Republic of Iraq
Ministry of Higher Education
and Scientific Research
Al -Nahrain University
College of Science
Department of Physics

Restoration of Digital Images Using an Iterative Tikhonov-Miller Filter

A Thesis

Submitted to the College of Science / Al-Nahrain University as a partial fulfillment of the Requirements for the Degree of Master of Science in Physics

By

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(2013)

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November 2016 Safar 1438

سُورَةُ ٱلْفَاتِحَةِ

بِسۡمِ ٱللَّهِ ٱلرَّحۡمَٰنِ ٱلرَّحِيمِ ١
ٱلۡحَمۡدُ لِلَّهِ رَبِّ ٱلۡعَٰلَمِينَ ٢
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ٱلدِّينِ ٤ إِيَّاكَ نَعۡبُدُ وَإِيَّاكَ
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عَلَيۡهِمۡ وَلَا ٱلضَّآلِّينَ ٧

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We, **the examining Committee** certify that we have read this thesis entitled‘‘**Restoration of Digital Images Using an Iterative Tikhonov-Miller Filter**’’and examined the student ‘‘**Hala Kadhim Hasan**’’ in its contents and that in our opinion, it is accepted for the Degree of Master of Science in Physics.

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ACKNOWLEDGMENTS

Indeed, all praise is due to Allah, I thank him for giving me the success and ease my mission for accomplish this project.

I feel deeply indebted to my supervisor, **Prof. Dr. Ayad A. Al-Ani** for proposing this project, and his support and guidance, and patience that continuously exhibited since first moment I have met.

Moreover, I would like to take this chance to express my appreciation and best wishes to Dean of the college of science**, Dr. Hadi M. Ali** and **To all of the Department faculty members** for their help and support. My gratitude is also to **Dr. Ali Talib** and **Dr. Hazim G. Dway** for their assistance to finish this project.

Finally affection to my father and mother for their love, help, and supports through my studies. and thanks to my sisters, my brother and friends to their help.

Abstract:

Digital images are applied in various fields such us: physics, computer, engineering, chemistry, biology and medication sciences. It have been known that any images acquired by optical or electronic means is likely to be degraded by the sensing environment. Image restoration, is one of digital image processing field, which is care about improving the degraded image. Image restoration may be linear or non-linear and blind or non-blind. The following research focusing on linear non-blind image restoration and assuming that the degradation model as a convolution of the original image with blurring function and distorded by additive noise. Image restoration algorithms are trying to "undo" the blurring function and the noise from the degraded image by deconvolving the blurring function and reducing the noise from the degraded image to produce an estimate image, which it approach to the original image. The image have been used, blurred by Gaussian blurring function with selected standard deviation values σ = 1,2 and degraded by additive Gaussian noise with selected signal to noise ratio values SNR= 5, 10 and 20. The degradation have been used for three type of images, these are gray image (Satellite image), sonar image (Embryo image) and color image (bird image). Iterative Tikhonov-Miller filter and Wiener filter have been used to restore the degraded images. Using Root Mean Square Error (RMSE) measuring it have been concluded that, Iterative Tikhonov-Miller filter has better performance for less degradation parameters, with high SNR and Wiener filter has better performance for more degradation parameters, with low SNR.

List of Symbols

Symbol	Description
A	Amplitude of wave
a	Brightness of light
a_i	Arbitrary constant
a_j	Arbitrary constant
B	Intensity function
b	Distance from the center to the vertical axis
C	Laplacian operator
$C_{ÛĐ}$	Correlation of two functions
c_n	Fourier series coefficients
D	Diagonal matrix of H
Đ(x)	Arbitrary function
D_a	Diameter of the lens aperture
d_i	Distance between the lens and the image plane
d_f	Focal length of lens
d_0	Distance from the lens
D^*	Complex conjugate of D
D^{-1}	Inverse of D
E	Energy of light
e_{rms}	Root mean square error
f(x, y)	Original image
$f^{(0)}$	illuminance from target
F(u,v)	Fourier transform of f(x,y)
$\hat{f}$	Restored image
$\hat{F}$	Fourier transform of $\hat{f}$
$\hat{f}_k$	First estimates
$\hat{f}_{k+1}$	Restored image after k iterations
g(x,y)	Degraded image
G(u,v)	Fourier transform of g(x,y)
Ǵ(x, y)	Target energy distribution
$g^{(0)}(m)$	Noise-free image
$H(u,v)$	Fourier transform of h(x,y)
H(x,y)	Point spread function
h	Plank's constant
H́	Transpose of H
I	Intensity of light
i(x,y)	illumination incident on the scene
j	Fourier transform
J	Criterion function
k	Number of iteration

L	Fourier series
l	Period
m	Point of image
m_a	Average brightness
n_G(b,s)	Gaussian noise
n_m	Multiplicative noise
n(x,y)	Additive noise in coordinate (x,y)
N(u,v)	Fourier transform of n(x,y)
P(a)	Probability distribution function
P(a)	Probability density function
q(x, y)	Impulsive noise
Q	linear operator
Q́	Transpose of Q
r(x,y)	illumination reflected by target
R_f	Correlation matrices of f
R_n	Correlation matrices of n
R_{COC}	Radius of circle of confusion
r_k	Steepest descent direction
s	Distance from the center to the horizontal axis
S	Toeplitz matrix
S_f	Power spectrum of f
S_n	Power spectrum of n
T	Exposure interval
Û(x)	Arbitrary function
v	Frequency of light
w(x,y, ξ,η)	Transformation function
w	Binary parameter
W	Eigenvector of H
W^{-1}	Inverse of W
x_i	Random variable
$\bar{x}$	Mean random variable x_i
X_{coc}	Characteristic function of the circle of confusion
y_i	Random variable
$\bar{y}$	Mean random variable y_i
Y(t)	Time-dependent vector
$Y_1(t)$	Time-dependent vector
$\acute{Y}$	Arbitrary point of the moving target
Z(x,y)	Image energy distribution
z	Circulant matrix
STD	Standard deviation
(ξ,η)	Spatial coordinates of target plane
(x, y)	Spatial coordinates of image plane

SNR	Signal to noise ratio
σ_t	Variances of the true image
σ_r	Variances of the recorded image
cov	Covariance
Δα	Resolving power
λ	Wavelength of light
σ_b	Standard deviation of Gaussian blurring function
σ_n	Standard deviation of Gaussian noise
α	Lagrange multiplier
ρ	Diagonal matrix of R_f
τ	Diagonal matrix of R_n
$\tilde{\sigma}$	Constant
ϑ	Diagonal matrix of C
ϑ^*	Complex conjugate of Ć
β	Control the convergence of the iterations
JPL	Jet Propulsion Laboratory

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CHAPTER ONE

General Introduction

1.1 Introduction

Concern in digital image process field started in 1920, at the point when digitized picture of world news events were first send by submarine cable among New York and London. Application of digital image process concept, began to be widespread in 1960's when digital computers began to supply the speed and storage ability needed for feasible implementation of digital image process algorithms. Digital image process field has, toughened vigorous growth and begin to incorporate into science like physics science, computer science, engineering science, information processing, chemistry science biology science and medication science **[1]**.

Digital image process is that, the field of applying variety of computer algorithms to improve digital images **[2]**. Advantage of digital image process is: amelioration of pictorial information for human interpretation, and processing of scene information for autonomous machine perception. A number of digital image process applications; in physics applications, pictures of experiments in such space as high-energy plasmas and electron microscopy habitually improved by computer techniques. in medication applications, as an example, physicians are power-assisted by computer procedures that enhance the distinction or code the intensity levels into color for easier interpretation of x-ray**[1]**. Another example, medical image also used for detection of tumors or different malady in patient. in remote sensing applications, pictures obtained by satellites are helpful in following of earth resources; geographical mapping like prediction of weather; flood, and fireplace control; and lots of different environmental applications. in outer space image applications, include recognition and analysis of objects contained in pictures got from deep space-probe missions **[3]**. In transportation applications, one in every of the key technological advances is that, the projection automatically driven vehicles, where imaging systems play important role in path designing, obstacle dodging and servo management, etc **[2]**.

Digital image processing can be categorized into the following fields **[3]**:

- Image enhancement
- Image restoration
- Image analysis
- Image reconstruction
- Image data compression

- **Image enhancement**

The aim of image enhancement, is to accentuate certain image features for subsequent analysis or image display. Examples include edge and contrast improvement, noise filtering, magnifying, and sharpening **[3]**.

- **Image restoration**

Image restoration mention as removal or minimizing of noted degradations in a picture. That includes deblurring of picture degraded by defect of optical system, or environment, correction of geometric distortion, and noise filtering **[3]**.

- **Image analysis**

Image analysis is interested with quantitative measurements from an image to produce the an overview of it, like measuring the dimension of blood cells in a medical image **[3]**.

- **Image reconstruction**

Image reconstruction from projections could be a special category of picture restoration problems, in such away, construct a two (or higher) dimensional target (or object) from one-dimension projections. Flat projections unit thus, obtained by viewing the target from many various angles. Reconstruction algorithms derive an picture of a thin axial slice of the target, giving an indoor scan otherwise untouchable whereas not acting full surgery. Such techniques are essentially in medical imaging (CT scanners), astronomy, geological exploration, radar imaging, and nondestructive testing of assemblies **[3]**.

- **Image data compression**

The amount of information associated with visual information is so huge, that its storage would need monumental storage capability. In spite of, the capacities of the many storage media unit are essential, their access speed unit generally reciprocally proportional to theirs capability. Image information compression field unit involved, reduction of the amount of bits required to store or transmit picture with none appreciable loss of knowledge **[3]**.

1.2 Image formation

light is propagated as electromagnetic waves, Light is, however, always detected in separate parts, light-weight quanta or photons, every of that represents an exact amount of energy. The energy E is proportional to the frequency v of light **[4]**:

$$E = h\, v \qquad (1.1)$$

The constant of proportion h is termed Plank's constant.

The intensity of light I, that is, the energy received per unit time per unit area, it is proportional to the square of amplitude "A";

$$I \propto |A|^2$$

Simple image refers to a two-dimensional light intensity function B(x, y), and the value of B at any pair of coordinates (x,y) is proportional to the brightness (or gray level*)* of the image at that point. The function B(x,y) may be described by two components: first, is the amount of source illumination incident on the scene being viewed, and the second, is the amount of illumination reflected by the target in the scene. These two component denoted by i(x,y) and r(x,y). Then intensity function B(x,y) given by **[1]**:

$$B(x,y) = i(x,y)\, r(x,y) \qquad (1.2)$$

i(x,y) is illumination component, r(x,y) is reflectance component, where x and y denote spatial (plane) coordinates.

The image formation (or modelling) system creates the image point (x,y) by acting upon the radiant energy propagating from the target. The tiniest attainable quantity of radiant energy transport is zero. However, the image formation system receives energy parts not solely from the target point (ξ,η) however from all different points within the target **[5]**.

Figure(1) shows the image point (x,y) create by operating of image formation system on the target point (ξ,η).

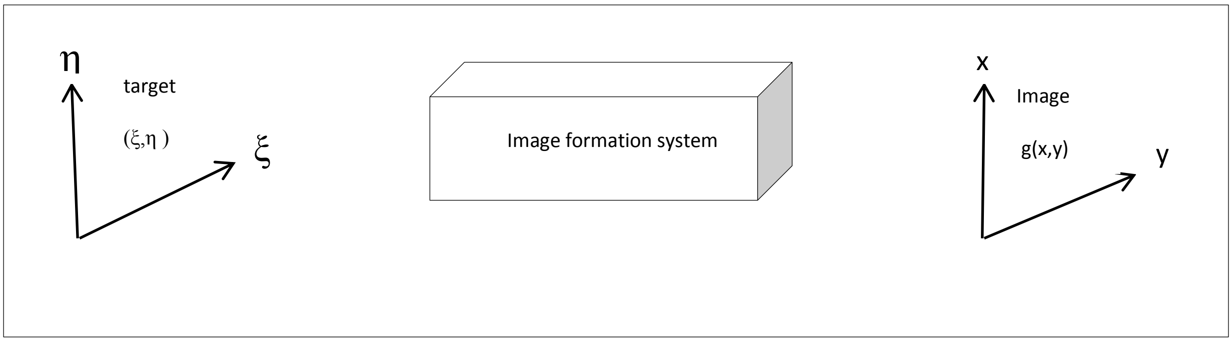


Figure (1-1) diagram of image formation system

Since the image formation system is chargeable for the distribution of energy within the target plane it is convenient to postulate a perform (or function) that describes the transformation of energy from target plane to image plane. The function should be mentioned to each coordinates (ξ,η) and (x,y), to account for mutation in distribution of energy in numerous plane. Named this function w, and also the energy within the image plane Z in term of w and also the target radiant energy distribution, Ɠ as:

$$Z(x,y) = w(x,y, \xi,\eta , Ɠ(\xi,\eta)) \quad (1.3)$$

Where Z (x,y), Ɠ (ξ,η) and w (x,y, ξ,η, Ɠ (ξ,η)) is image energy distribution, target energy distribution, transformation function respectively.

For linear image formation model, eq.(1.3) becomes:

$$w(x,y, \xi,\eta , Ɠ(\xi,\eta)) = w(x,y, \xi,\eta) Ɠ(\xi,\eta)$$

Image detection and recording are dividing into two major technologies **[5]**: photochemical and photoelectronic. Photochemical technology is exemplified by photographic film; photoelectronic technology is exemplified by tv camera. chemistry ways have the advantage of mixing image detection and recording into one compact entity the film. Photoelectronic systems, on the opposite hand, typically need separation of image detection method from the image recording process; but, Photo electronic systems sense image in a very fashion that produces them ideally fitted to conversion to digital.

The most basic demand for computer processing of picture is that the picture should be obtainable in digital form **[3]**. Conversion the image to digital by digitization, that divide image into tiny fields, every of that, is assigned a value for its intensity **[5]**. Digitizing the coordinate values is termed sampling, and digitizing the amplitude values is termed quantization **[6]**. An example of digitizers devise such as, microdensitometer and vidicon cameras **[1]**. Then digital image is an image f(x,y) that has been discretized spatial plane and in brightness, we have a tendency to considering a digital image as a matrix whose row and column indices define as point within the image and corresponding matrix component mention as the grey level at that point **[1]**. A digital image "f(x,y)" can be represented in matrix form **[1]**:

$$f(x,y) \approx \begin{bmatrix} f(0.0) & f(0.1) & \dots & f(0.N-1) \\ f(1.0) & f(1.1) & \dots & f(1.N-1) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ f(M-1.0) & f(M-1.1) & \dots & f(M-1.N-1) \end{bmatrix}_{M\times N} \quad (1.4)$$

Every component of matrix array is termed as image component, element, pixel, M is rows and N is columns of matrix **[1]**.

Pictures is categorized consistent with their supply **[6]**: Gamma-Ray Imaging, X-ray Imaging, Imaging within the Ultraviolet Band, Imaging within the Infrared Bands and visual Bands, Imaging within the Radio Band, Imaging within the Microwave Band. An example on how picture acquisition, radio waves are used in magnetic resonance imaging (MRI). This technique places a patient in a powerful magnet and passes radio waves through his or her body in short pulses. Each pulse causes a responding pulse of radio waves to be emitted by the patient’s tissues. The location from which these signals originate and their strength are determined by a computer, which produces a two-dimensional picture of a section of the patient.

1.3 Statistical Properties of Images

It is a quite common in digital image process to use statistic description of image by many parameters as follows **[7]** .

1- Probability density function (pdf)

The probability that a brightness in a region falls between *a* and *a*+Δ*a*, by given the probability distribution function " P(*a*)", can be expressed as *p*(*a*) Δ*a* where *p*(*a*) is the probability density function **[7]**:

$$p(a)\ \Delta a = \left(\frac{dP(a)}{da}\right)\ \Delta a \qquad (1.5)$$

2- Average

The average brightness of a region " m_a" is defined as the sample mean of the pixel brightnesses within that region. The average, m_a, of the brightnesses over the R pixels within a region (R) is given by **[7]**:

$$m_a = \frac{1}{M\times N}\sum_{(M,N)\in R} a[M,N] \qquad (1.6)$$

3- Standard deviation

In terms of image process it shows what quantity variation or "dispersion" exists from the average (mean, or expected value). Low standard deviation indicates that the information points tend to be terribly near the mean, whereas high standard deviation indicates that the information points are spread out an oversized vary of values **[8]**.

Mathematically standard deviation "STD" is given by **[7]**:

$$STD = \sqrt{\frac{1}{\Lambda-1}\sum_{M,N\in R}(a[M,N]-m_a)^2} \qquad (1.7)$$

4- Signal to Noise Ratio (SNR)

Signal to noise ratio describe the impact of the noise on the image, given by **[9]**:

$$\text{SNR} = \sqrt{\frac{\sigma_t^2}{\sigma_r^2} - 1} \tag{1.8}$$

Where σ_t and σ_r are the variances of the true image and the recorded image, respectively.

5- Correlation

Correlation of two functions f(x) and X(x) is mentioned as the integral of the multiplicative of $\hat{U}^*$ (x) with $Đ(x + \Delta x)$, the latter shifted over far Δx **[10]**.

$$C_{\hat{U}Đ}(\Delta x) = \int_{-\infty}^{\infty} \hat{U}^*(x)\, Đ(x + \Delta x) dx \tag{1.9}$$

Where $C_{\hat{U}Đ}$ is correlation of two functions, $\hat{U}^*$ and $Đ(x + \Delta x)$ are arbitrary function.

The superscripted asterisk implies that the complex conjugated worth is taken. Thus for every shift Δx the correlation is calculated. This implies that if a structure moves as a full besides random and offset correlations, a correlation going to be found at the shift corresponding to the translation **[10]**.

6- Covariance

In statistics, covariance could be defined as a measure of how much two random variables amendment along. If the larger values of one variable primarily correspond with the larger values of the opposite variable, and also the same holds for the smaller values, i.e. the variables tend to indicate similar behavior, the covariance could be a positive range. Within the opposite case, once the larger values of one variable primarily correspond to the smaller values of the other, i.e. the variables tend to indicate opposite behavior, the covariance is negative.

Covariance sign leanings appears in the linear relationship between the variables. Mathematically covariance between two variable x_i and y_i is given as **[8]**.

$$\mathrm{cov}(x,y) = \frac{1}{N}\sum(x_i - \bar{x})(y_i - \bar{y}) \qquad (1.10)$$

x_i and y_i are random variables, $\bar{x}$ and $\bar{y}$ is mean random variable of x_i and y_i respectively**.** $\mathrm{cov}(x,y)$ is covariance

7- Root Mean Square Error (RMSE)

It is one of the criteria to measure the image quality. The error between an input f(x,y) and the output g(x,y) is given by **[11]**:

$$e(x,y)= g(x,y) - f(x,y), \qquad (1.11)$$

The squared error averaged over the image array "e^2" is given by **[11]**:

$$e^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} e^2(x,y) \qquad (1.12)$$

Then one get,

$$e^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [g(x,y) - f(x,y)]^2 \qquad (1.13)$$

Then the Root Mean Square error " RMSE " of the image is **[11]**:

$$e_{rms} = [\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [g(x,y) - f(x,y)]^2]^{1/2} \qquad (1.14)$$

1.4 Literature Review

Digital computer techniques in image restoration and enhancement had their first fruitful application at the JPL of the California Institute of Technology in 1960. As part of the program to land a man on moon, it was decided to land unmanned spacecraft initially, which would television back images of the moon's surface and test the soil for later manned landing, unfortunately, the limitation on weight and power supply made it impossible to launch a "perfect" TV camera system on the unmanned craft. Consequently, JPL measure the degradation properties of the cameras before they were launch and then used computer processing to remove as well as possible, the degradation from the received moon image **[5]**.

In 1977, **Hsieh and Harry [12]** were studied Iterative methods for both unconstrained and constrained solutions to the normal equation and presents a theoretical analysis and computational technique for constrained least squares image restoration using spline basis functions.

In 1988, **Reginald L.Lanfendijk and Jan Biemind and Dick E.Boekee [13]** were proposed a regularized iterative image restoration algorithm, in which ringing reduction methods are included by making use of the theory of the projections onto convex sets and the concept of norms in a weighted Hilbert space.

