cm, bed diameter of 8.89 cm, tortousity of 1.29, permeability of 1.23E-08m^2 [115]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	ΔP (pa) (Ergun equation)
0.014	73.37296	66.325684	201.965
0.02	220.1189	135.35854	392.1399
0.027	366.8499	246.69094	692.5879
0.033	513.5958	368.51362	1017.467
0.039	660.3418	514.70084	1404.517
0.046	953.8187	716.04667	1934.652
0.052	1100.565	915.02372	2456.406
0.059	1467.415	1177.9577	3143.696

Table A.36 Pressure drop versus velocity for glass sphere of particles diameter of 0.42cm, pore diameter of 0.171cm, bed porosity of 0.3818, packing height of 20 cm, bed diameter of 7.62 cm, tortousity of 1.3078, permeability of 7.13E-11m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	4700	2804.8424	6239.893
0.0609	15455	11182.615	21233.64
0.0914	32145	25188.434	45079.87
0.1218	53165	44730.459	77622.23
0.1523	82835	69937.255	119074.8
0.1827	114360	100643.53	169165.8
0.2132	151455	137051.31	228224.7
0.2436	193490	178921.84	295864.3
0.2741	242325	226530.59	372529.5
0.3046	295485	279749.02	458012.4

Table A.37 Pressure drop versus velocity for glass sphere of particles diameter of 1.01cm, pore diameter of 0.51 cm, bed porosity of 0.3892, packing height of 20 cm, bed diameter of 7.62 cm, tortousity of 1.3, permeability of 2.46E-09m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	1115	623.7907	1313.864
0.0609	3770	2486.9886	4881.434
0.0914	8035	5601.8517	10726.18
0.1218	13910	9947.9545	18809.78
0.1523	21020	15553.89	29185.41
0.1827	29055	22382.898	41785.04
0.2132	39565	30479.905	56691.55
0.2436	51310	39791.818	73807.2
0.2741	64910	50379.898	93244.6
0.3046	78510	62215.56	114951.2

Table A.38 For Glass spheres of particles diameter (dp_1=0.42 and dp_2=0.51with dp_{eff}=0.46cm),fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.1689cm, bed porosity of 0.382, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.3075 , permeability of 6.83E-11 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	5685	2856.5955	7105.272
0.0609	18545	11388.949	24331.3
0.0914	38945	25653.194	51791.41
0.1218	64910	45555.796	89305.54
0.1523	97055	71227.691	137120.5
0.1827	137235	102500.54	194922.7
0.2132	181745	139580.09	263092.5
0.2436	231815	182223.18	341182.9
0.2741	289305	230710.38	429707.5
0.3046	351125	284910.77	528426.3

Table A.39 Pressure drop results for Acrylic balls of particles diameter (dp_1=0.655,dp_2=1.27 with dp_{eff}=1.016 cm), fractions of (x_1=0.25,x_2=0.75), pore diameter of 0.338 cm, bed porosity of 0.388, packing height of 49.53 cm, bed diameter of 8 cm, tortousity of 1.303, permeability of 6.4E-10 m^2 [114]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.00867	200.9772	89.130969	385.6205
0.01108	301.4658	145.5252	575.735
0.01589	552.6873	299.57756	1064.223
0.0191	753.6644	432.84499	1470.072
0.02392	1055.13	678.59487	2199.134
0.02874	1507.329	979.36324	3072.54
0.03235	1859.039	1241.0454	3822.321
0.03516	2210.749	1465.9721	4461.618
0.03837	2612.703	1745.9555	5252.386
0.04399	3265.879	2294.7658	6790.599
0.04881	3969.299	2824.7779	8265.44
0.05402	4622.475	3461.0446	10026.07
0.05844	5376.14	4049.8577	11648.16
0.06165	5828.338	4507.1229	12904.03
0.06807	6883.469	5495.0115	15608.25

Table A.40 For Glass spheres of particles diameter (dp_1=0.42 and dp_2=0.61with dp_{eff}=0.4975cm),fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.1906cm, bed porosity of 0.3829, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.307, permeability of 1.01E-10 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	4820	2418.6575	5805.826
0.0609	16380	9642.9359	20096.77
0.0914	34620	21720.363	42966.85
0.1218	58110	38571.744	74266.09

0.1523	88400	60307.941	114200.4
0.1827	124255	86786.423	162508
0.2132	165050	118181.39	219506.5
0.2436	210180	154286.97	284822.4
0.2741	266435	195340.71	358885.2
0.3046	323925	241231.76	441494.2