In 1991, **Aggelos K.Katsaggelos and Jan Biemond and Ronald W.Schafer and Russell M.Mersereau [14]** were proposed adaptive and ono-adaptive algorithm based on wiener algorithm.

In 1992, **Christakis Charalambous and Farah Kamel Ghaddar and Kypros Kouris [15]** were present two methods for the recovering of Nuclear Medicine images that has been degraded while being processed. The restoration problem is formulated as a constrained optimization problem .The first algorithm reduces the problem to the computation of few discrete Fourier transforms and has the ability to control the degree of sharpness and smoothness of the restored image where the input parameter can be interactively chosen by the observer. The second algorithm with weight matrices included enables the handling of edges and flat regions in the image in a pleasing manner for the human visual system. In this case the iterative conjugate gradient method is used in conjunction with the discrete Fourier transform to minimize the Lagrangian function.

In 1994, **Byung-Eul Jun and Dong-Jo Park [16]** were develop A new steepest descent least mean square adaptive filter algorithm.

In 1995, **Ayad A. Al-ani [17]** was adapted Wiener filter and the maximum entropy method to restore optical astronomical images.

In 1997, **Ebtesam Fadhl [18]** was adopting wiener and homomorphic as non-iterative restoration method , and an iterative restoration method based on the least –square criterion.

In 2000, **Michael K. Ng and Robert J. Plemmons and Felipe Pimentel [19]** Were proposed A new approach of blind deconvolution, such as Constrained total least squares image deconvolution algorithm with Neumann boundary conditions.

In 2006, **Ayad A. Al-ani [20]** was adapted in many image restorations an iterative Wiener filter. To estimate the power spectral density of the original image from degraded image using an iterative method. The adapted filter was designed for restoring astronomical images that are blurred with space-invariant point spread function and corrupted with additive noise .The result using an adaptive filter were compared, quantitatively, using mean square error (MSE). His result shows that this method has better performance for restoring the degraded images, especially for high signal to noise ratio.

In 2006, **Michael R. Charest Jr and Michael Elad and Peyman Milanfar [21]** were proposed methods for image denoising such as, Osher et al.'s Method, iterative regularization method, Iterative ”Twicing” Regularization Method and Iterative Unsharp Regularization Method, with using the Bilateral Filter and Total Variation Filter.

In 2008, **Jun Ma [22]** was formulated a new approach to medical image reconstruction from projections in emission tomography, Similar to the Richardson-Lucy algorithm.

In 2008, **Mohammed Khudier [23]** was adopt different types of restoration filters

Such as, Inverse filter, Wiener filter, Constrained Least-Squares Filter and terative restoration.

In 2011, E**. Loli Piccolomini *and* F. Zama [24]** were proposed an iterative algorithm based on Constrained Least Squares Regularization Algorithm to restore the image.

In 2013, **Amudha.J and Sudhakar.R and Ramya.M [25]** were proposed three restoration algorithms namely, Local Polynomial Approximation Intersection of Confidence Interval rule and Sparse Prior Deconvolution Algorithm and Richardson-Lucy Deconvolution Algorithm.

In 2013, **P. Sureka and G. Sobiyaraj and R. Suganya and T.N.Prabhu [26]** were proposed iterative image restoration such as wiener filter to restore the degraded face images, which improved the recognition performance and the quality of the images.

In 2014, **Fernando Pazos and Amit Bhaya [27]** were adapted Iterative gradient descent algorithms such as Barzilai-Borwein Algorithm and the Conjugate Gradient Algorithm to restore the images.

In 2015, **Aswathi V M and James Mathew [28]** were studied and compared various image restoration techniques of medical images, such as iterative restoration algorithms, etc.

1.5 Aim of this research

The goal of this work to improve the quality of the degraded images by using an iterative linear restoration filter such as Tikhonov-Miller regularized restoration filter using a prior information about the degradation phenomena, i.e. type blurring function and the noise, when we stopping the iteration and effect of the regularize parameter on the result. Compared the obtained results from Tikhonov-Miller filter and another non iterative filter such as Wiener filter by Root Mean Square Error (RMSE) quantitative test measure.

1.6 Thesis Layout

The thesis is organized as follows:

Chapter two define image restoration and classify image restoration according to blurring operator is unknown or known: Blind image restoration and Non-Blind image restoration. Then shows some type of degradation such as degradation in the case of blurring only and degradation in the case of noise only, and their sources. Then define some important operator and matrices.

Chapter three, show what is the image restoration filters effective it's depending, and image restoration algorithm will be presented such as; unconstrained or a constrained algorithm.

Chapter four, discuss the achieved results from our work and conclude some information from it. Also, suggested for future works

Figure(1-2) represented the structure of the thesis including all chapters and what it's contained.

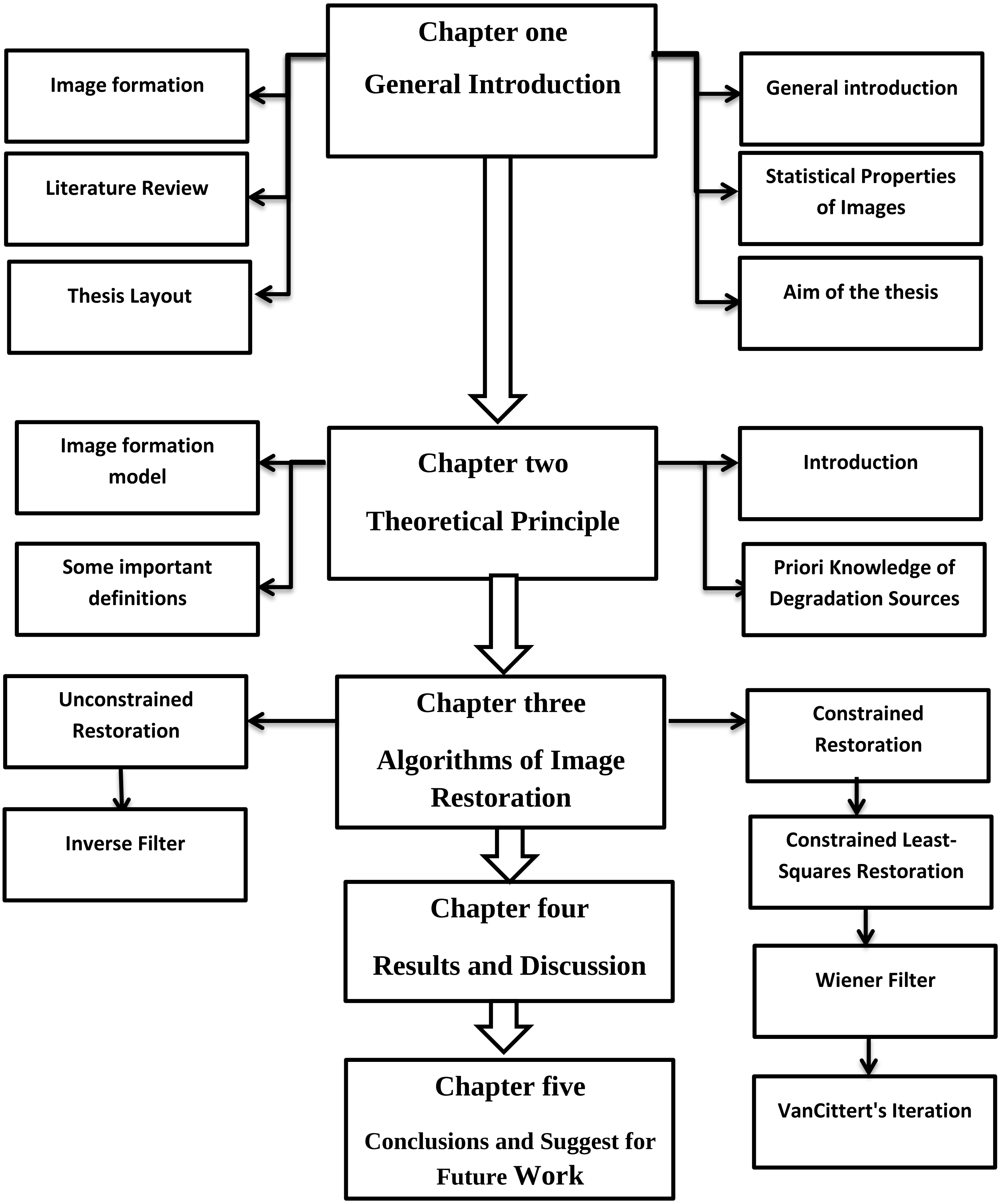


Figure (1-2) Block diagram describing the structure of the thesis

CHAPTER TWO

Theoretical fundamental

2.1 Background:

Image restoration is mention as improving the image quality that is degraded by blurring and noise function. This degradation is as a result of numerous defects that injury the standard of a picture **[29]**.

The restoration of the image is divided into two phase process section called degradation phase and restoration phase, mentioned every phase below **[30]**:

1- Degradation phase

During this phase, the original image is degraded with blurring and the extra noise. The resultant image of this section is called a degraded image.

2-Restoration phase

At this phase, the restoration of degraded image using many filters, and an estimated image of the original image is produced as an output.

Image Restoration kind are often can be divided into two kinds: Blind and Non-Blind. Blind restoration, the one within which operation of blurred unknown factor, therefor it create an estimate of the blurring operator and so using that estimate we have to deblur the image. Non-Blind restoration is that the one within which the blurring operator is noted, then try remove blur from the degraded image using the noted of blurring function.

2.2 Mathematical Model of Degradation:

The degradation method model consists of two portion, the blurring perform (or function) and also the noise function. In the case of additive noise, the degradation model in spatial domain is given by **[1]**:

$$g(x,y) = H(x,y) \circledast f(x,y) + n(x,y) \quad (2.1)$$

Where g(x,y) is degraded image, f(x,y) is original image, n(x,y) is additive noise, H(x,y) is circulant matrix.

And the model in the Frequency domain is given by:

$$G(u,v) = H(u,v) \bullet F(u,v) + N(u,v) \quad (2.2)$$

For: u,v=0,1,2,3,4………,N-1, and (u,v) is an spatial frequency coordinates, G(u,v), H(u,v), F(u,v) and N(u,v) is Fourier transform of g(x,y), H(x,y), f(x,y), n(x,y), respectively.

2.3 Priori Knowledge of Degradation Sources:

A picture may be a signal carrying data about physical target that is not directly noticeable. Generally the data consists of a degraded illustration of the original target, one will roughly distinguish two sources of degradation: the method (or process) of image formation and also the method of image recording **[31]**. The degradation result from method of image formation is sometimes denoted by blurring and may be a type of band limiting of target, while the degradation caused by recording method is sometimes denoted by noise, which is because of activity error, count errors, etc.

2.3.1. Degradation in case of blurring only

As a result of diffraction limitation, the image of a point target (or some extent supply is not any longer point). It is a patch of light intensity known as the point spread function PSF **[11]**.

Then the blurring model given as **[1]**:

$$g\,(x,y) = H(x,y)\, f\,(x,y) \tag{2.3}$$

Where g (x,y) is blurred image. H is said to be a linear operator if **[32]**:

$$H\,[a_i\, f_i(x,y) + a_j\, f_j(x,y)] = a_i\, H\,[f_i(x,y)] + a_j\, H\left[f_j(x,y)\right]$$

Where a_i, a_j are arbitrary constants and $f_i(x,y)$, $f_j(x,y)$ images (of the same size), respectively. The above equation indicates that the output of a linear operation due to the sum of two inputs is the same as acting of sum of individual inputs **[32]**. Point spread function could be define as a two dimension impulse response produced from a point source of light passing through the degradation system in the absence of noise **[33]**.

Blur model can be categorized into two types **[17]**:

i-Space-Invariant Point Spread Function (SIPSF)

ii-Space-Variant Point Spread Function (SVPSF)

i-Space-Invariant Point Spread Function (SIPSF)

Point spread function changes solely the position of the input however simply changes the placement of the output with keeping identical perform, This characteristic seems within the linear system **[17]**.

ii-Space-Variant Point Spread Function (SVPSF) this kind changes shape and position, i.e. the PSF depends on the location of the target, this property related to nonlinear system **[17]**.

We will conjointly classify the blur sorts as follows **[23]**:

1- Liner motion blur 2- Defect of optical system 3-Inhomogeneous optical media.

1- Liner motion blur

The best example of blurring is that result from relative motion, throughout exposure between the camera and target. Suppose $f^{(0)}(m)$ be illuminance from target, which might result in the image plane of camera within the absence of relative motion between the camera and target. The blurring is space-invariant solely within the case where the target and image planes area unit parallel, and also the motion is translation. If we tend to assume that these conditions are applied, then the translation motion is delineate by a time-dependent vector **[31]**:

$$\mathrm{Y(t)} = \{\mathrm{Y_1(t)\,.\,Y_2(t)}\}, \qquad 0 \le \mathrm{t} \le \mathrm{T}$$

Where $\mathrm{Y_1}$, $\mathrm{Y_2}$ are time-dependent vectors and T is the exposure interval.

This vector defines the trail, with regard to a fixed coordinated system within the image plane that is delineate by the origin of a coordinated mounted with regard to the moving target. One can assume that, at time equal to 0, the origins of two systems coincide. Then the flight delineate by an arbitrary point $\acute{Y}$ of the moving target is given by $\acute{Y}$+Y(t), in order that , if m may be a given point within the mounted fixed image plane, this point is reached at time t, by the point of moving target that, at time equal to zero, is given by $\acute{\mathrm{Y}} = \mathrm{m} - \mathrm{Y(t)}$. If the noise-free image "$g^{(0)}(m)$" at the point m is the result of the addition of the contribution all points $\acute{Y} = m - Y(t)$ passing through m in the interval [0,T], i.e. **[31]**:

$$g^{(0)}(m) = \frac{1}{T} \int_0^T f^{(0)}[(m - Y(t)]dt \quad (2.4)$$

The factor $(1/T)$ is introduced as a normalization factor.

2- defect of optical system

The medium through that the waves should propagate whereas passing from the target to the imaging system is itself optically imperfectness. Since the particular resolution that a faraway from than the theoretical optical phenomenon limit that obtained by Rayleigh criterion given by **[23]**:

$$\Delta\alpha = 1.22 \frac{\lambda}{D_a} \quad (2.5)$$

Where $\Delta\alpha$ is resolving power and λ is the wavelength of light, D_a is the diameter of the lens aperture.

Another defect of optical system is aberration. One of aberration type is out-of-focus, within the case of a lens associate point of target is focused If the distance from the lens, d_0, satisfy the lens conjugation law **[31]**:

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{d_f} \quad (2.6)$$

Wherever d_i is that the distance between the lens and the image plane and d_f is that the focal length of lens. If condition of eq.(2.6) is not satisfied, then the image of the point, forever in line with geometrical optics, is a disc, known as the circle of confusion (COC). The radius of COC may depend on the wavelength through the refractive index of the lens, operate of $d_0, d_{i,} d_f$ and of effective lens diameter. Finally, as issues the intensity distribution among the COC it follows from geometrical optics that it is around uniform over COC so the image of an out-of-focus point, situated on the optical axis, is given by:

$$H(x) = \frac{1}{\pi\ R^2{}_{COC}}\ X_{coc}(x) \qquad (2.7)$$

Where H(x) is point spread function, R_{COC} is the radius of COC and X_{coc} is the characteristic function of the COC.

3- Inhomogeneous optical media

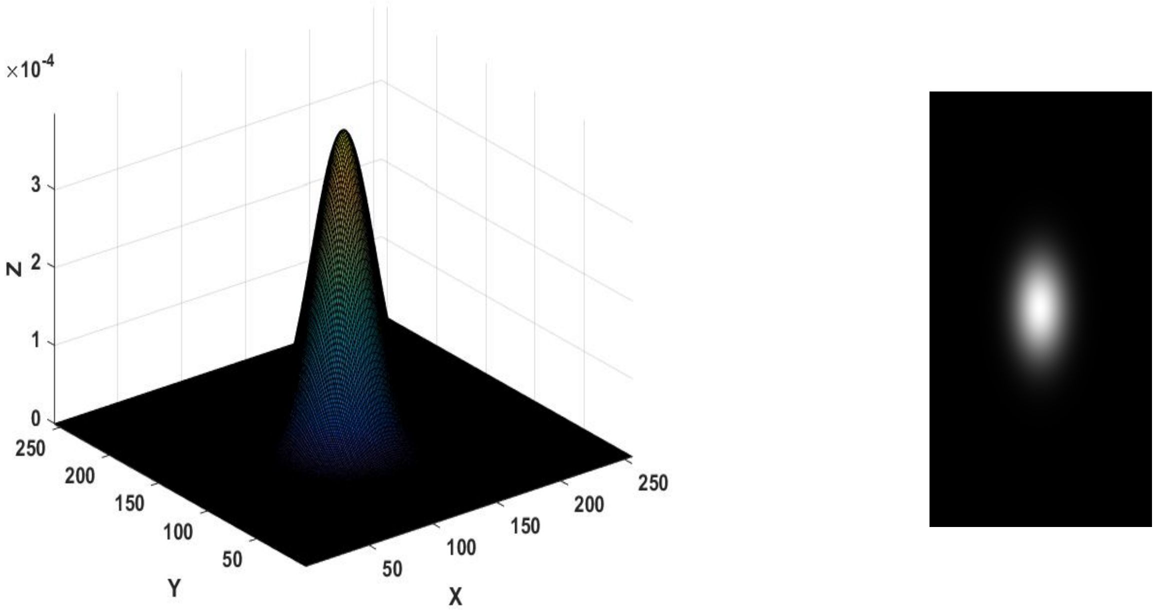
Atmospheric region turbulence is due to random variation within the index of refraction of medium between the target and imaging system. This means, the wave-front transmission of point source origin through a turbulent medium (e.g. the planet atmosphere) causes turbulence in phase and degrades the quality of the shaped image. The blur may be describe by Gaussian function **[34]**:

$$H(x,y) = \frac{1}{\sqrt{2\pi\sigma_b}} e^{-(x^2+y^2)/2\sigma^2} \qquad (2.8)$$

Where σ_b is standard deviation of the Gaussian function.

Although, the blur introduced by region turbulence depends on many parameter such as temperature, wind speed, exposure time **[35]**.

Figure (2-1) shows perspective a plot and image display of Gaussian blurring function **[6]**



(a) (b)

Figure (2.1) represent (a) Perspective a plot of Gaussian blurring function (b) Gaussian blurring function displayed as an image

2.3.2 Degradation in the case of noise only

The noise embedded in a picture manifests in several types. These types may uncorrelated or correlated, also it may signal dependent or freelance. The information regarding to the imaging system and visual perception of The image assists in generation of noise model and the estimating of the applied statistical characteristics of noise embedded in a picture is a crucial as a result of it helps in removing the noise from the beneficial image signal **[36]**.

In the case of additive noise, the noise model equation is given by **[37]**:

$$g(x,y) = f(x,y) + n(x,y) \tag{2.9}$$

Where g(x,y) is noisy image.

There is four important classes of noise **[36]**:

1- Additive noise

Additive noise is a sort of distortion that happens within the image. And it is caused by dispersion within the atmosphere caused by the scattering of electromagnetic waves in a distinct directions and that it due to the presence of minutes of dust or impurities with comparatively tiny diameters within the air and imaging system, inflicting a block light-weight of sunshine in an area within the image and a rise in light in different areas. The mathematical form of the Gaussian noise is given by **[38]**.

$$n_G(b,s) = \frac{1}{\sqrt{2\pi \quad \sigma_n{}^2}} e^{-(b^2+s^2)/2\sigma^2} \tag{2.10}$$

Where n_G (b,s) is Gaussian noise, b is the distance from the center to the vertical axis and s is the distance from the center to the horizontal axis, σ_n represents the standard deviation of Gaussian noise.

2. Multiplicative noise

The granularity noise from photographic plates is actually multiplicative in nature. The spots noise from imaging system as ultrasound imaging etc. is multiplicative in nature, which can be modeled as **[36]**

$$g(x, y) = f(x, y) * n_m(x, y) \tag{2.11}$$

Where $n_m (x, y)$ is the multiplicative noise.

3. Impulse noise

Very often the sensors generate a loud noise pulsed. generally noise generated from a digital (or analog) image transmission system is impulsive in nature, which may be modeled as **[36]**.

$$g(x, y) = (1 - N)f(x, y) + N.q(x, y) \tag{2.12}$$

where $q(x, y)$ is the impulsive noise and N is a binary parameter that assumes the values of either 0 or 1. Impulse noise is also simply detected from the distorted image due to the distinction anomalies. Once the noise impulses are detected, these are replaced by the signal samples **[36]**.

4. Quantization noise

The quantization noise is actually a signal dependent noise. This noise is characterized by the size of signal quantization interval Such noise produces image-like artifacts and will turn out false contours round the target. The quantization noise additionally removes the image details that are of low-contrast **[36]**.

2.4 Some important definitions

2.4.1 Fourier transform

An essential to transform the digital image to organize its information. So we will define Some of transformations kind such as Fourier transform, possibly, the most powerful tool in signal analysis **[4]**.

Fourier series can represent light composed of a discontinuous set of wavelengths λ/n_n, n_n = 1, 2, ⋯ , That is, a discontinuous set of colors. Complex Fourier series L(x) formulas is given by **[39]**:

$$\mathrm{L(x)} = \sum_{-\infty}^{\infty} \mathrm{c_d} \mathrm{e}^{i n_n \pi \mathrm{x/l}}$$

$$\mathrm{c_d} = \frac{1}{2\mathrm{l}} \int_{-\mathrm{l}}^{\mathrm{l}} \mathrm{L(x)}\, \mathrm{e}^{-i n_n \pi \mathrm{x/l}} \quad \mathrm{dx} \tag{2.13}$$

Where x is coordinates of particle, l is period, $\mathrm{c_d}$ is Fourier series coefficients.

Fourier transform extend or modify Fourier series to cover the case of a continuous spectrum of wavelengths of light containing a continuous set of frequencies . An integral part is to limit sum, which is replace the Fourier series (any amount of terms) by Fourier integrated. Then the Fourier integration can be used to represent the functions of non-rotating, for example, a flash of light, or voltage pulse and one not to be repeated, or sound that does not repeat. It represents an integral part of Fourier also a continuous range of frequencies, for example, a full range of Light colors rather than a discontinuous group. the formulas corresponding to eq.(2.9) for a continuous range of frequencies is given by **[39]**.

$$L(x) = \int_{-\infty}^{\infty} j(\in) e^{i\propto x}\, d\in$$

(2.14)

$$j\,(\in) = \frac{1}{2\pi} \int_{-\infty}^{\infty} L(x) e^{-i\in x}\, dx$$

Compare eq.(2.13) and eq.(2.14); j(∈) corresponds to c_d, ∈ corresponds to d and $\int_{-\infty}^{\infty}$ coresponds to $\sum_{-\infty}^{\infty}$. The two functions L(x) and j(∈) are called a pair of Fourier transforms. Usually, j(∈) is called the Fourier transform of L(x), and L(x) is called the inverse Fourier transform of j(∈).

2.4.2 Toeplitz and Circulant matrices

Toeplitz matrix S is a matrix containing the constant elements along the diagonal diameter and the sub diagonals. This means the elements S(M, N) depend only on the difference (M – N), i.e., S(M, N) $= S_{M-N}$. Thus N X N Toepliz matrix is of the form **[3]**:

$$S = \begin{bmatrix} S_0 & S_{-1} & \cdots & \cdot & S_{-N+1} \\ S_1 & S_0 & S_{-1} & \cdot & S_{-N+2} \\ S_2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & S_{-1} \\ S_{N-1} & \cdots & S_2 & S_1 & S_0 \end{bmatrix} \quad (2.15)$$

And is completely defined by the (2N-1) elements. Toeplitz Matrices describing inputs and outputs transformation of one-dimensional linear shift systems and correlation matrices of stationary sequences.

The so-called matrix z circulant if both it is rows (or columns) is a circular shift of the previous row (or column), i.e:

$$z = \begin{bmatrix} z_0 & z_1 & z_2 & \cdots & z_{N-1} \\ z_{N-1} & z_0 & z_1 & . & z_{N-2} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ z_2 & . & . & . & z_1 \\ z_1 & z_2 & \cdots & z_{N-1} & z_0 \end{bmatrix} \qquad (2.16)$$

Circulate matrices describing the behavior of the input and output of one dimensional linear periodic systems and correlation matrices of periodic sequences.

For large matrix, Circulants are good approximation to Toeplitz forms, the Fourier series signal processing become immediately applicable **[5]**.

2.4.2 Convolution

The mathematical description of the convolution process between function h(x,y) and function f(x,y) is given by **[3]**:

$$g(x,y) = h(x,y) \otimes f(x,y) \triangleq \int_{-\infty}^{\infty} h(x - \grave{x},\ y - \grave{y})\, f(\grave{x}, \grave{y}) dx\, dy \qquad (2.17)$$

While the Discrete representation of Eq.(2.17) is given by:

$$g(x,y) = h(x,y) \otimes f(x,y) \triangleq \sum\sum_{\grave{y},\grave{x}=-\infty}^{\infty} h(x - \grave{x},\ y - \grave{y})\, f(\grave{x}, \grave{y}) \qquad (2.18)$$

The Fourier transform of Eq.(2.18) is given by:

$$G(u,v) = H(u,v)\ F(u,v) \tag{2.19}$$

This means that the Fourier transform of convolution process in spatial domain leads to a simplest multiplication in frequency domain.

CHAPTER THREE

Algorithms of Image Restoration

3.1 Background:

Any image acquired by optical or electronic means is likely to be degraded by the sensing environment **[3]**. Image restoration algorithms trying to"undo" the blurring function and the noise from the degraded image by deconvolving the blurring function and reducing the noise from the degraded image to produce an estimate image, which is approach to the original image **[40]**. The effectiveness of image restoration filter depends on extent and the accuracy of the knowledge of degradation process, and it also depends on the filter design criterion . One of criterion usually used is Root Mean Square Error and maximum entropy criterion sometimes used **[3]**.

3.2 Algebric Approach to Restoration

The aim of image restoration is to estimate an original image, given a degraded image and some knowledge or assumption about blurring and noise function. Central to the algebraic approach is the concept of seeking an estimate of original image, which minimizes a predefined criterion of performance. Because of simplicity, the approach attention on least-squares criterion function. The well-known restoration methods are result of considering either an unconstrained or a constrained approach to least-square restoration problem **[1]**. According to the affected of blurring function (PSF), whether its space invariant or space variant image restoration can be classified into **[23]**:

I- Linear restoration: associated with space invariant PSF

II- Non-Linear restoration: associated with space variant PSF

The Linear restoration techniques, in turn, can be classified into the following **[23]**:

I- Unconstrained linear restoration techniques

II- Constrained linear restoration techniques

The constrained techniques are classified into **[23]**:

I- Direct solution: also known as, linear, non-iterative, or one shot solution.

II- Indirect solution: also known as iterative solution.

3.2.1 Unconstrained Restoration

The noise term of the degradation model is given by **[1]**:

$$n = g - Hf \qquad (3.1)$$

In the absence of any information about n, a meaningful criterion function is to seek an estimate of original image, denoted by $\hat{f}$ such that H $\hat{f}$ approximates g in a least-squares sense by assuming that the norm of the noise $\|n\|^2$, is as small as possible. In other word , find $\hat{f}$ such that:

$$\|n\|^2 = \left\|g - H\,\hat{f}\,\right\|^2 \qquad (3.2)$$

Where $\hat{f}$ is restored image.

$\hat{f}$ is minimum, where, by definition, $\|n\|^2 = \acute{n}n$ and $\left\|g - H\,\hat{f}\,\right\|^2$=$(g - H\,\hat{f})'$ $(g - H\,\hat{f})$ are squared norms of n and $(g - H\,\hat{f})$, respectively. From Eq.(3.2), it could be equivalent view this problem as one of minimizing the criterion function

$$J(\hat{f}) = \left\|g - H\,\hat{f}\,\right\|^2 \qquad (3.3)$$

Where J is a criterion function, a said from the requirement that it minimize Eq.(3.3), $\hat{f}$ is not constrained in any way.

Minimization of Eq.(3.3), is straightforward. Simply by differentiate J with respect to $\hat{f}$, and set the result equal to the zero vector; that is:

$$\frac{\partial J(\hat{f})}{\partial \hat{f}} = 0 = -2\acute{H}(g - H\hat{f}) \qquad (3.4)$$

Where $\acute{H}$ is transpose of H.

Solving Eq.(3.4) for $\hat{f}$ yields:

$$\hat{f} = (\acute{H}\,H)^{-1}\,\acute{H}\,g \qquad (3.5)$$

By letting M = N so that H is square matrix, and assuming that H^{-1} exists, Eq.(3.5) Reduce to:

$$\hat{f} = H^{-1}(\acute{H})^{-1}\acute{H}g \qquad (3.6)$$

$$= H^{-1}g$$

3.2.2 Invers Filtering

Image restoration techniques by considering the unconstrained result given in eq.(3.6), can be derive by using elementary matrix theory, in which, H may be expressed in the form **[1]**,

$$H = W\,D\,W^{-1} \qquad (3.7)$$

$$\acute{H} = W\,D^{*}\,W^{-1} \qquad (3.8)$$

Where D is a diagonal matrix of H, D^* is the complex conjugate of D, W is eigenvector of H, W^{-1} is the inverse of W

Eq.(3.7) and Eq.(3.8) shows the Fourier transform of the block elements of H and Ḧ. By substitution Eq.(3.7) in Eq.(3.6) we get:

$$\hat{f} = H^{-1}\, g$$

$$= (W\, D\, W^{-1})^{-1}\, g$$

$$= W\, D^{-1}\, W^{-1} g \qquad (3.9)$$

Premultiplying both sides of eq.(3.9) by W^{-1} yields:

$$W^{-1}\, \hat{f} = D^{-1}\, W^{-1} g \qquad (3.10)$$

Where $W^{-1}\ \hat{f}$ and $W^{-1}g$ is recognize as discrete Fourier transform of $\hat{f}$ and g. Then Eq.(3.10) can be written in the form:

$$\hat{F}(u, v) = \frac{G(u,v)}{H(u,v)} \qquad (3.11)$$

This relation is the formal solution to the Inverse filtering problem. The reconstructed signal f is determined by it is Fourier transform $\hat{F}$, which in turn is obtained from the system function H and the Fourier transform G of degraded signal. The system function of the inverse filter is, thus, **[4]**:

$$H_{inv} = 1/H \tag{3.12}$$

Since the effect of the blur is described by a convolution, the inverse process, the reconstruction of signal before blurring is called a deconvolution. Unfortunately, the above direct method turns out to be useless in practice. Not unexpectedly, the reason turns out to be noise and incomplete knowledge of the system function H.

A more serious difficulty arise in the present of noise, multiplying both sides of Eq.(2.1) by W^{-1} and using Eq.(3.7) we get **[1]**.

$$W^{-1}\, g = D\, W^{-1}\, \hat{f} + W^{-1}\, n \tag{3.13}$$

Then Eq.(3.13) can be written in the form:

$$\hat{F} = \frac{G(v,v)}{H(u,v)} - \frac{N(u,v)}{H(u,v)} \tag{3.14}$$

In general, the noise may very well possess large component at high frequencies (u,v), while g and h normally will be dominated by low-frequency components. For large values, we have G(u,v) = 0 and H(u,v) = 0. The latter relation have, as evidenced by Eq.(3.14), devastating effect.

Since $F(u,v) \approx -N(u,v) / H(u,v)$, Where both the numerator is large and the denominator small (in absolute term). This qualitative argument shows that the noise will be amplified considerably in the inverse filtering, and the directly reconstructed signal will be worthless. The same conclusion is obtained by noting that, since (low pass) filtering remove noise, inverse filtering must add noise. As is evident, an incompletely known function H(u,v) may give rise to the same effect, since small, erroneous, values of H(u,v) will imply large value of F(u,v) **[4]**.

3.2.3 Constrained Restoration

Let Q be a linear operator on $\hat{f}$. Consider the least-squares restoration problem as one of minimizing functions of the form $\left\|Q\hat{f}\right\|^2$, subject to the constraint $\left\|g - H\hat{f}\right\|^2 = \|n\|^2$. This approach introduces considerable flexibility in the restoration process because it produces different solutions for different choices of Q **[1]**.

The addition of an equality constraint in the minimization problem can be handled without difficulty by using the method of Lagrange multipliers The procedure is to express the constraint in the form $\alpha(\left\|g - H\hat{f}\right\|^2 - \|n\|^2)$ and then append it to the function $\left\|Q\hat{f}\right\|^2$. In other words, seek $\hat{f}$ that minimizes the criterion function.

$$J(\hat{f}) = \left\|Q\hat{f}\right\|^2 + \alpha(\left\|g - H\hat{f}\right\|^2 = \|n\|^2) \qquad (3.15)$$

where α is a constant called the Lagrange multiplier and Q linear operator. Once the constraint has been appended, minimization is carried out in the usual way. Differentiating Eq.(3.15) with respect to $\hat{f}$ and setting the result equal to the zero vector yields:

$$\frac{\partial J(\hat{f})}{\partial \hat{f}} = 0 = 2\,\acute{Q}\,Q\,\hat{f} - 2\alpha\,\acute{H}(g - H\,\hat{f}) \qquad (3.16)$$

The solution is obtained by solving eq.(3.16) for $\hat{f}$;that is:

$$\hat{f} = (\acute{H}\, H + \gamma\, \acute{Q}Q)^{-1}\acute{H}g \tag{3.17}$$

Where $\gamma = \frac{1}{\alpha}$. This quantity must be adjusted so that the constraint is satisfied.

3.2.4 Least-Mean-Square (Wiener) Filter

Let R_f and R_n as denoted the correlation matrices of f and n as, respectively, and defined by **[1]**:

$$R_f = K\{f\,\hat{f}\} \tag{3.18}$$

$$R_n = K\{n\,\acute{n}\}, \tag{3.19}$$

Where $K\{.\}$ is denotes the expected value operation. Assuming that the correlation between any two pixels is a function of the distance between the pixels and not their position, Using ρ and τ to define, R_f and R_n respectively, then we have:

$$R_f = W\,\rho\,W^{-1} \tag{3.20}$$

$$R_n = W\,\tau\,W^{-1} \tag{3.21}$$

Where ρ and τ are diagonal matrix of R_f and R_n respectively. the Fourier transform these correlations is called the power spectrum (or spectral density) of f(x, y) and n(x, y), respectively, and will be denoted in the following discussion by S_f (u, v) and S_n(u,v), by supposing that:

$$\acute{Q}Q = {R_f}^{-1}R_n \tag{3.22}$$

R_f and R_n are correlation matrices of f and n respectively.

and substituting eq.(3.22) in eq. (3.17) we obtain:

$$\hat{f} = (\acute{H}\, H + \gamma {R_f}^{-1}R_n)^{-1}\acute{H}\, g \tag{3.23}$$

Using eq.(3. 7), eq. (3. 8), eq. (3.20) and eq. (3.21) yields in Eq.(3.23):

$$\hat{f} = (WD^{*}DW^{-1} + \gamma W\, \rho^{-1}\tau\, W^{-1})^{-1}\, W\, D\, ^{*}W^{-1}g \tag{3.24}$$

After multiplying both sides by W^{-1} and some matrix manipulations this equation reduce to the form:

$$W^{-1}\acute{f} = (D^{*}D + \gamma\, \rho^{-1}\tau\,)^{-1}D^{*}W^{-1}g \tag{3.25}$$

Then Eq.(3.25) can be written in the form:

$$\hat{F} = \left[\frac{H^{*}(u,v)}{|H(u,v)|^{2} + \gamma\ \ [S_n(u,v)/S_f(u,v)]}\right] G(u,v) \tag{3.26}$$

When $\gamma = 1$, the term inside the outer brackets in Eq. (3.26) reduces to so-called Wiener filter. If γ is variable we refer to this expression as the parametric Wiener filter. In the absence of noise, $S_n(u, v) = 0$, Wiener filter reduce to ideal inverse filter. When $S_f(u, v)$ and $S_n(u, v)$ are not known it is sometimes useful to approximate eq.(3.26) by the relation followed:

$$\hat{F} \approx \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma\ \tilde{\sigma}}\right] G(u, v) \tag{3.28}$$

Where $\tilde{\sigma}$ is constant.

3.2.5 Constrained Least-Squares Restoration

Constrain least squares suggested by phillips **[1]**, to formulate a criterion of optimality based on a measure of smoothness such as, for example, minimizing some function of second derivative.

Given a discrete function f(x), x = 0, l , 2, ... , it may approximate its second derivative at a point x by the expression **[1]**:

$$\frac{\partial^2 f(x)}{\partial x^2} \approx f(x+1) - 2f(x) + f(x-1) \tag{3.29}$$

A criterion based on this expression, then, might be to minimize $[\partial^2 f(x)/\partial x^2]^2$ over x; that is **[1]**:

Minimizing $\{ \sum_x [f(x+1) - 2f(x) + f(x-1)]^2 \}$

or in a matrix notation,

$$\ddot{\omega} = \begin{bmatrix} 1 & & & & & & \\ -2 & 1 & & & & & \\ 1 & -2 & 1 & & & & \\ & 1 & -2 & & 1 & & \\ & & & \ddots & & & \\ & & & & 1 & -2 & 1 \\ & & & & & 1 & -2 \\ & & & & & & 1 \end{bmatrix} \tag{3.30}$$

Where $\ddot{\omega}$ is smoothing matrix.

In the case of two-dimensional case considering a direct extension of eq.(3.29).

In this case the criterion is to **[1]**:

$$\text{minimize} \left[\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right]^2 \tag{3.31}$$

Where the derivative function is approximated by the expression **[1]**:

$$\frac{\partial^2 f(x)}{\partial x^2} \approx f(x+1,y) - 2f(x,y) + f(x-1,y)+f(x,y+1) - 2f(x,y) + f(x,y-1)$$

$$\approx f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y) \tag{3.32}$$

The derivative function given in eq.(3.31) is recognized as Laplacian operator "C", and given by:

$$C(x,y) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{3.33}$$

the above smoothness criterion in a block circulant matrix:

$$C_j = \begin{bmatrix} C(j,0) & C(j,N-1) & \dots & C(j,1) \\ C(j,1) & C(J,0) & \cdots & C(j,2) \\ \vdots & & & \\ C(j,N-1) & C(j,N-2) & \dots & C(j,0) \end{bmatrix} \quad (3.34)$$

Since C is block circulant, it diagonalized by the matrix W:

$$\vartheta = W^{-1} C\, W \quad (3.35)$$

Where ϑ is Diagonal matrix of C

The optimal solution is given by eq.(3.17) with $Q \approx C$; that is:

$$\hat{f} = (\acute{H}\, H + \gamma\, \acute{C}\, C)^{-1} \acute{H}\, g \quad (3.36)$$

By using eq.(3.7), eq. (3.8), eq.(3.35), eq.(3.36) in eq.(3.36) gave as:

$$\hat{f} = (W\, D^{*}\, D\, W^{-1} + \gamma\, W\, \vartheta^{*}\, \vartheta\, W^{-1})^{-1} W\, D^{*}\, W^{-1}\, g \quad (3.37)$$

After multiplying both side by W^{-1} and some matrix manipulations. This equation reduces to

$$W^{-1}\hat{f} = (D^{*}\, D + \gamma\, \vartheta^{*}\, \vartheta)^{-1} D^{*}\, W^{-1}\, g \quad (3.38)$$

eq.(3.38) could be express by the form:

$$\hat{F}(u,v) = \left[\frac{H^*}{|H(u,v)|^2+\gamma|C(u,v)|^2}\right] \quad (3.39)$$

For u,v = 0, 1, 2, ... N - I, Where $|H(u,v)|^2 = H^*(u,v)H(u,v)$

3.3 VanCittert's Iteration

The simplest of the iterative restoration methods have a long history. It goes back at least to the work of VanCittert in the 1930's, and may in fact have even older antecedents **[41]**.

3.3.1 Formulation of the Algorithm

The advantage iterative procedures that are no matrix inverses need to be implemented, and the additional deterministic constraints can be incorporated into the solution algorithms **[41]**.

by applying the method of successive approximations to compute a solution to Eq. (2.1) given as **[41]**:

$$\hat{f} = 0$$

$$\hat{f}_{k+1} = \hat{f}_k + \beta(g - H\,\hat{f}_k) \quad (3.40)$$

Where k is number of iteration, $\hat{f}_{k+1}$is restored image after k+1 iterations, $\hat{f}_k$ is restored image after k iterations, β is control the convergence of the iterations.