Table A.41 For Glass spheres of particles diameter (dp_1=0.51,dp_2=0.79with dp_{eff}=0.6198cm), fractions of (x_1=0.5,x_2=0.5), pore diameter of 0.26cm, bed porosity of 0.385, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.306, permeability of 2.8E-10 m^2[117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	2905	1562.4617	3508.787
0.0609	9890	6229.3722	12472.71
0.0914	21020	14031.435	26950.73
0.1218	35235	24917.489	46847.93
0.1523	54400	38959.152	72295.19
0.1827	76035	56064.349	103125.7
0.2132	102615	76345.614	139542.2
0.2436	131670	99669.955	181305.9
0.2741	165670	126190.82	228691.6
0.3046	202145	155836.61	281571

Table A.42 Pressure drop results for Acrylic balls of particles diameter (dp_1=0.655,dp_2=1.27 with dp_{eff}=0.7056 cm), fractions of (x_1=0.8,x_2=0.2), pore diameter of 0.21 cm, bed porosity of 0.385, packing height of 48.26 cm, bed diameter of 8 cm, tortousity of 1.306, permeability of 1.4E-10 m^2 [3]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.01108	721.9312	277.1708	1238.4

0.01389	928.1973	435.59286	1776.625
0.0167	1134.463	629.67249	2405.54
0.0187	1340.729	790.13194	2910.291
0.02071	1546.995	968.78403	3461.314
0.02232	1805.374	1124.8044	3935.447
0.02392	2063.752	1292.4681	4439.194
0.02513	2269.945	1425.8568	4836.438
0.02633	2476.138	1565.7947	5250.339
0.02673	2527.704	1613.8961	5392.008
0.02713	2579.271	1662.7253	5535.528
0.02834	2785.537	1813.5789	5977.192
0.02954	2991.803	1970.9818	6435.514
0.03155	3326.985	2247.8741	7236.401
0.03255	3494.576	2393.1425	7654.196
0.03355	3662.167	2542.9591	8083.559
0.03496	3907.108	2760.3432	8704.101
0.03636	4152.049	2986.6417	9347.316
0.03777	4396.99	3221.8545	10013.2
0.03917	4641.931	3465.9818	10701.77
0.04038	4886.872	3682.3289	11310.01
0.04158	5131.813	3905.2253	11934.9
0.04279	5376.754	4134.671	12576.46
0.04399	5621.695	4370.6661	13234.67
0.04519	5892.655	4613.2106	13909.55
0.0464	6163.616	4862.3044	14601.07
0.0476	6434.576	5117.9475	15309.26
0.04881	6705.537	5380.14	16034.1

Table A.43 Pressure drop results for Acrylic balls of particles diameter (dp_1=0.655,dp_2=1.27 with dp_{eff}=0.7056 cm), fractions of (x_1=0.8,x_2=0.2), pore diameter of 0.21 cm, bed porosity of 0.3878, packing height of 48.26 cm, bed diameter of 8 cm, tortousity of 1.306, permeability of 1.4E-10 m^2 [3]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0303	1029.888	774.77387	1635.007
0.0606	3510.99	3099.0955	6031.274
0.0909	7490.112	6972.9648	13188.8
0.1211	12171.43	12375.934	23070.31
0.1511	19661.54	19267.201	35602.25
0.1817	29960.44	27861.184	51173.43
0.2121	37450.55	37963.92	69431.5
0.2424	48217.58	49585.528	90395.32
0.2726	60857.15	62710.666	114037.6
0.303	72560.46	77477.387	140606.7

Table A.44 Pressure drop results for glass of particles diameter (0.51, 0.61, 0.79 and dp_{eff}=0.617cm), pore diameter of 0.258cm, bed porosity of 0.385, packing height of 15.15cm, bed diameter of 7.64cm, tortousity of 1.305 and permeability of 1.36E-10 m^2 [116]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0303	2574.725	1470.646	3358.1
0.0606	9362.638	5882.584	12011.84
0.0909	18725.28	13235.814	25961.23
0.1211	32769.23	23491.523	45134.03
0.1511	49153.85	36572.261	69388.46
0.1817	65538.47	52885.028	99476.06
0.2121	91285.72	72061.653	134715.2
0.2424	117033	94121.343	175142.8
0.2726	142780.2	119034.98	220706.4
0.303	170868.1	147064.6	271884.9