Inspected iterative restoration algorithms need a first estimate to start their iterations, and it may be estimated as follows **[40]**:

- The recorded image

An obvious choice of the first estimate is to use the degraded image.

- the mean of the recorded image

Kaufman (1987) has argued that starting from a uniform first guess is a good approach shot if no other sensible guess as a first estimate can be obtained. We have used the mean of degraded image as a uniform first estimate.

- The original object

Ideally, the restoration algorithms should produce the original object. Utilizing the original target as a first estimate enables us to test whether this is the solution found by these algorithms under realistic circumstances.

- The result of Tikhonov-Miller restoration

The iterative algorithms based on the Tikhonov functional should converge more quickly on their solutions if a first estimate close to the final solution is provided.

3.3.2 Deblurring Procedure

deblurring is equivalent to applying the VanCittert procedure to the identity **[41]**:

$$f = f + \beta H^{t}(g - Hf) \qquad (3.41)$$

yielding the iteration gives **[41]**:

$$\hat{f}_{k+1} = \hat{f}_k + \beta H^{t}(g - H\hat{f}_k) \qquad (3.42)$$

An alternative way to derive Eq. (3.42) is from the viewpoint of minimizing the norm of the residual $(g - H\hat{f})$ because for a good restoration result the blurred estimate $H\hat{f}$ should be approximately equal to the observed image *g*. Iterative minimization of the criterion function using eq.(3.3), by the method of steepest descent yields **[28]**:

$$\hat{f}_{k+1} = \hat{f}_k + \beta r_k = \hat{f}_k - \tfrac{1}{2}\beta \nabla J\,(f)\big|_{\hat{f}_k} \qquad (3.43)$$

$$= \hat{f}_k + \beta H^t(g - H\hat{f}_k)$$

Where $r_k = -\frac{1}{2}\nabla_f J(f)|_{\hat{f}_k}$ is called the steepest descent direction which points in the direction of the negative gradient of the objective function at $\hat{f}_k$.

3.3.3 Iterative Tikhonov-Miller Solution

Iterative Tikhonov-Miller Solution usually used because of **[41]**:

1- the iterative computation of the Tikhonov-Miller regularized solution might be easier than the direct evaluation of (3.36)

2- the use of an iterative scheme allows for additional constraints to be imposed on the solution .

We use the method of the steepest descent to minimize the objective function $J(\hat{f})$ in (3.15). This gives the following iterations:

$$\hat{f}_{k+1} = \hat{f}_k + \beta r_k = \hat{f}_k - \frac{1}{2}\beta \nabla_f J(f)|_{\hat{f}_k}$$

$$= \hat{f}_k - \beta\left((H^T H + \alpha C^t C)\hat{f}_k - H^t g\right)$$

$$=(I-\alpha\beta C^t C)\hat{f}_k + \beta H^t(g - H\hat{f}_k) \quad (3.44)$$

This iteration reduces to the (deblurred) VanCittert iteration if $\alpha = 0$ (no Tikhonov-Miller regularization).

the term $(I-\alpha\beta C^t C)$ in Eq.(3.44) behaves like a low-pass filter, suppressing the noise in the iterates. The optimal value of β at the iteration k is given by **[41]**:

$$\beta_k = \frac{\|r_k\|^2}{\|H\, r_k\|^2 + \alpha \|C\, r_k\|^2} \qquad (3.45)$$

the optimized method of steepest descent will converge faster than the method of steepest descent with a fixed value for β. Although the optimization procedure for β increases the convergence speed of the algorithm, the improvements are usually moderate and may not justify the efforts involved.

A drawback of these iterative methods is the fact that the quality of the restored image does not necessarily increase monotonically. So it seems thus reasonable to stop the iteration when $g - H\hat{f} \approx n$ [42].

3.3.4 Image Restoration Algorithm

The procedure has been used to applying restoration algorithm as follow:

1- Read binary or color original image with a size of 256 × 256 pixel.

2- Generating Gaussian function (size 256 × 256) with selected standard deviations values σ =1,2.

3- Generating random Gaussian noise with selected signal to noise ratio values SNR=5,10,20

4- Convolve original image with Gaussian function to produce a blurred image.

5- Add Gaussian noise to the blurred image to produce a degraded image.

6- Select a three value of regularizing parameter α = 0.25, 1,2 for Tikhonov-Miller regularized restoration filter.

7- Estimate original image using two restoration filter these are, Tikhonov-Miller regularized restoration filter and Wiener filter.

8- Calculate the Root Mean Square Error (RMSE) of restored images obtained by filters. Figure (4-1) shows flowchart of the program.

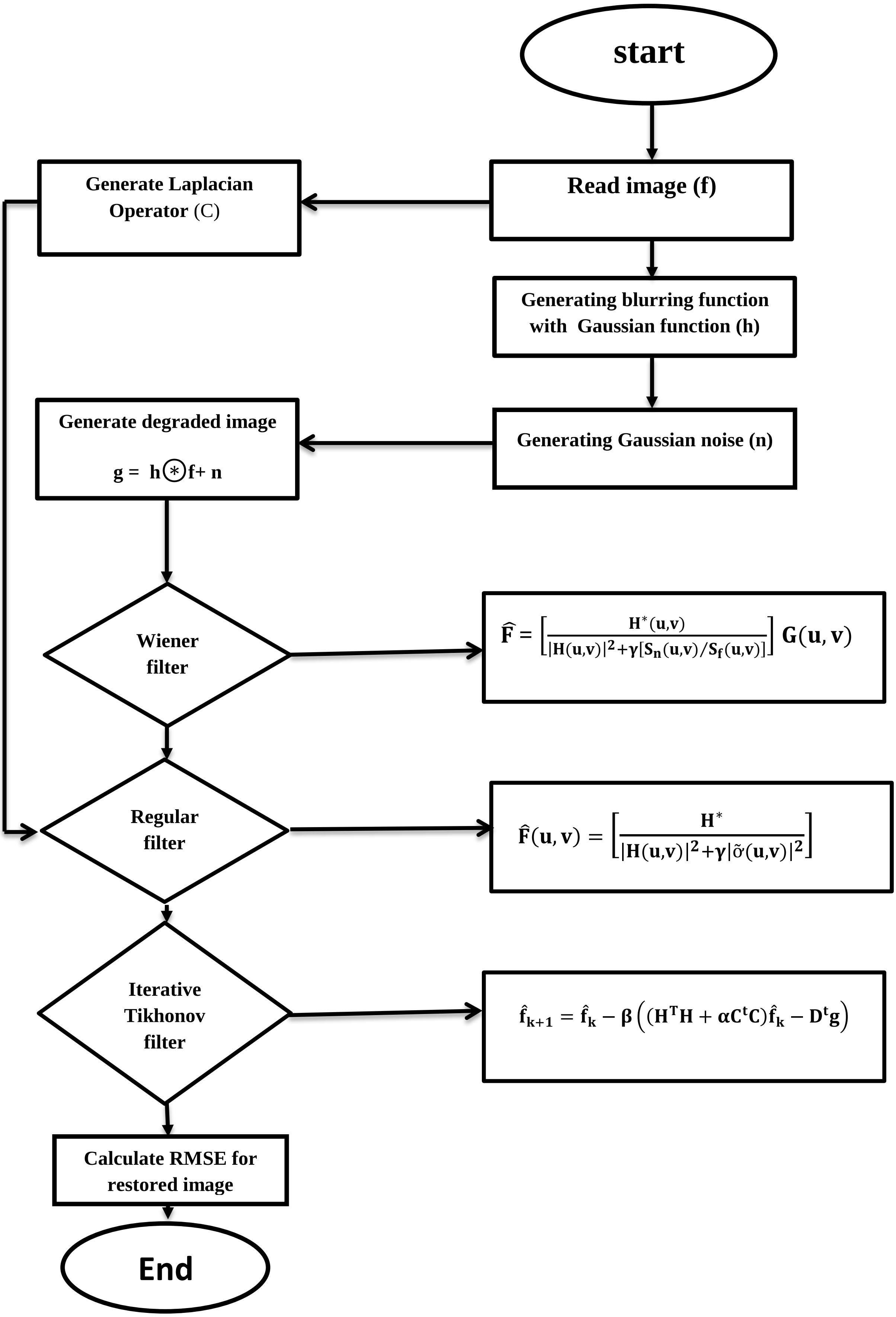


Figure (4-1) Flowchart of Image Restoration Algorithm

CHAPTER FOURE

Results and Discussion

4.1 Introduction

This chapter is conducted to present the result and discussion that have been a chivied in this study. The effectiveness of image restoration filter depends on the accuracy of the knowledge of degradation process, and it also depend on the filter design criterion. RMSE is one of the most important criterion, that is used to measure the quality of image. The research assumed that, the degradation model, as a convolution of the original image with blurring function and distorded by additive noise. Image restoration algorithms could be define as, a trying to "undo" the blurring function and the noise from the degraded image by deconvolving the blurring function and reducing the noise from the degraded image to produce an estimate image, which is approach to the original image.

The numerical result of research focuses on a linear non-blind image restoration, in which the blurring function is known and it is a space invariant. Matlab program have been used to apply a written algorithm on three type of images, these are gray image (Satellite image), sonar image (Embryo image) and color image (bird image) are 256*256 pixels in size, to blur images with Gaussian blurring function with selected standard deviation values " σ =1,2 " and degraded it with additive Gaussian noise with selected signal to noise ratio "SNR= 5,10,20" to produce degraded images. By applying another written algorithm on degraded images to restore it, using Tikhonov-Miller filter and Wiener filter. Display the relation between Root Mean Square Error and signal to noise ratio for σ = 1,2 for the degraded image, the restored image using Wiener filter. Display the relation between Root Mean Square Error and SNR, number of iteration with different SNR and number of iteration with different regularizing parameters "α = 0.25, 1,2" for restored image using Iterative Tikhonov-Miller filter.

4.2 Test and Result

Constrained images restoration techniques in frequency domain are adopted to restore the degraded images using Matlab program, which is:

- Non-iterative solution using Wiener filter.
- Iterative solution using Iterative Tikhonov-Miller filter.

Also, Gaussian blurring function were adopted with different standard deviation values " σ ", σ = 1 and 2. and Gaussian noise, with different noise selected signal to noise ratio" SNR =5, 10, and 20", with zero mean.

4.2.1 Degraded images

Three images has been selected, these are gray image (Satellite image), sonar image (Embryo image) and color image (bird image) are 256*256 pixels in size, each of them convolved with Gaussian blurring function for selected standard deviation values " σ = 1,2 ", and degraded by additive Gaussian noise at selected signal to noise ratio "SNR =5,10,20" to produce a degraded images and the result display in the following tables.

Tables ((4-1)-(4-3)) describe the result value of Root Mean Square Error (RMSE) of images after degradation for selected standard deviation values " σ = 1,2 " and selected signal to noise ratio values "SNR =5,10,20". the result value of Root Mean Square Error of restoring images using Iterative Tikhonov-Miller filter with selected regularizing parameter values α= 0.25, 1,2 and result value of Root Mean Square Error of restoring images using Wiener filter. Where RMSE decrease or increase with parameter value of σ, SNR and α.

Table (4-1) display the result value of RMSE for Satellite degraded, restored Satellite image.

	σ	SNR=5 RMSE	SNR=10 RMSE	SNR=20 RMSE
Degraded image	1	41	36	29
	2	44	37	30
Wiener filter	1	26	27	28
	2	22	21	23
Restore image Using Tikhonov Filter when α=0.25	1	35 9 iter	21 15 iter	18 13 iter
	2	32 4 iter	24 4 iter	18 2 iter
Restore image Using Tikhonov Filter when α=1	1	26 22 iter	21 16 iter	18 14 iter
	2	31 5 iter	23 3 iter	17 2iter
Restore image Using Tikhonov Filter when α=2	1	29 13 iter	26 8 iter	18 13 iter

Table (4-1) shows the RMSE of Satellite restored images with different type of filters

Table (4-2) display the result value of RMSE for Embryo degraded, restored Embryo image.

	σ	SNR=5 RMSE	SNR=10 RMSE	SNR=20 RMSE
Degraded image	1	55	44	23
	2	64	50	27
Wiener filter	1	32	36	35
	2	29	35	34
Restore image Using Tikhonov Filter when α=0.25	1	21 20 iter	14 19 iter	7 17 iter
	2	25 4 iter	15 3 iter	7 2iter
Restore image Using Tikhonov Filter when α=1	1	21 21iter	13 20 iter	8 17 iter
	2	24 5 iter	15 5 iter	7 3 iter
Restore image Using Tikhonov Filter when α=2	1	21 21 iter	13 22 iter	7 17
	2	23 5	14 3 iter	7 3 iter

Table (4-2) shows the RMSE of Embryo restored image with different type of filters

Table (4-3) display the result value of RMSE for bird degraded, restored bird image.

	σ	SNR=5 RMSE	SNR=10 RMSE	SNR=20 RMSE
Degraded image	1	42	35	18
	2	50	36	20
Wiener filter	1	17	15	17
	2	15	13	10
Restore image Using Tikhonov Filter when α=0.25	1	17 24 iter	12 19 iter	8 16
	2	23 5 iter	17 5 iter	9 3 iter
Restore image Using Tikhonov Filter when α=1	1	17 24 iter	12 19 iter	8 14 iter
	2	22 5 iter	15 4 iter	8 3 iter
Restore image Using Tikhonov Filter when α=2	1	18 25 iter	13 22 iter	8 15 iter
	2	23 4 iter	16 4 iter	9 3 iter

Table (4-3) shows the RMSE of Bird restored image with different type of filters

- **Satellite image**

Satellite image of Baghdad zoo is 256*256 pixels in size, convolved with Gaussian blurring function for selected standards deviation values " σ = 1,2 ", and degraded by additive Gaussian noise at selected signal to noise ratio "SNR =5,10,20" to produce degraded Satellite image as shown in figures (4-1) and (4-2).

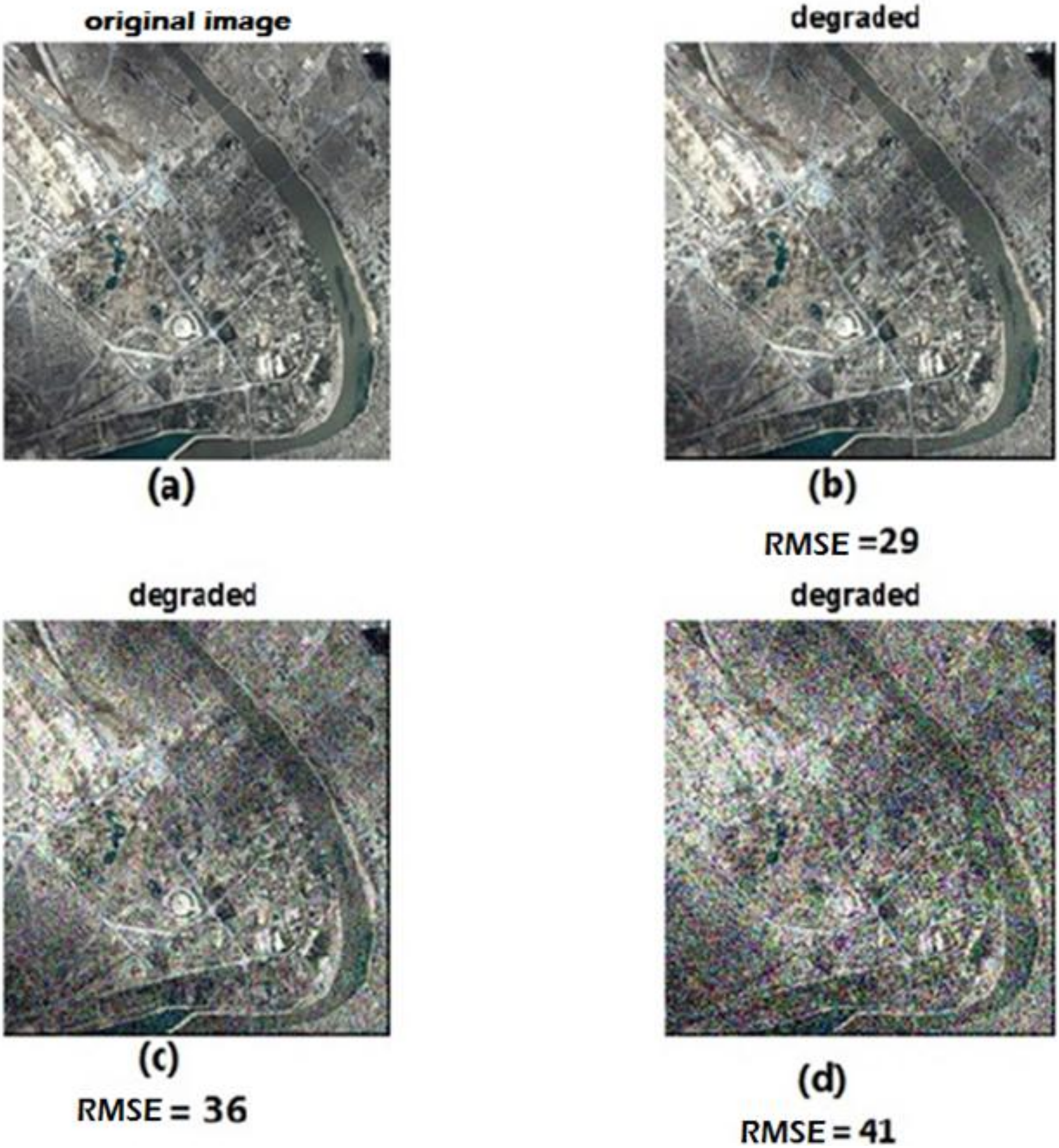


Figure (4-1) Present (a) original image. (b)-(d) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 20, 10 and 5 respectively.

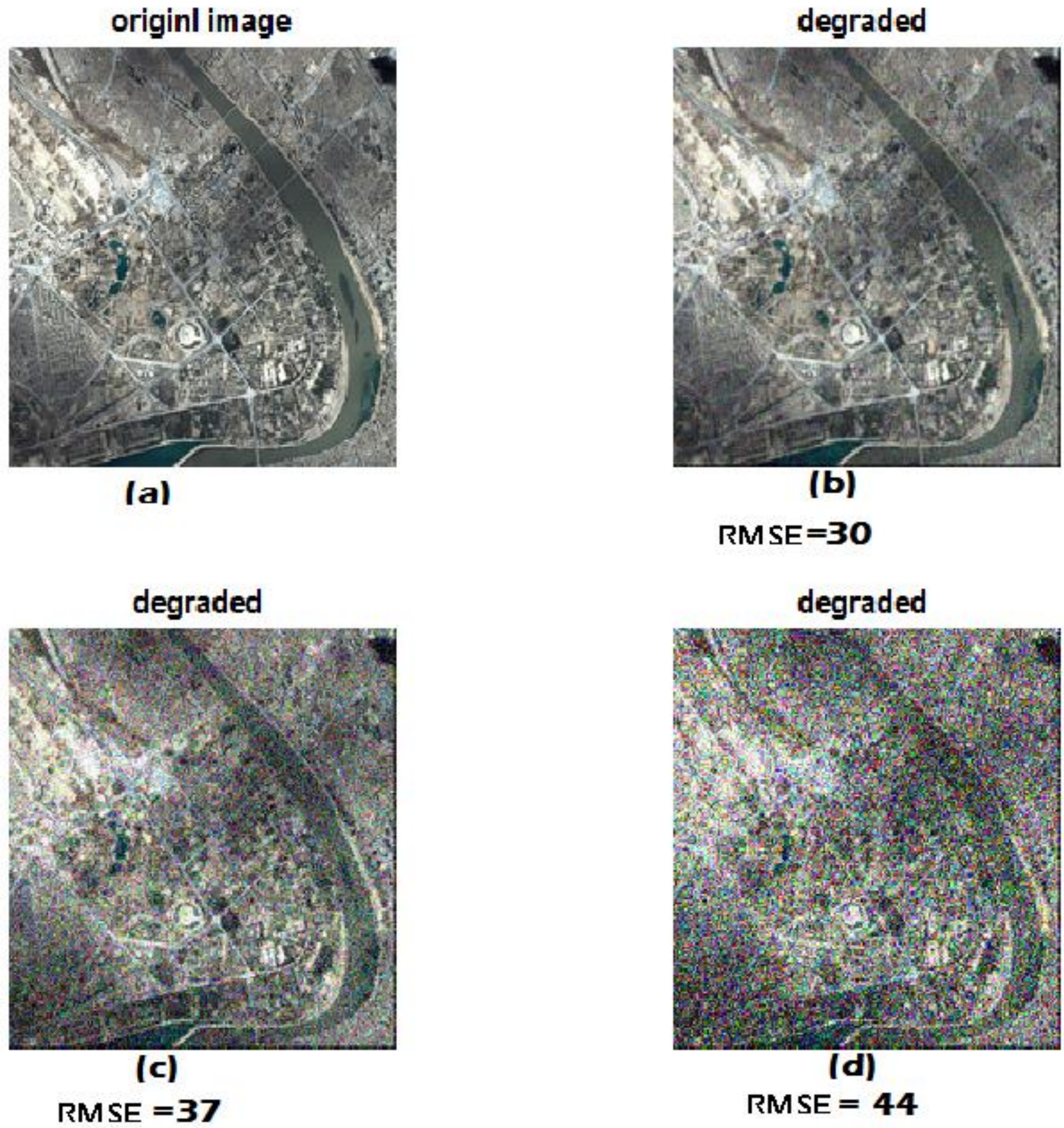


Figure (4-2) Present (a) original image. (b)-(d) degraded images, Gaussian function of σ = 2 and Gaussian noise with SNR = 20, 10 and 5 respectively

- **sonar image**

Sonar image of Embryo is 256*256 pixels in size, convolved with Gaussian blurring function for selected standard deviation values " σ = 1,2 ", and degraded by additive Gaussian noise at selected signal to noise ratio "SNR =5,10,20" to produce degraded sonar image as shown in figures (4-3) and (4-4).

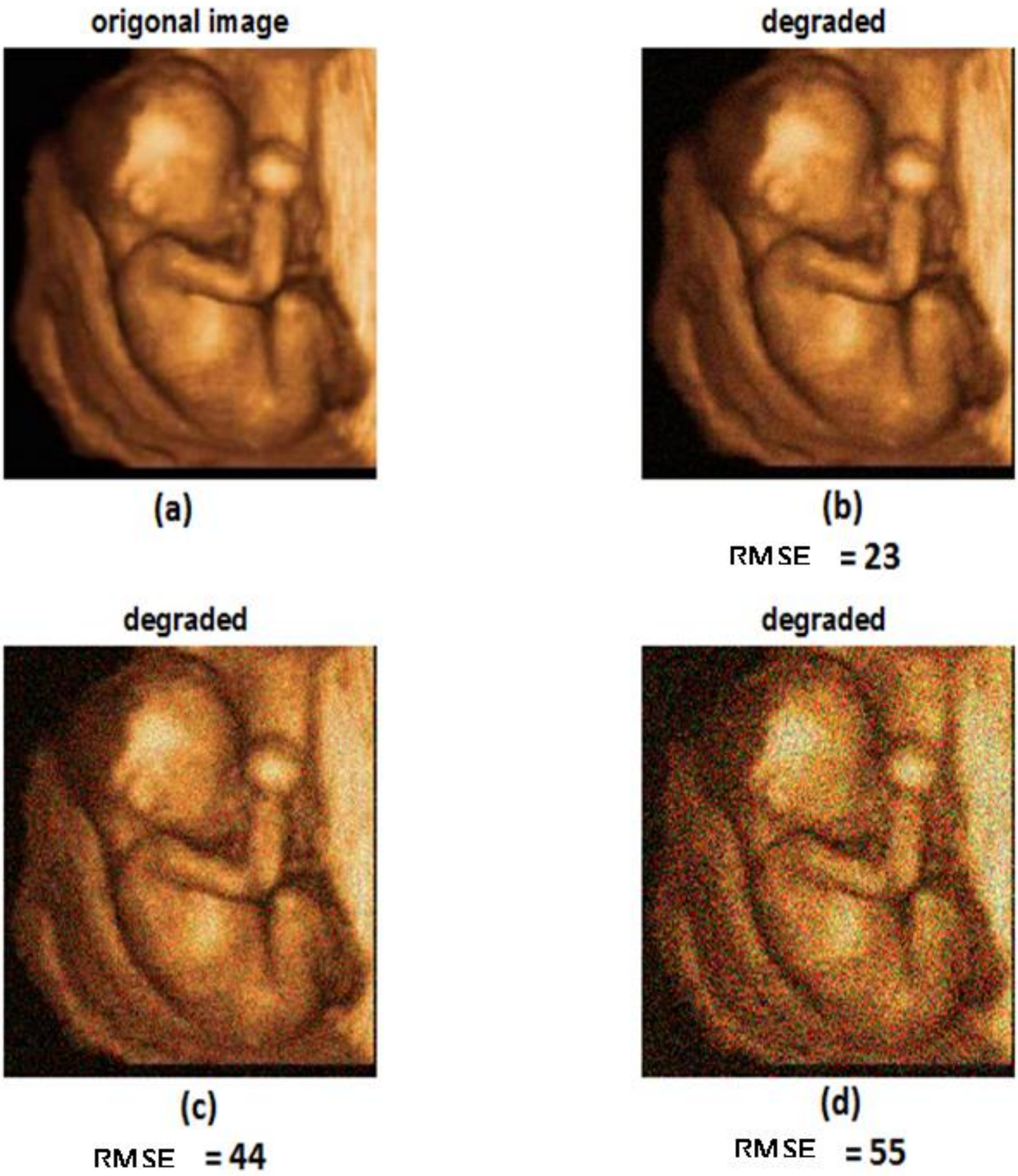


Figure (4-3) Present (a) original image. (b)-(d) degraded images, Gaussian function of σ = 1 and Gaussian noise with SNR = 20, 10 and 5 respectively.

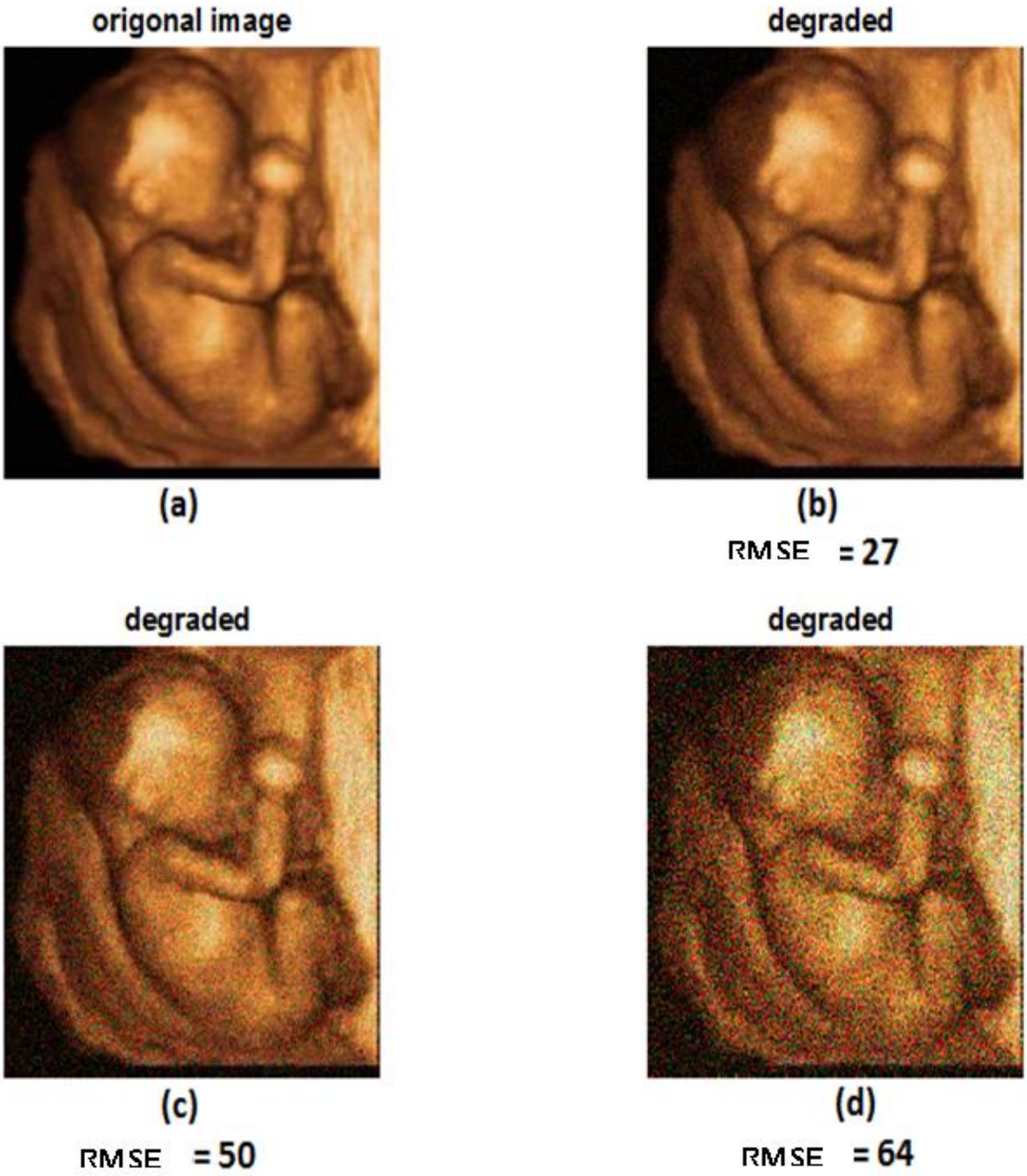


Figure (4-4) Present (a) original image. (b)-(d) degraded images, Gaussian function of σ = 2 and Gaussian noise with SNR = 20, 10 and 5 respectively

- **Color image**

Color image of bird image is 256*256 pixels in size, convolved with Gaussian blurring function for selected standard deviation values " σ = 1,2 ", and degraded by additive Gaussian noise at selected signal to noise ratio "SNR =5,10,20" to produce degraded color image as shown in figures (5) and (6).

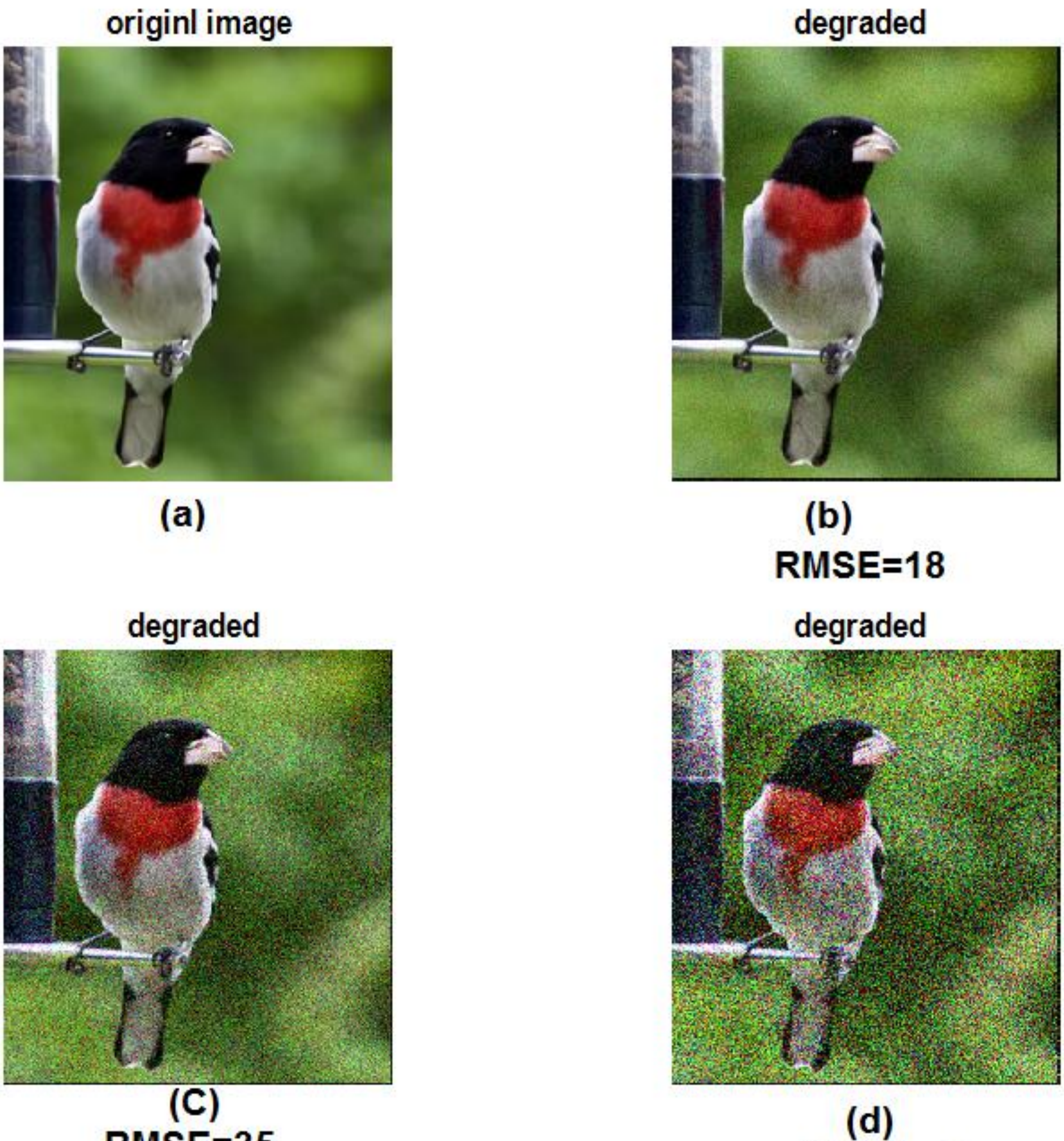


Figure (4-5) Present (a) original image. (b)-(d) degraded images Gaussian function of σ = 1 and Gaussian noise with SNR = 20, 10 and 5 respectively.

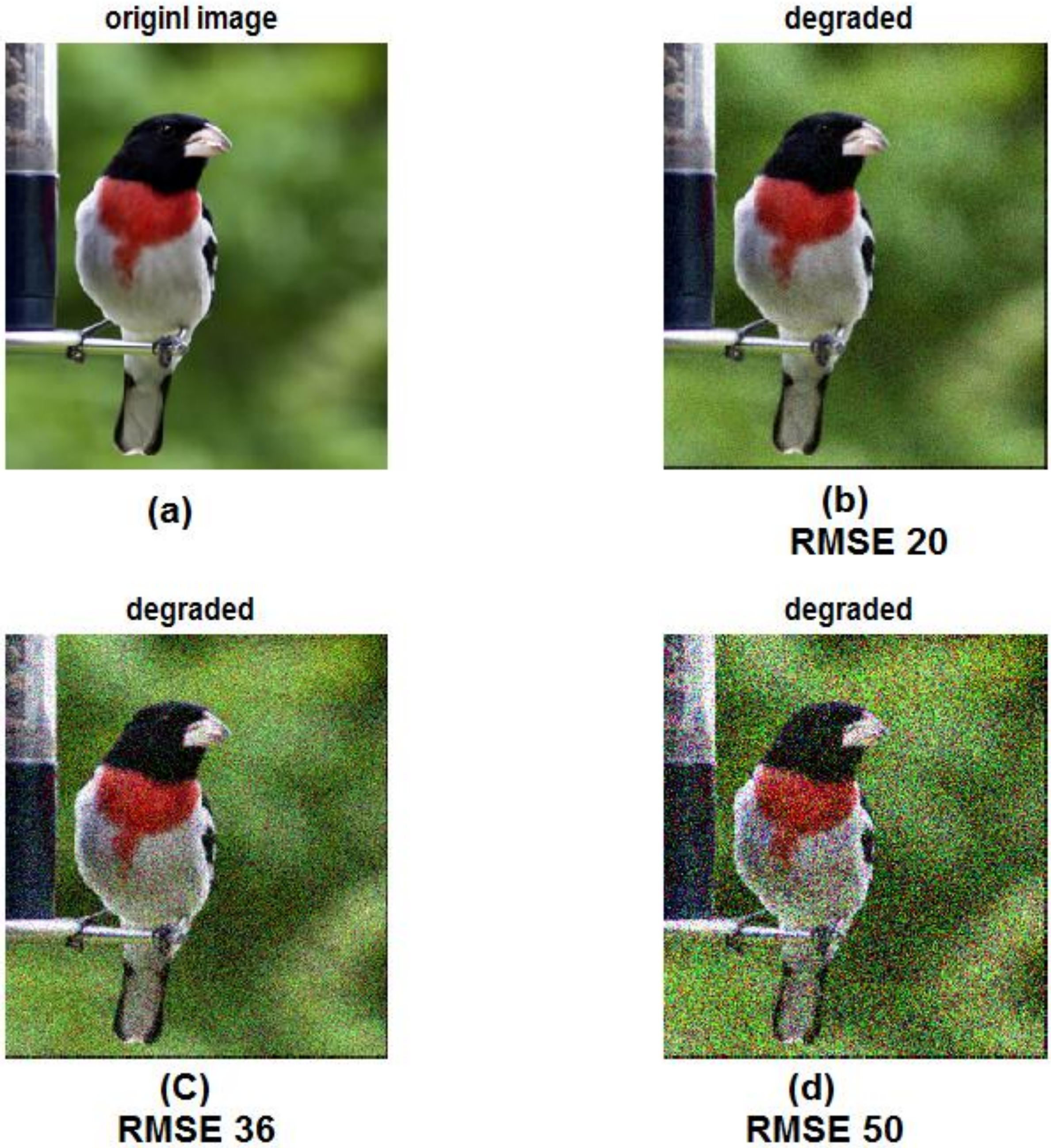


Figure (4-6) Present (a) original image. (b)-(d) degraded images, Gaussian function of σ = 2 and Gaussian noise with SNR = 20, 10 and 5 respectively.

4.2.2 Restored images

The degraded images that have shown in figure ((4-1)-(4-6)) have been restored using Iterative Tikhonov-Miller filter and Wiener filter as follow:

I- Iterative Tikhonov-Miller filter

Iterative Tikhonov-Miller filter improved three selected images as follow:

- **satellite image**

Iterative Tikhonov-Miller filter improved degraded satellite images, with using α =1, which have generally minimum result value of RMSE.