Table A.45 Pressure drop results for particles diameter (0.421, 0.5099, 0.7955and dp_{eff}=0.536cm), pore diameter of 0.22cm, bed porosity of 0.383, packing height of 15.15cm, bed diameter of 7.64cm, tortousity of 1.307 and permeability of 1.7E-10 m^2 [116]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0303	3230.112	1931.9768	4336.016
0.0606	11703.3	7727.9074	15215.52
0.0909	24810.99	17387.792	32638.52
0.1211	39791.21	30860.641	56515.15
0.1511	60857.15	48044.711	86669.59
0.1817	86604.42	69474.674	124035.4
0.2121	115628.6	94666.865	167765.4
0.2424	148865.9	123646.52	217905.9
0.2726	185380.3	156375.37	274392
0.303	224703.3	193197.68	337817.2

Table A.46 Pressure drop results for particles diameter (0.42, 0.51, 0.61 and dp_{eff}=0.502 cm), pore diameter of 0.1881 cm, bed porosity of 0.383, packing height of 20 cm, bed diameter of 7.62 cm, tortousity of 1.307 and permeability of 9.7E-11 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	5070	2463.6969	6048.326
0.0609	16690	9822.5033	20937.03
0.0914	35235	22124.832	44764.06
0.1218	58725	39290.013	77373.18
0.1523	88400	61430.975	118978.9
0.1827	122400	88402.53	169308.4
0.2132	161345	120382.12	228692.9
0.2436	208325	157160.05	296742.8
0.2741	260250	198978.28	373906
0.3046	314035	245723.9	459973.1

Table A.47 Pressure drop results for particles diameter(0.42, 0.61, 1.01 and dp_{eff}=0.5627cm), pore diameter of 0.217cm, bed porosity of 0.383, packing height of 20cm, bed diameter of 7.62cm, tortousity of 1.306 and permeability of 1.5E-10 m^2 [117]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0305	3835	2024.9236	4900.836
0.0609	12980	8073.1597	17194.89
0.0914	27820	18184.499	36963.06
0.1218	47600	32292.639	64075.7
0.1523	72325	50490.394	98711.19
0.1827	102000	72658.437	140642.4
0.2132	136615	98942.609	190145.2
0.2436	174945	129170.55	246895.1
0.2741	218835	163541.14	311265.2
0.3046	263960	201961.58	383081.2

Table A.48 Pressure drop results for particles diameter (0.6015,0.7955,0.998 and dp_{eff}=0.7651cm), pore diameter of 0.381cm, bed porosity of 0.3861, packing height of 15.15cm, bed diameter of 7.64cm, tortousity of 1.305 and permeability of 9.5E-10m^2 [116]

U (m/s)	ΔP (pa) (Experiments)	ΔP (pa) (Present work)	ΔP (pa) (Ergun equation)
0.0303	1357.583	923.31178	1862.188
0.0606	4213.187	3693.2471	6798.266
0.0909	9830.77	8309.806	14808.23
0.1211	16384.62	14748.621	25850.45
0.1511	25747.25	22961.066	39842.92
0.1817	37450.55	33202.667	57219.55
0.2121	47749.45	45242.277	77587
0.2424	64134.07	59091.954	100966.4
0.2726	77241.76	74733.414	127327.4
0.303	93626.38	92331.178	156946.9

Table A.49 Wall correction factor results

Type of packing	Dr (m)	d_p (m)	L (m)	Sc/S	f_w (Present model)
Pea gravel[112]	0.0889	0.0127	0.4128	0.000135	1.000211
Pea gravel[113]	0.0889	0.019	0.673	0.000317	1.000159
Pea gravel[113]	0.1461	0.019	0.673	0.000542	1.000095
Pea gravel[112]	0.1524	0.0127	0.3937	0.000238	1.000182
Acrylic ball[118]	0.1524	0.0127	0.5588	0.000322	1.000158
Marbles [113]	0.1524	0.0127	0.3302	0.000206	1.000191
Marbles [113]	0.0762	0.0042	0.2	1.99E-05	1.000244
Acrylic ball[118]	0.08	0.0127	0.4775	0.000138	1.000211
Marbles [113]	0.08	0.0127	0.4826	0.000139	1.00021
Pea gravel[112]	0.0889	0.0127	0.5334	0.000171	1.000201
Pea gravel[112]	0.0889	0.0127	0.2604	9E-05	1.000224
Pea gravel[113]	0.0889	0.0127	0.5207	0.000167	1.000202
Pea gravel[112]	0.1524	0.0127	0.508	0.000296	1.000165
Pea gravel[112]	0.1524	0.0127	0.2794	0.00018	1.000198
Acrylic ball[118]	0.1524	0.0889	0.3429	0.001486	0.999825
Acrylic ball[118]	0.1524	0.0127	0.4699	0.000277	1.000171
Marbles [113]	0.0889	0.0191	0.4572	0.000222	1.000186
Marbles [113]	0.08	0.0064	0.4572	6.61E-05	1.000231
Marbles [117]	0.08	0.0073	0.508	8.33E-05	1.000226
Marbles [117]	0.08	0.0071	0.4826	7.72E-05	1.000228
Marbles [113]	0.0889	0.0127	0.4826	0.000156	1.000205
Marbles [118]	0.08	0.0073	0.4953	8.13E-05	1.000227
Marbles [118]	0.0764	0.0089	0.1515	3.36E-05	1.00024
Acrylic ball[118]	0.0764	0.0075	0.1515	2.85E-05	1.000242
Acrylic ball[118]	0.0764	0.0067	0.1515	2.56E-05	1.000243
Acrylic ball[118]	0.0764	0.0059	0.1515	2.25E-05	1.000244
Acrylic ball[118]	0.0764	0.0069	0.1515	2.6E-05	1.000243
Acrylic ball[118]	0.0764	0.0062	0.1515	2.35E-05	1.000243
Acrylic ball[118]	0.0764	0.0055	0.1515	2.09E-05	1.000244
Glass [117]	0.0764	0.0055	0.1515	2.09E-05	1.000244
Glass [117]	0.0764	0.0046	0.1515	1.75E-05	1.000245