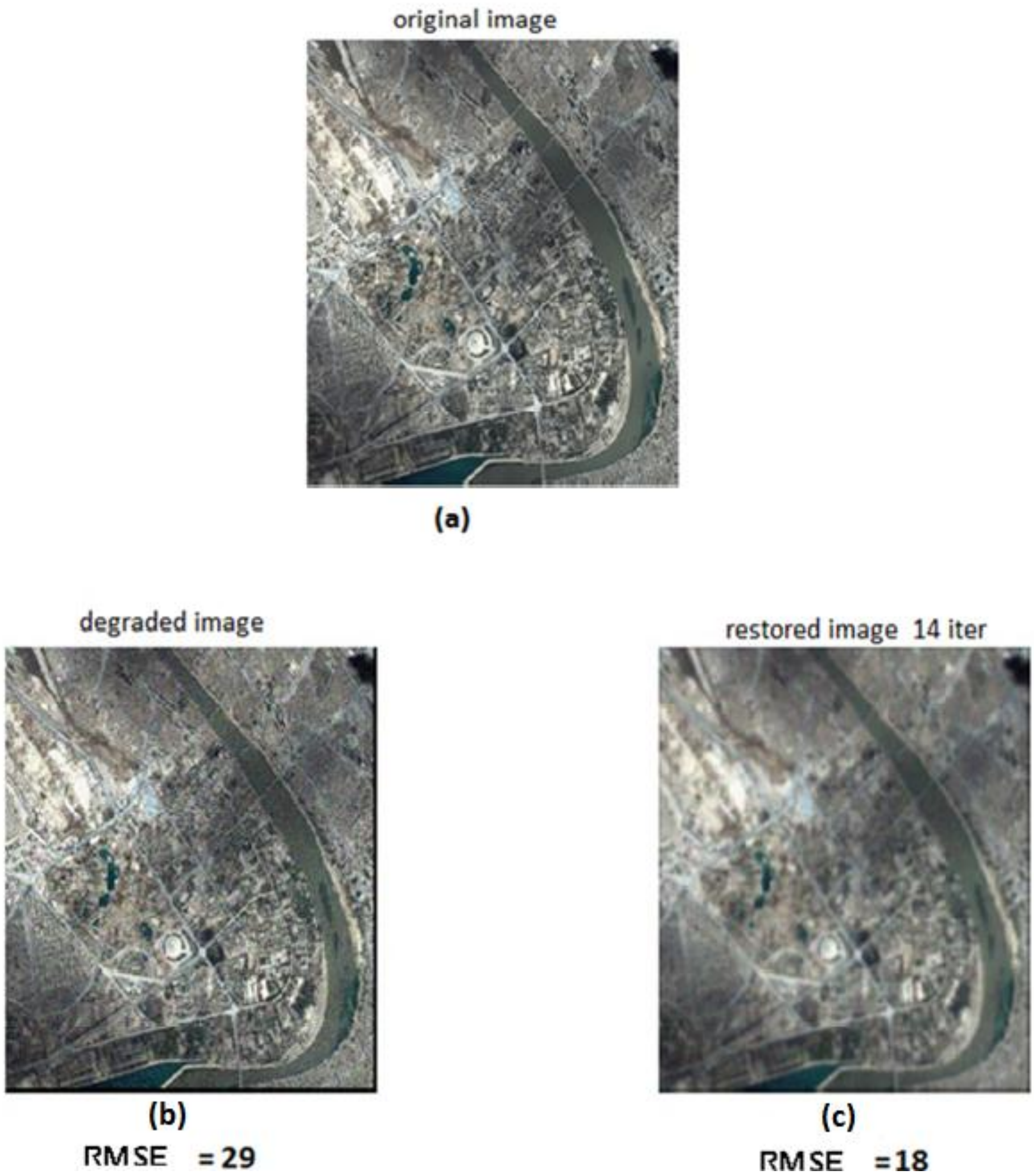


Figure (4-7) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 20. (c) restored image.

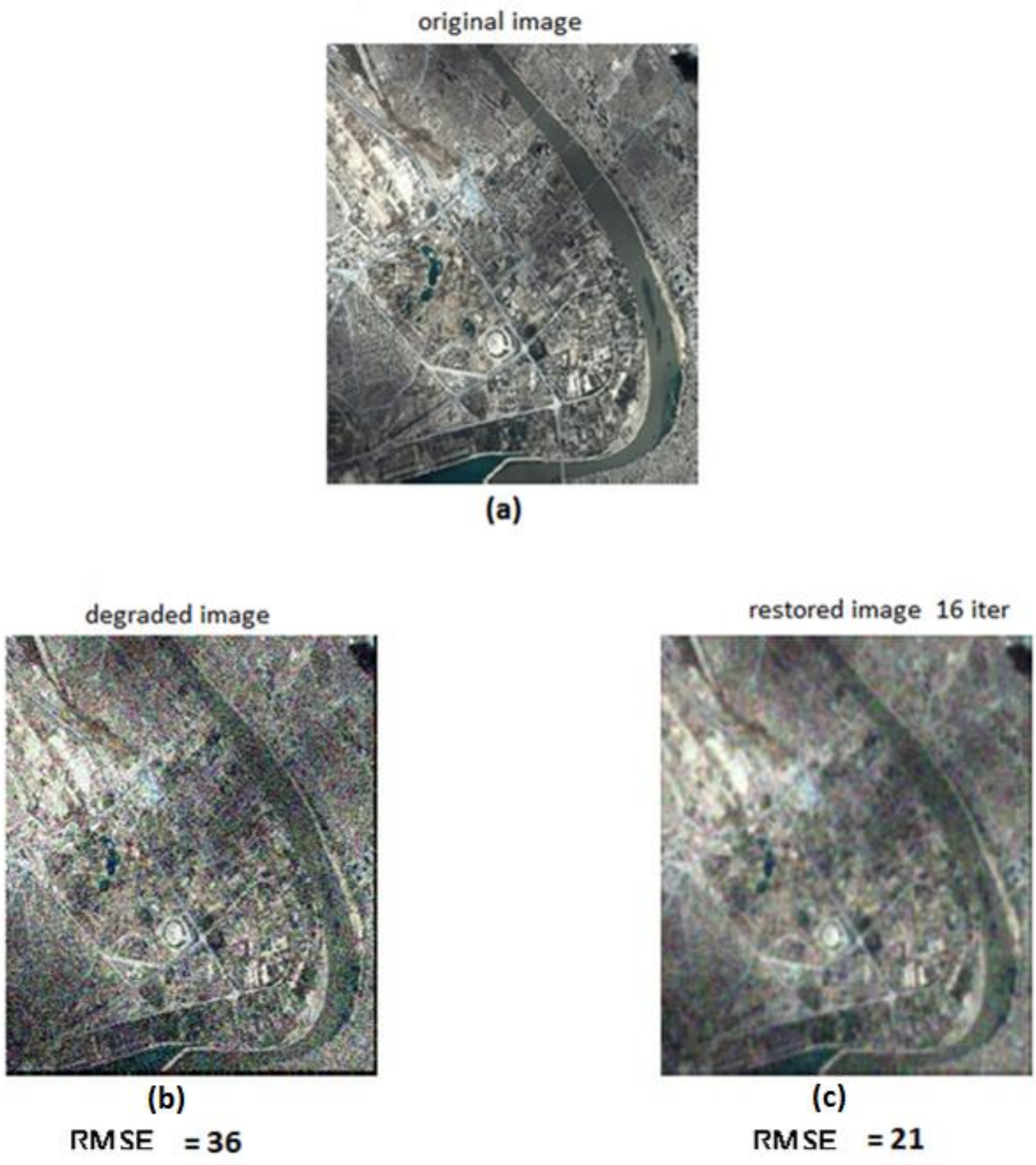


Figure (4-8) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 10. (c) restored image.

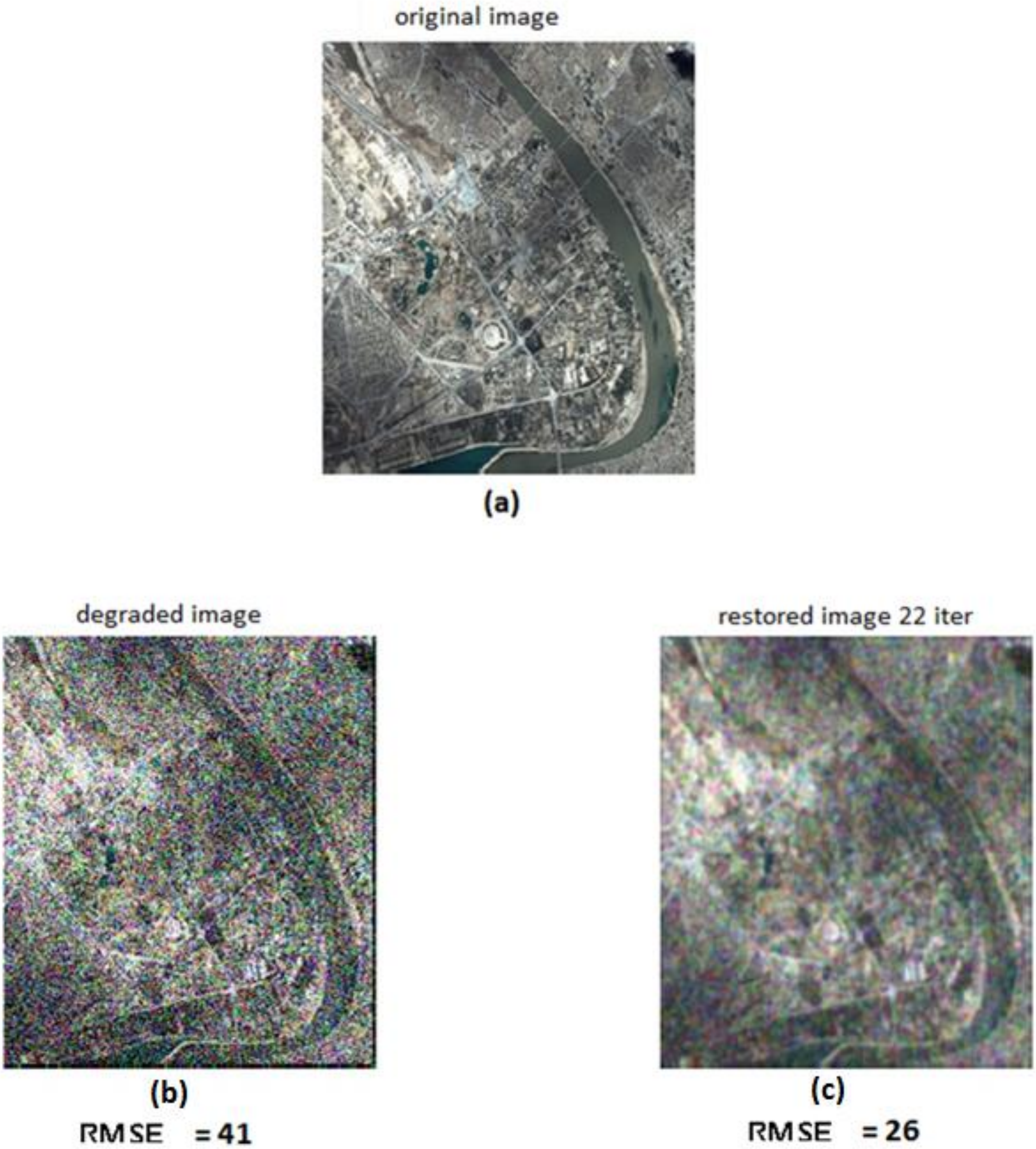


Figure (4-9) Present (a) original image. (b) degraded image, Gaussian function of $\sigma = 1$ and Gaussian noise of SNR = 5. (c) restored image.

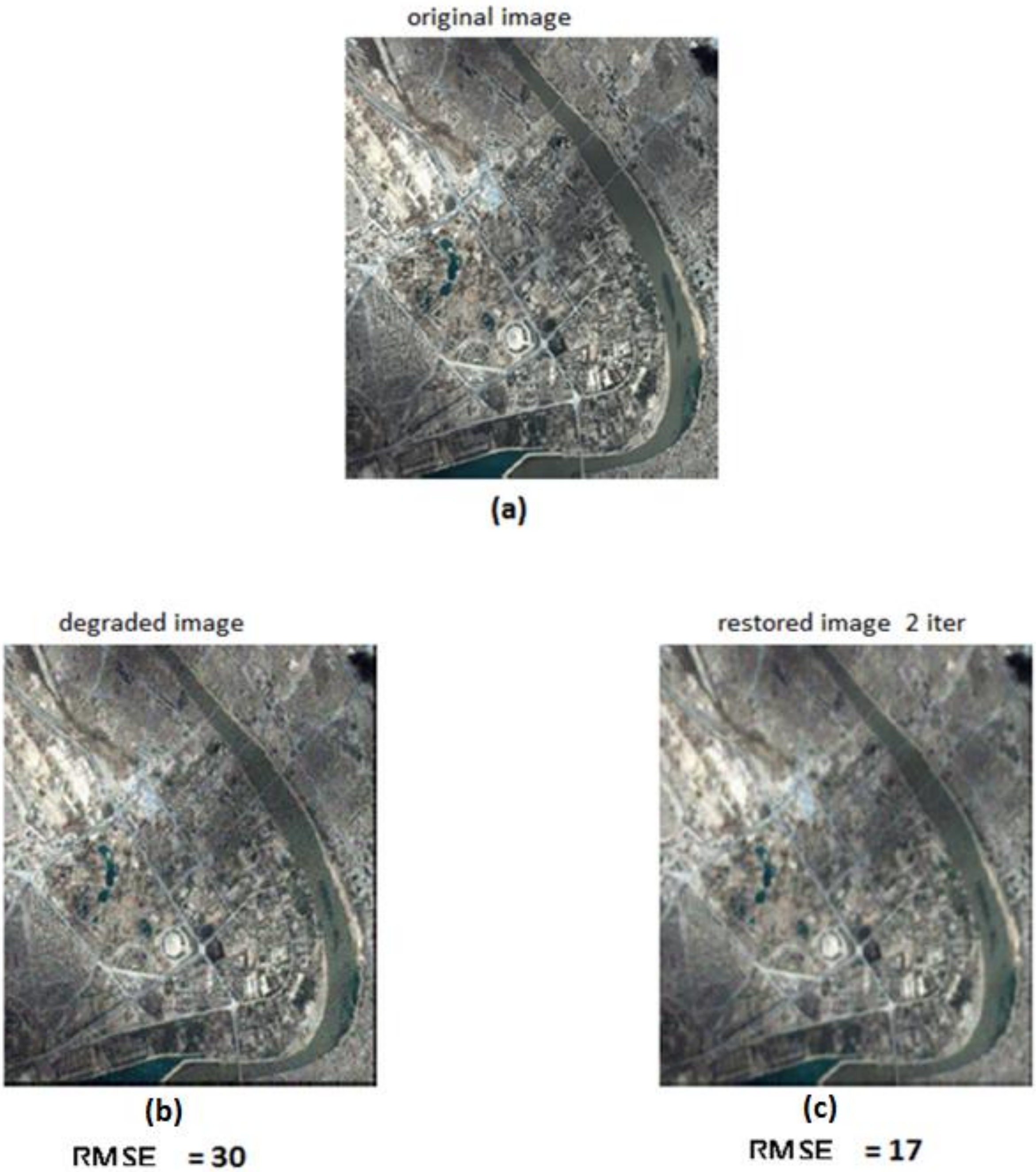


Figure (4-10) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 20. (c) restored image.

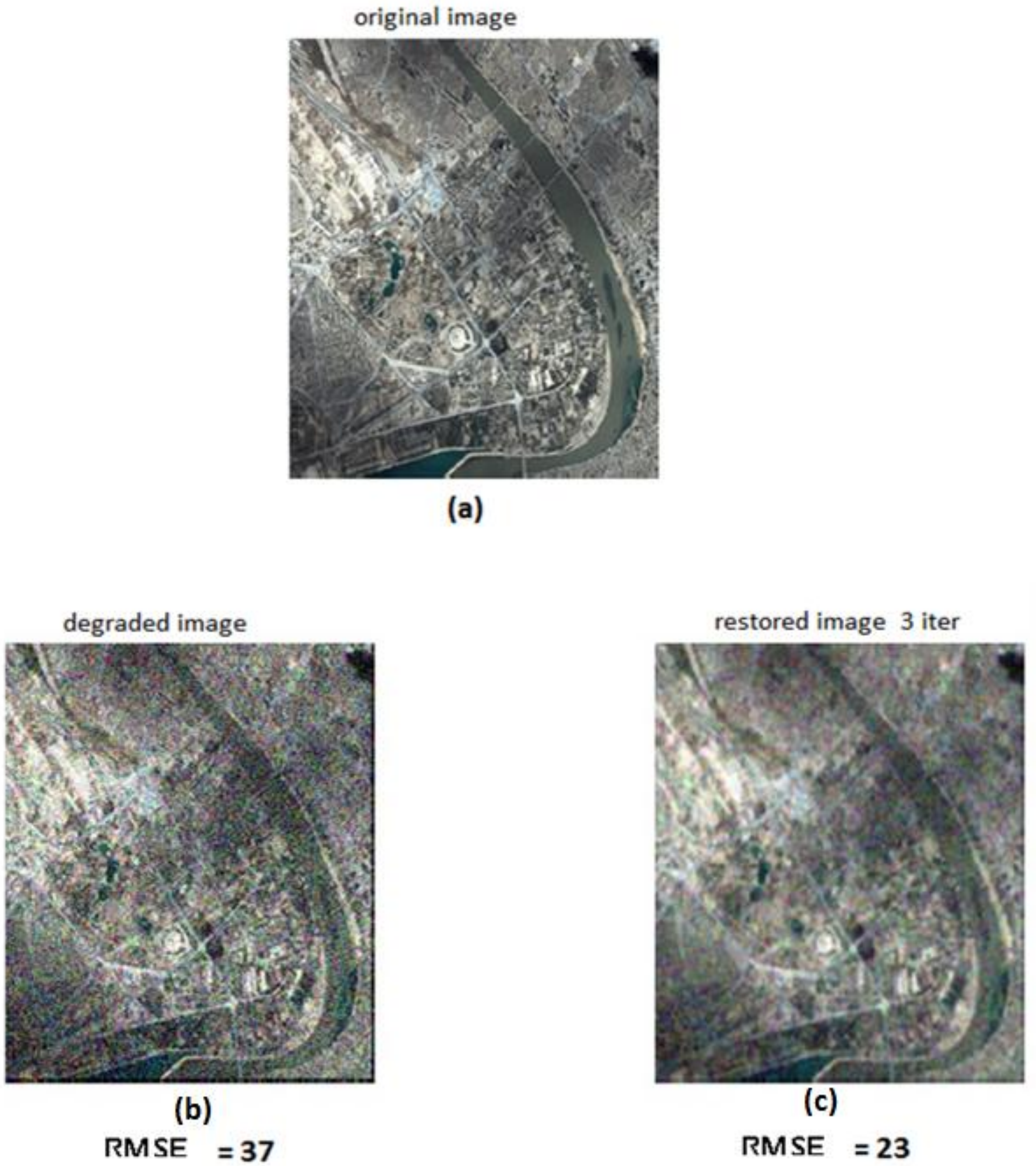


Figure (4-11) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 10. (c) restored image.

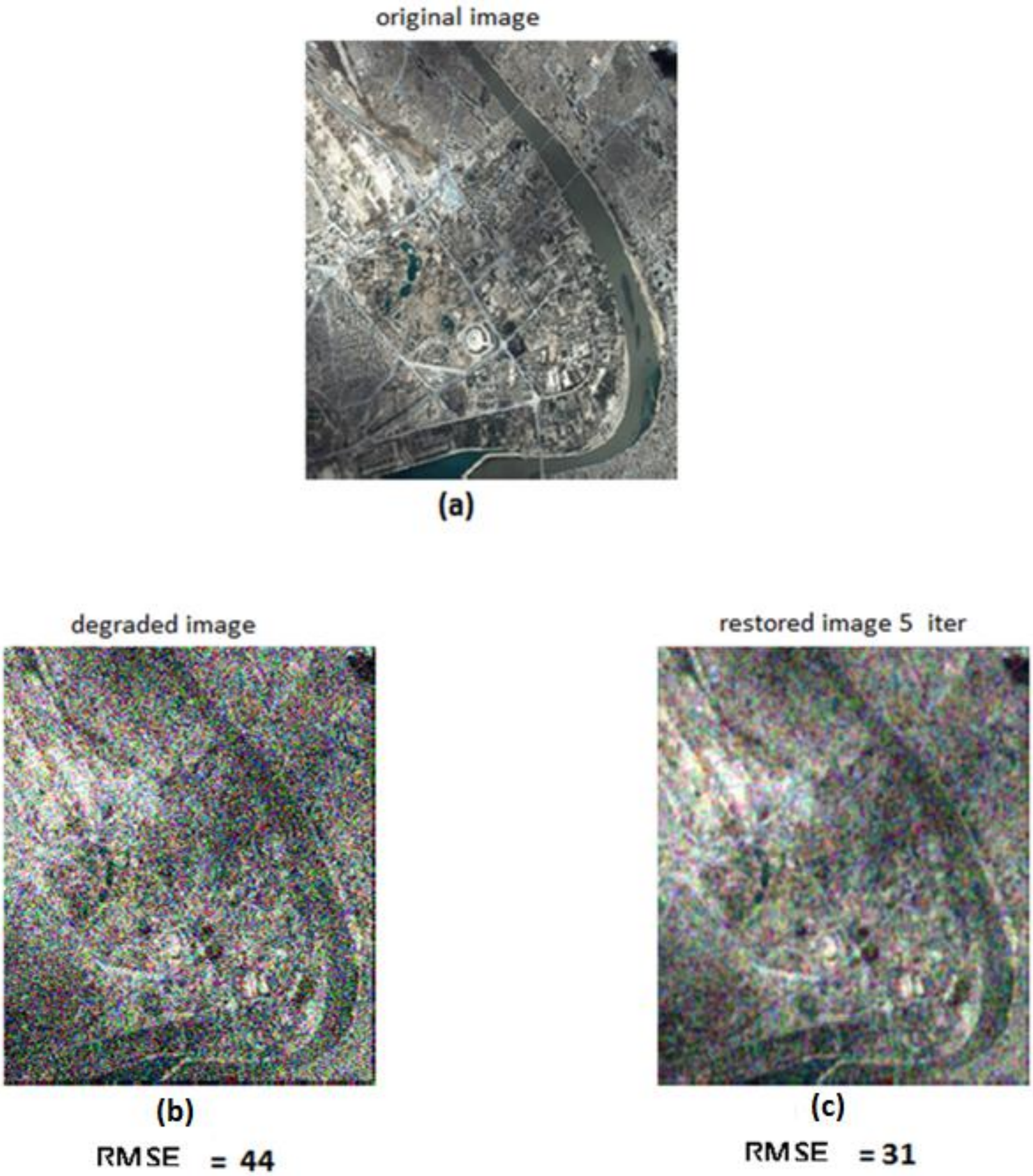


Figure (4-12) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 5. (c) restored image.