Glass [117]	0.0762	0.0046	0.2	2.18E-05	1.000244
Glass [117]	0.0762	0.0069	0.2	3.27E-05	1.000241
Glass [116]	0.0764	0.0071	0.1515	2.69E-05	1.000242
Glass [116]	0.0764	0.0065	0.1515	2.46E-05	1.000243
Glass [116]	0.0764	0.0065	0.1515	2.46E-05	1.000243
Glass [116]	0.0764	0.006	0.1515	2.26E-05	1.000244
Glass [116]	0.0764	0.0056	0.1515	2.13E-05	1.000244
Glass [116]	0.0764	0.0061	0.1515	2.33E-05	1.000243
Glass [116]	0.0764	0.0057	0.1515	2.15E-05	1.000244
Glass [116]	0.0764	0.0054	0.1515	2.03E-05	1.000244
Glass [116]	0.0764	0.005	0.1515	1.9E-05	1.000245
Glass [116]	0.0762	0.005	0.2	2.38E-05	1.000243
Glass [116]	0.0762	0.0056	0.2	2.67E-05	1.000242
Glass [116]	0.0762	0.0077	0.2	3.66E-05	1.00024
Glass [116]	0.0762	0.0054	0.2	2.54E-05	1.000243
Glass [116]	0.0762	0.0057	0.2	2.69E-05	1.000242
Glass [116]	0.0762	0.006	0.2	2.84E-05	1.000242
Glass [116]	0.0762	0.0065	0.2	3.07E-05	1.000241
Glass [116]	0.0762	0.0062	0.2	2.93E-05	1.000242
Glass [116]	0.0762	0.0065	0.2	3.1E-05	1.000241
Glass [117]	0.0762	0.005	0.2	2.36E-05	1.000243
Glass [117]	0.0762	0.0055	0.2	2.6E-05	1.000243
Glass [117]	0.0762	0.0059	0.2	2.82E-05	1.000242
Glass [117]	0.0762	0.0062	0.2	2.94E-05	1.000242
Black marbles [115]	0.0826	0.0127	0.4128	0.000125	1.000214
Marbles [112]	0.08	0.0071	0.4826	7.72E-05	1.000228
Black marbles [115]	0.0889	0.0127	0.4699	0.000152	1.000207
Black marbles [115]	0.0825	0.003	0.4128	2.99E-05	1.000241
Black marbles [115]	0.08	0.0115	0.4064	0.000108	1.000219
Marbles [112]	0.08	0.0102	0.4953	0.000114	1.000217
Marbles [112]	0.08	0.0091	0.4826	9.93E-05	1.000222
Glass [117]	0.0762	0.0089	0.2	4.23E-05	1.000238
Glass [117]	0.0762	0.0068	0.2	3.22E-05	1.000241
Glass [116]	0.0762	0.0076	0.2	3.61E-05	1.00024