- **Embryo image**

Iterative Tikhonov filter, in which optimum regularization parameter" α" have value $\alpha = 2$ which have generally best value of RMSE, use to Restore the degraded

Embryo image, blurring with Gaussian blurring function for selected standard deviations values and distorted with Gaussian noise at different noise ratio values "SNR =5,10,20, as shown in the following figures.

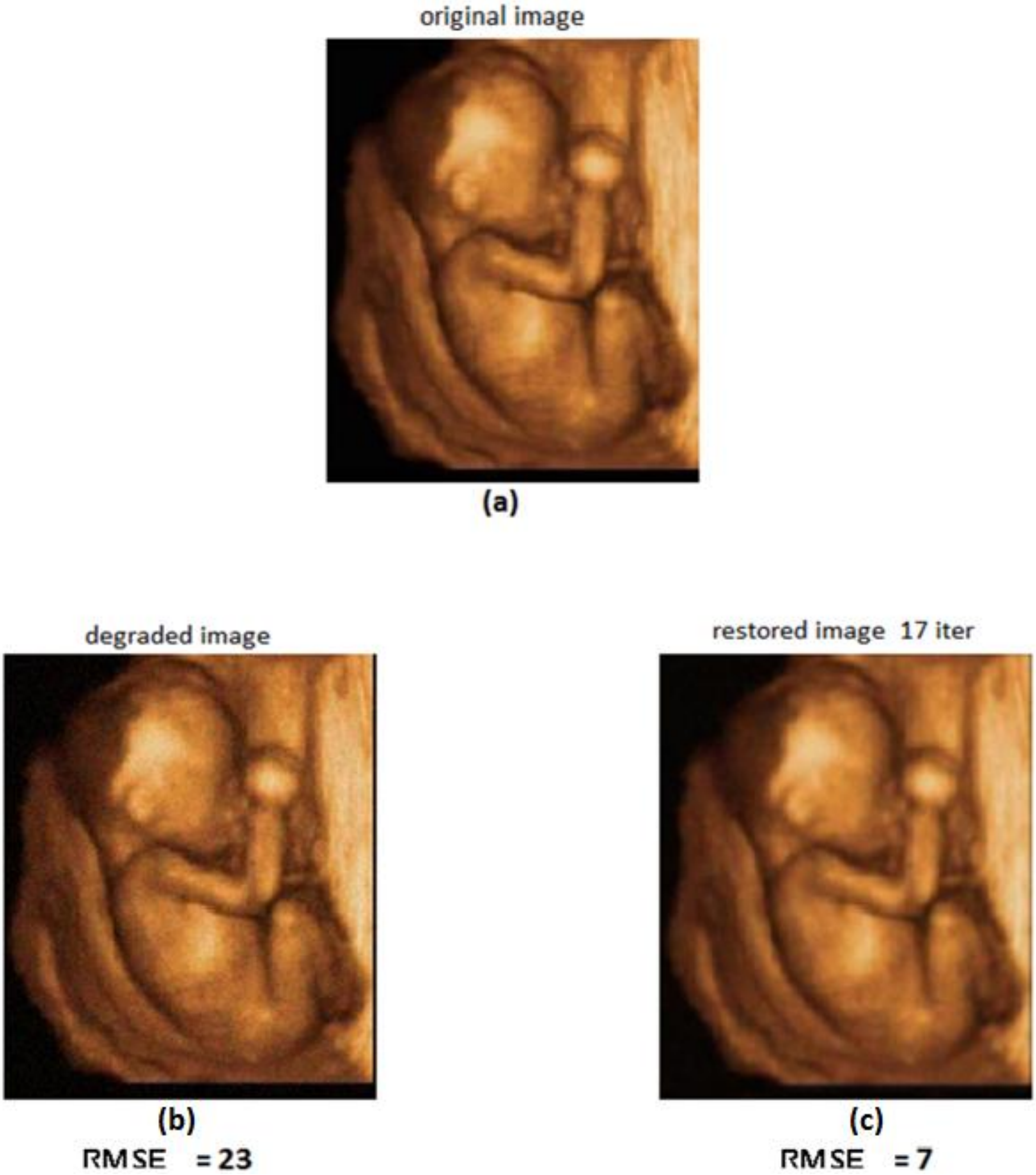


Figure (4-13) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 20 (c) restored image.

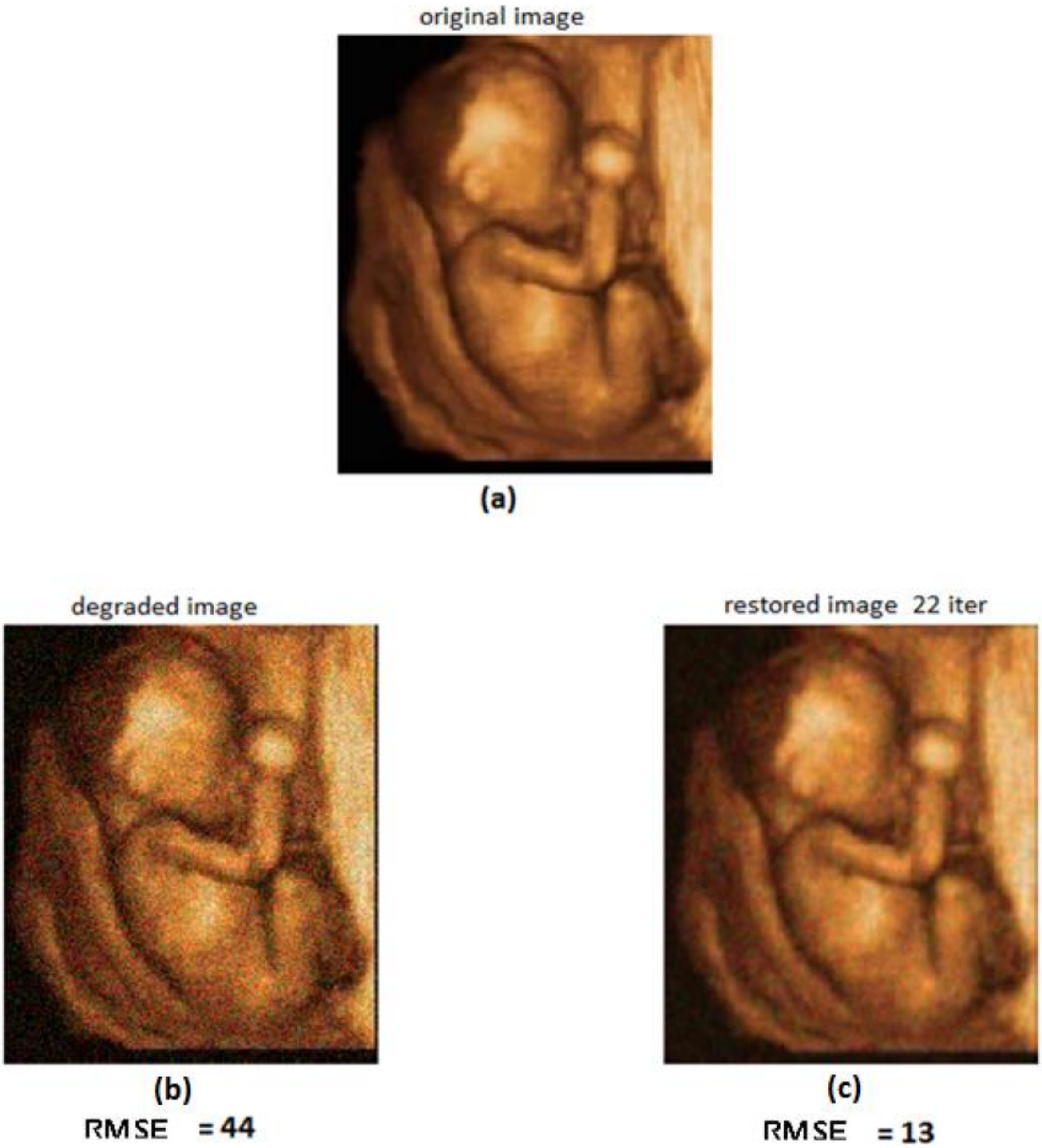


Figure (4-14) Present (a) original image. (b) degraded image Gaussian function of σ = 1 and Gaussian noise of SNR = 10 (c) restored image.

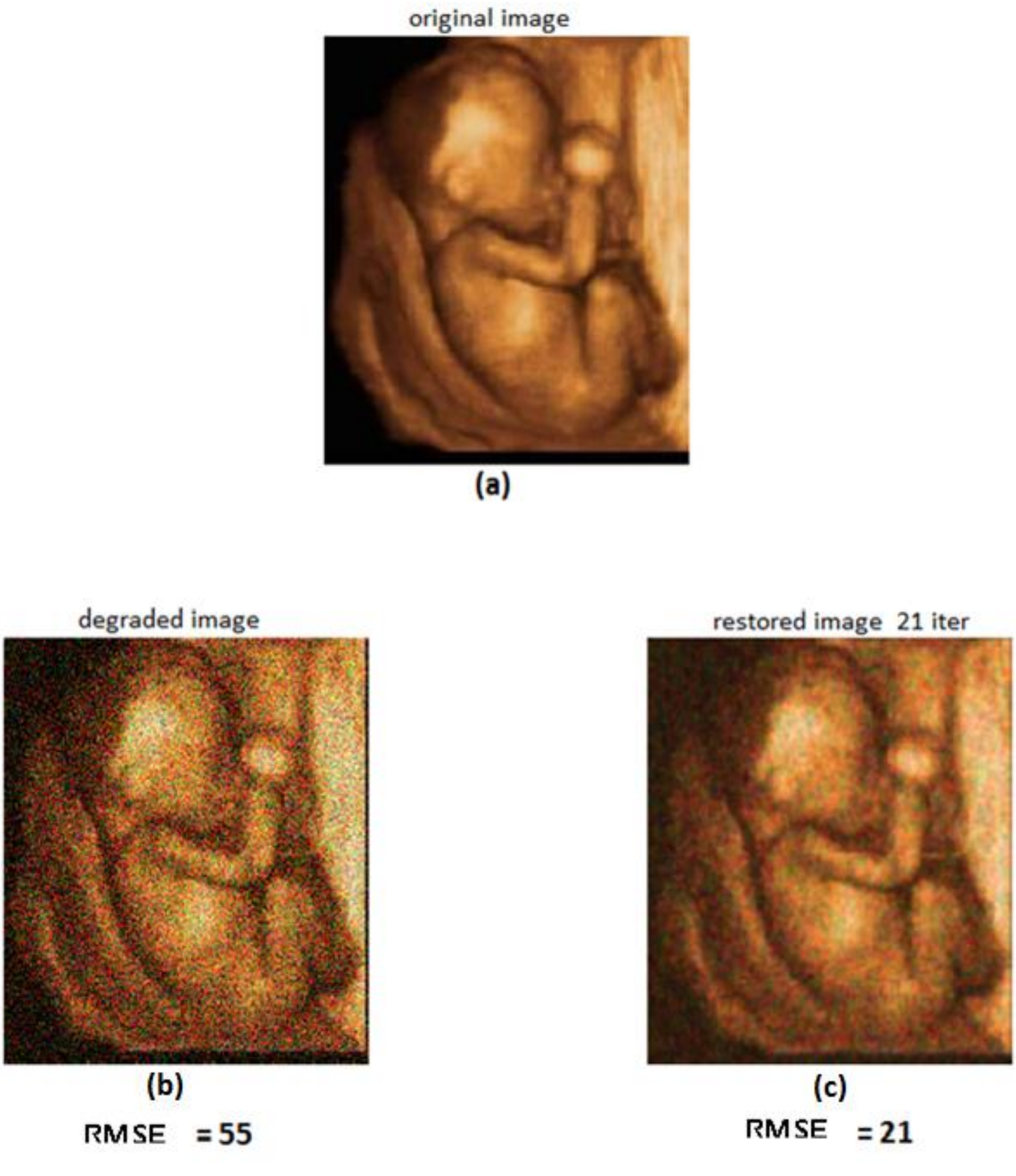


Figure (4-15) Present (a) original image. (b) degraded image, h Gaussian function of σ = 1 and Gaussian noise of SNR = 5 (c) restored image.

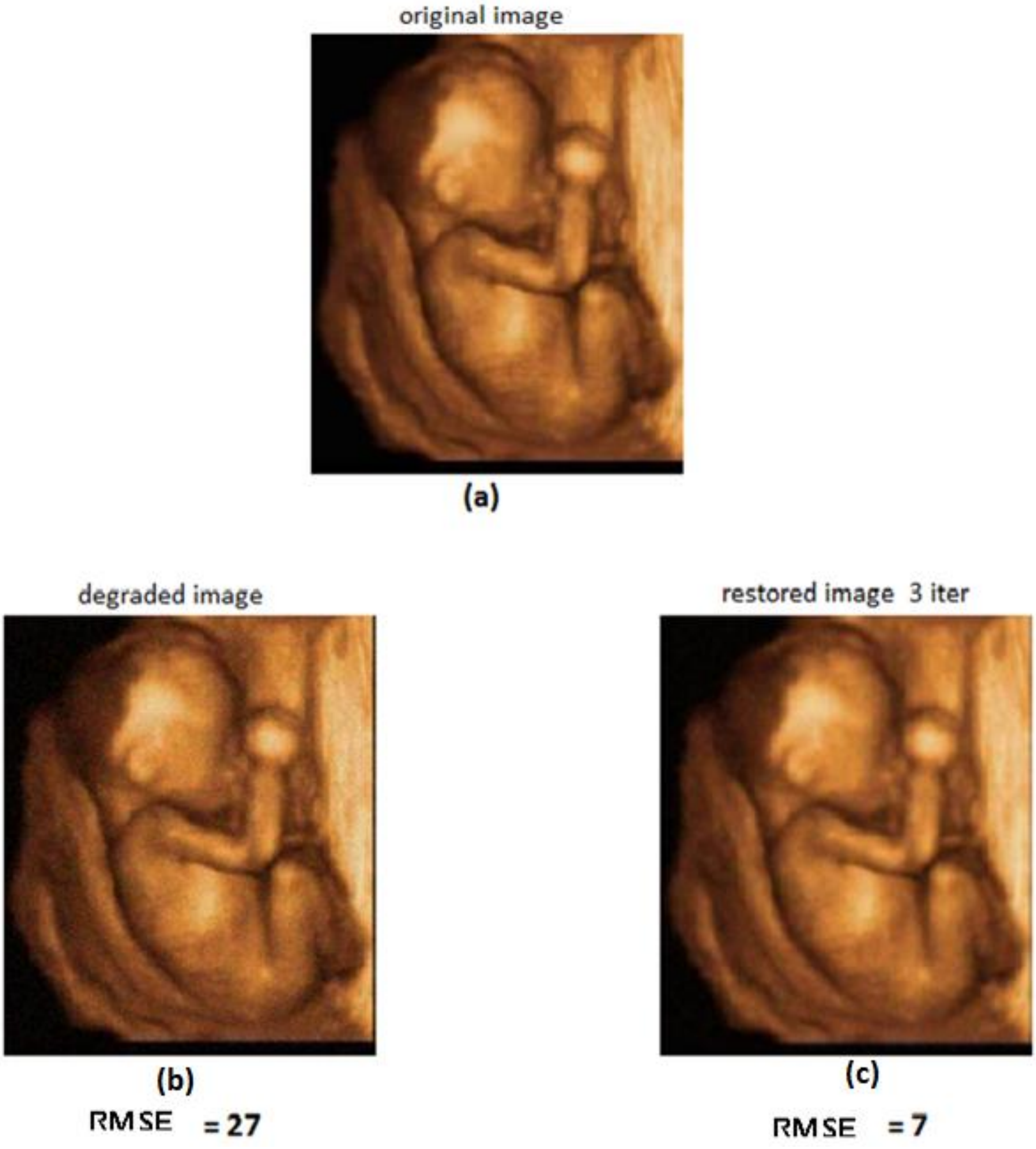


Figure (4-16) Present (a) original image. (b) represent the degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 20. (c) restored image.

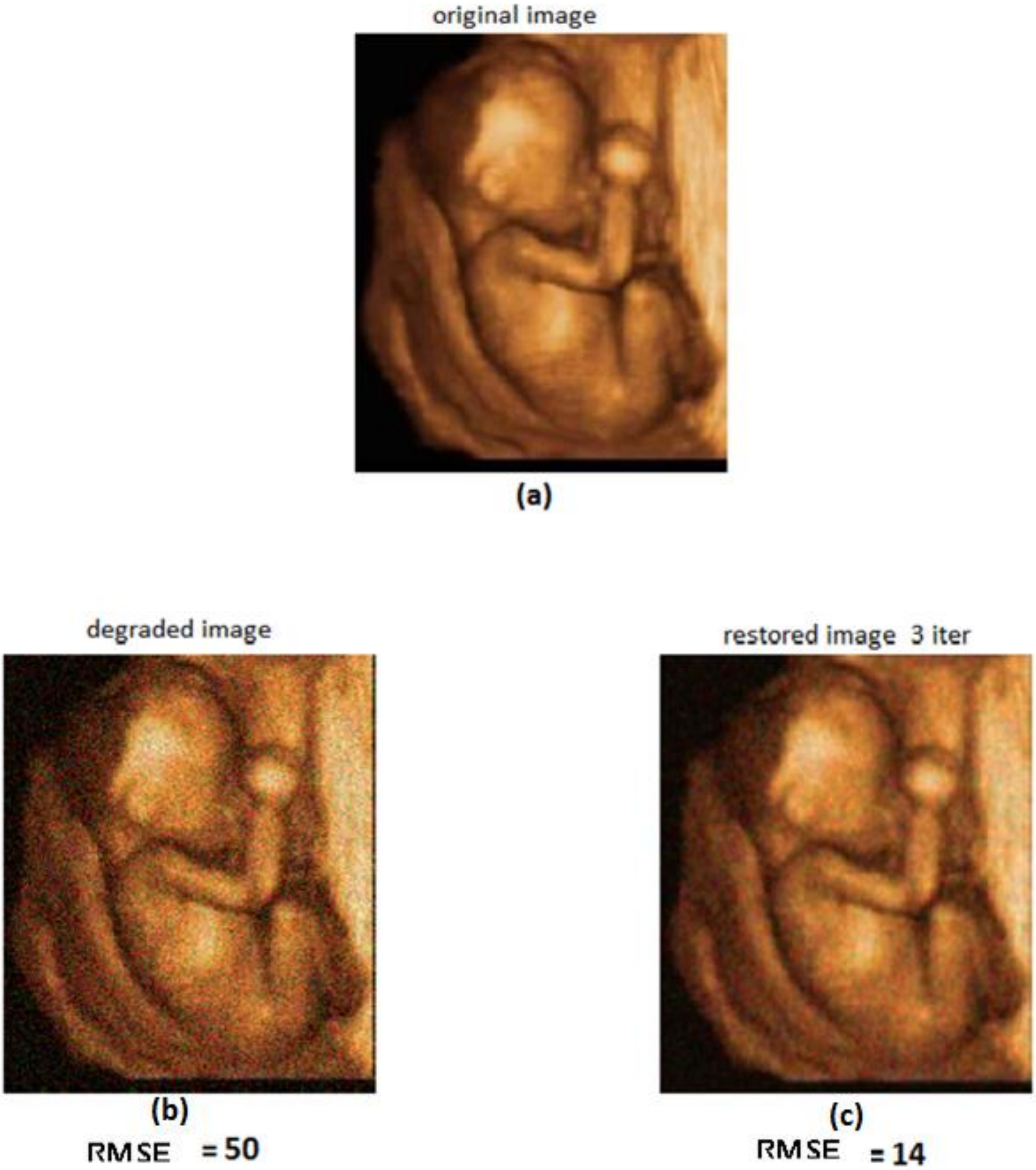


Figure (4-17) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 10 (c) restored image.