الخلاصة

تم صياغة معادلات شبه عملية مطورة لحساب قطر الفراغ والمسامية بالاعتماد على العوامل المؤثرة عليهما للحشوات الكروية الشكل الاحادية، الثنائية والثلاثية. ان العوامل المؤثرة على قطر الفراغ وجدت لتكون قطر الحشوة،مسامية الحشوة ونفاذية المائع للمرور. اما العوامل المؤثرة على مسامية الحشوة في العمود الحشوي للحشوات الكروية فوجدت انها قطر الحشوة وقطر العمود الحشوي. تم استخذام انواع واشكال مختلفة من الحشوات وبحجوم مختلفة، ومن ثم دراسة كل واحد منها بشكل منفصل. السرع المستخدمة في هذا العمل (٠,٠٠٦-٠,٠٣)م/ثا ، قطر العمود الحشوي (٠,٠٧٦٢-٠,١٥٢٤)م ، قطر الجسيمة (٠,٠٠٣-٠,٠٠١)م، ارتفاع العمود الحشوي (٠,١٥-٠,٦٥)م والمسامية (٠,٣-٠,٥)م.

النتائج المستحصلة من المعادلات المطورة لحساب مسامية الحشوة تم مقارنتها مع معادلة فرناس لحساب المسامية ومع النتائج المستحصلة من المصادر الموثقة، وقد اظهرت المقارنة تطابق جيد بين نتائجنا والنتائج.

قورنت النتائج المستحصلة من معادلة لطيف المطورة لحساب قطر الفراغ بين الحشوات الثلاثية مع معادلة ملنكتون وكورك المطورة ومع النتائج المستحصلة من المصادر الموثقة، وقد اظهرت المقارنة ان المعادلتين مطابقتين للنتائج ، وعليه يمكن استخدام كلا المعادلتين للحشوات الثلاثية.

معادلات شبه عملية مطورة لحساب هبوط الضغط عند جريان الماء في عمود حشوي تم صياغتها، وذلك بتطوير معادلة فوركايمر لادخال تاثير المعادلات المفروضة لحساب قطر الفراغ بين الحشوة ومسامية الحشوة، لشكل معين من الحشوات، ولاي شكل.

العوامل المختلفة التي تؤثر على هبوط الضغط عند جريان الموائع في عمود حشوي درست كل على حده، هذه العوامل هي سرعة جريان الموائع، مسامية الحشوة، نفاذية المائع، قطر الفراغ بين الحشوات، قطر الحشوة ،طول الحشوة في العمود الحشوي، قطر العمود الحشوي ودراسة تاثير جدار العمود الحشوي.

قارنا النتائج المستحصلة لهبوط الضغط خلال عمود حشوي مع عدد كبير من النتائج المستحصلة من المصادر الموثقة،و هذه المقارنة اعطت تطابق جيد جدا.

معادلة شبه عملية لحساب النفاذية صيغت لكل نوع من الحشوات باستخدام النتائج المستحصلة من التجارب. معادلة النفاذية المفروضة قورنت مع النتائج المستحصلة من التجارب ومع معادلة كارمن - كوزيني.

معادلة ماكسويل لحساب المسار المتعرج خلال العمود الحشوي طورت لغرض تحقيق معادلات الفراغ والمسامية وهبوط الضغط.

معادلات لحساب معامل تصيحيح الاحتكاك الناتج عن تاثير الاحتكاك بالجدار صيغت وتم ذلك بدراسة كافة العوامل التي ولدت هذا الاحتكاك.

شكر وتقدير

في البداية اشكر الله عز وجل الذي وفقني لاكمال متطلبات هذا البحث.

كما اتقدم بجزيل الشكر الى رئيس قسم الهندسة الكيمياوية، و جميع اساتذة قسم الهندسة الكيمياوية لمساعدتهم القيمة لي طيلة فترة الدراسة ولمدهم يد العون لي خلال اعداد هذه الرسالة.

واتقدم بشكري وامتناني الى رئاسة جامعة النهرين،لمساعدتهم ودعمهم الدائم لي طيلة فترة الدراسة.

وشكري الجزيل الى جميع زملائي و زميلاتي الذين مدو يد العون عند حاجتي اليها في البحث.

ولا انسى ان اتقدم بجزيل الشكر والتقدير الى من ساندني وساعدني على تخطي الصعوبات خلال فترة البحث الى الذين لا مثيل لهم في الدنيا الى أبي وأمي الأعزاء، زوجي الحبيب،أخوتي وأخواتي.

م. مريم يوسف غضبان

بحث مميزات العمود الحشوي باستعمال جسيمات كروية

رسالة
مقدمة إلى كلية الهندسة في جامعة نهرين
وهي جزء من متطلبات نيل درجة ماجستير علوم
في الهندسة الكيمياوية

من قبل

مريم يوسف غضبان
(بكالوريوس علوم في الهندسة الكيمياوية 2005)

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