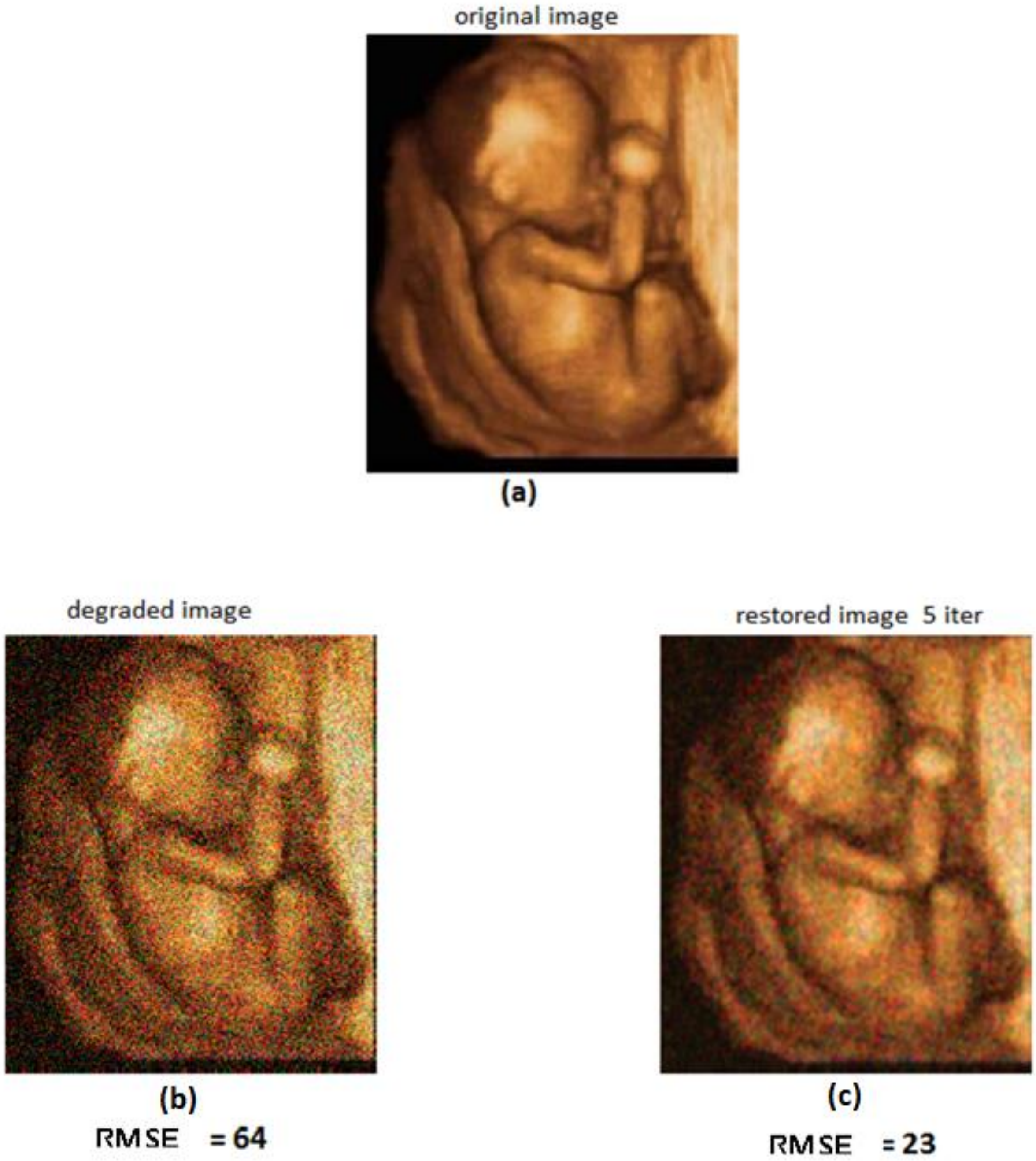


Figure (4-18) Present (a) original image (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 5 (c) restored image.

- **bird image**

Iterative Tikhonov filter, in which optimum regularization parameter" α" have value α =1 which have generally best value of RMSE, use to Restore the degraded bird image, blurring with Gaussian blurring function for different standard deviation values and distorted with Gaussian noise at different noise ratio as shown in the following figures.

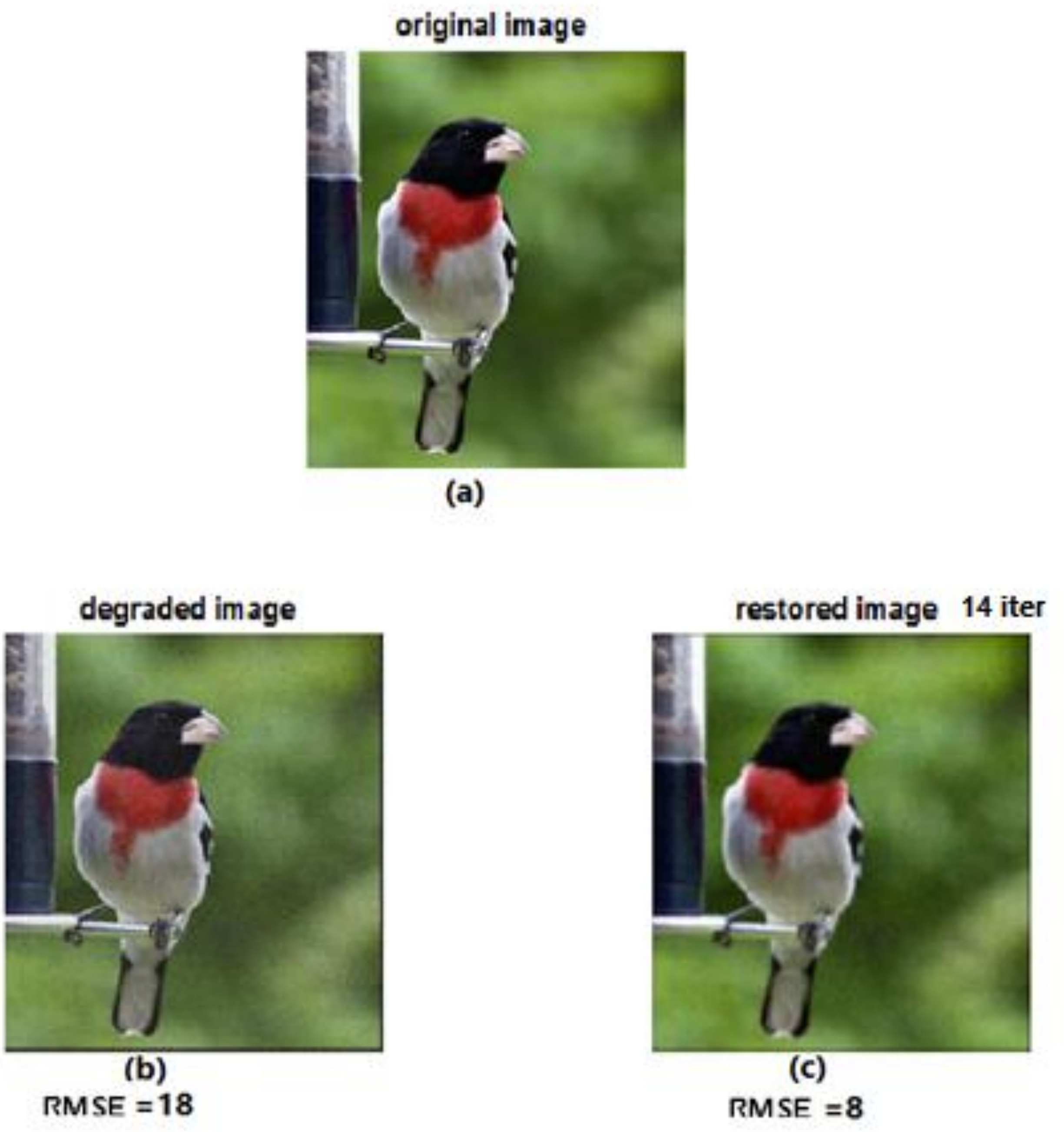


Figure (4-19) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 20. (c) restored image.

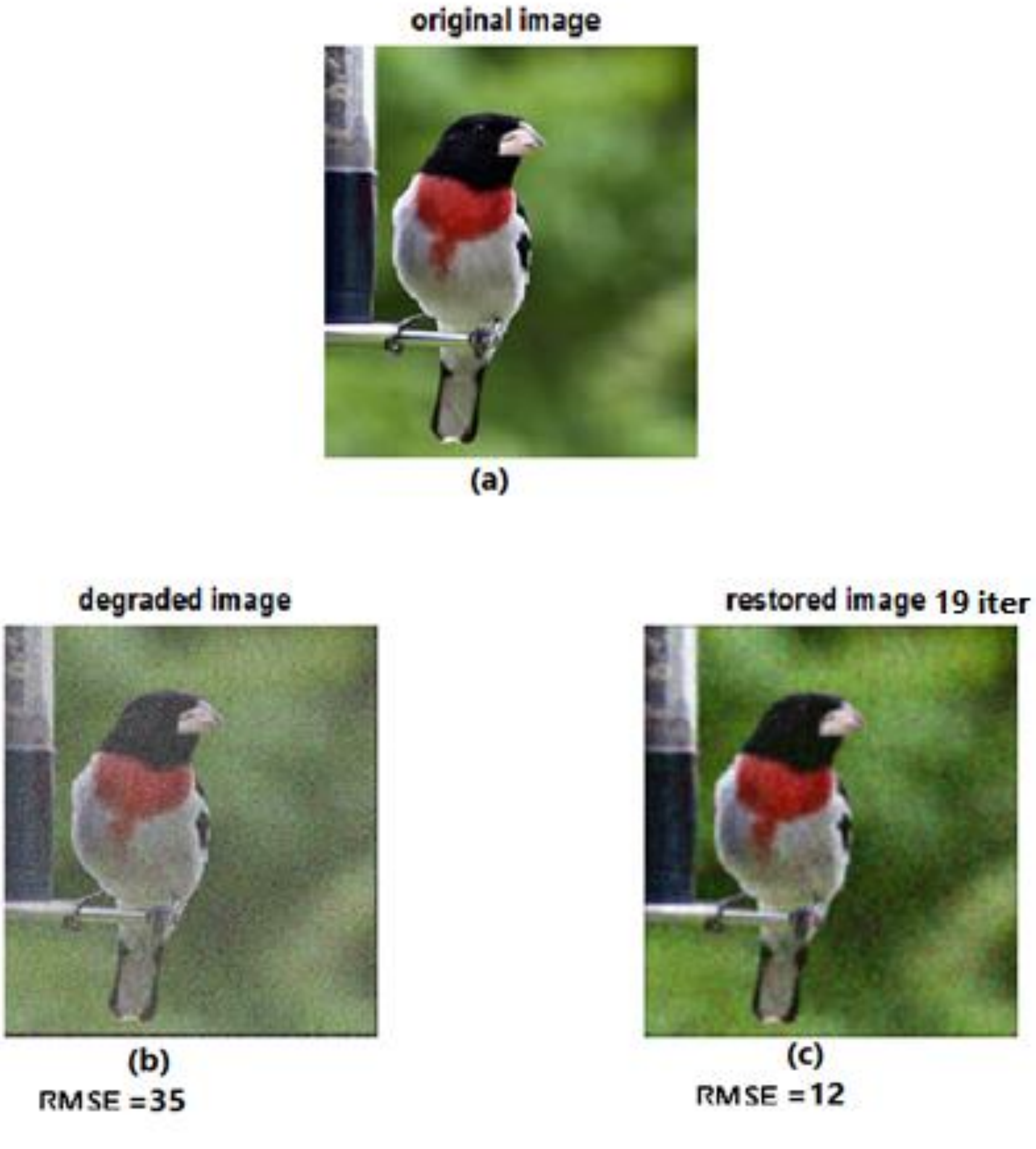


Figure (4-20) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 10. (c) restored image using.

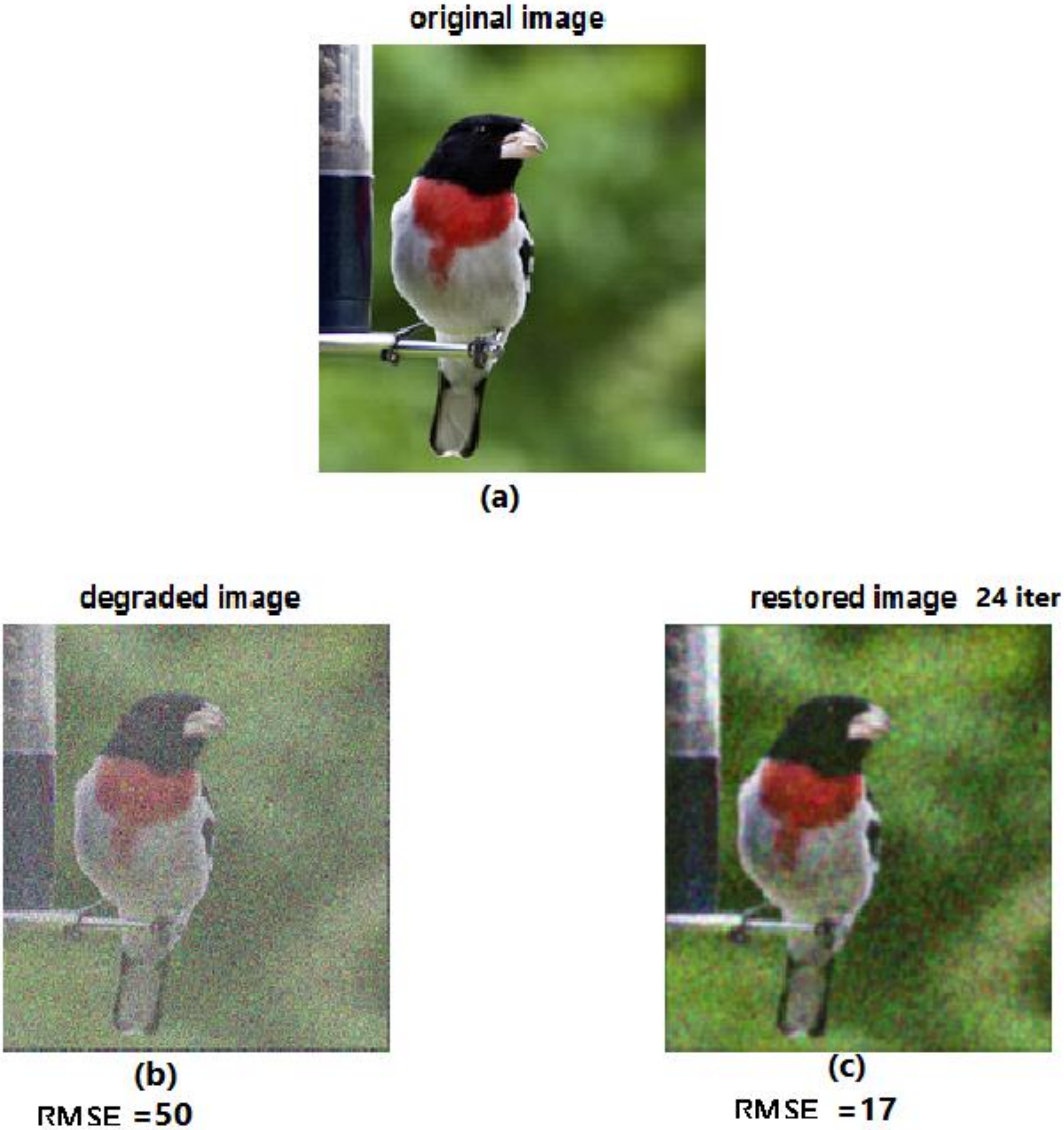


Figure (4-21) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 5. (c) restored image.

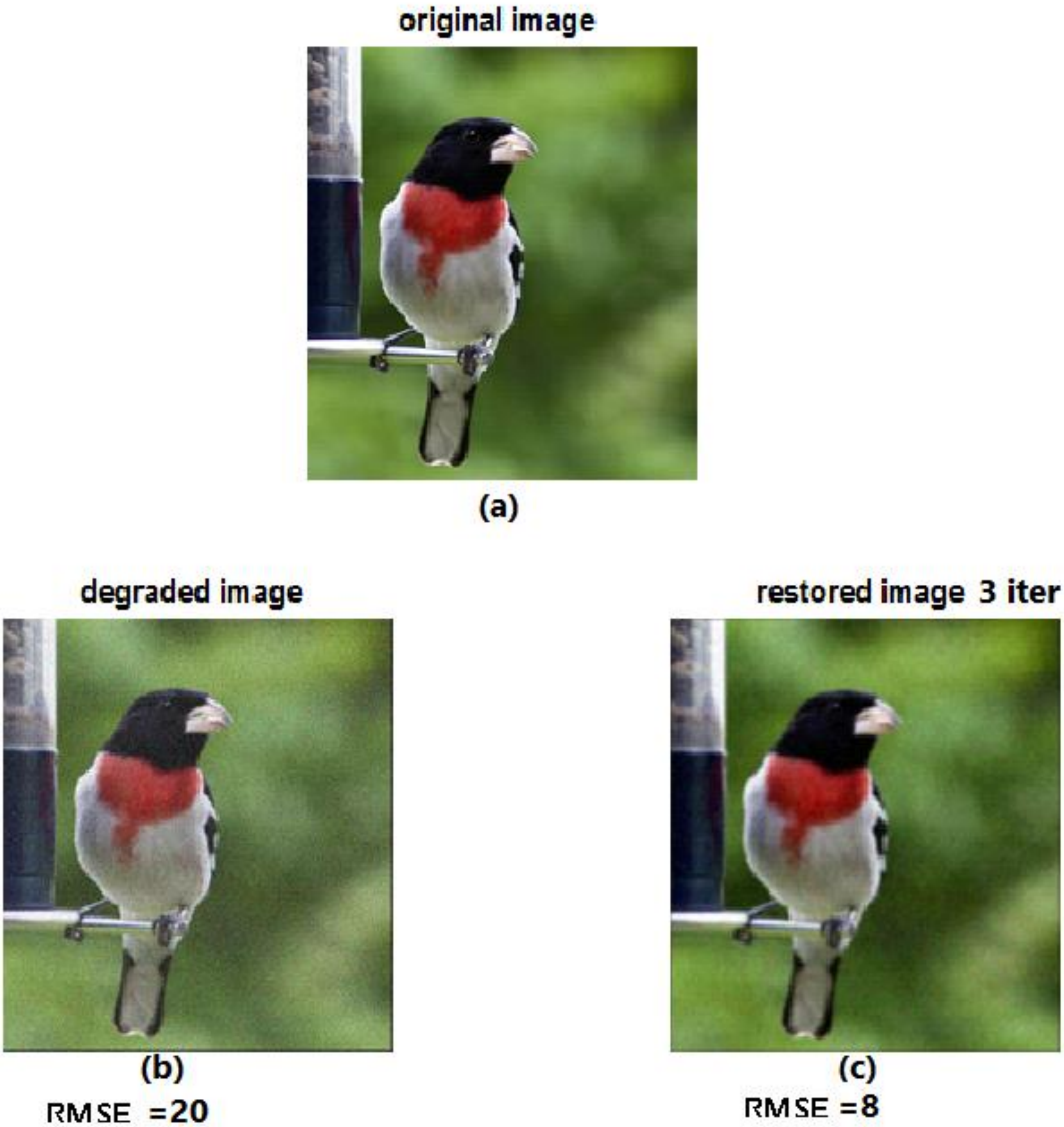


Figure (4-22) Present (a) original image. (b) degraded image, Gaussian function of $\sigma = 2$ and Gaussian noise of SNR = 20. (c) restored image.

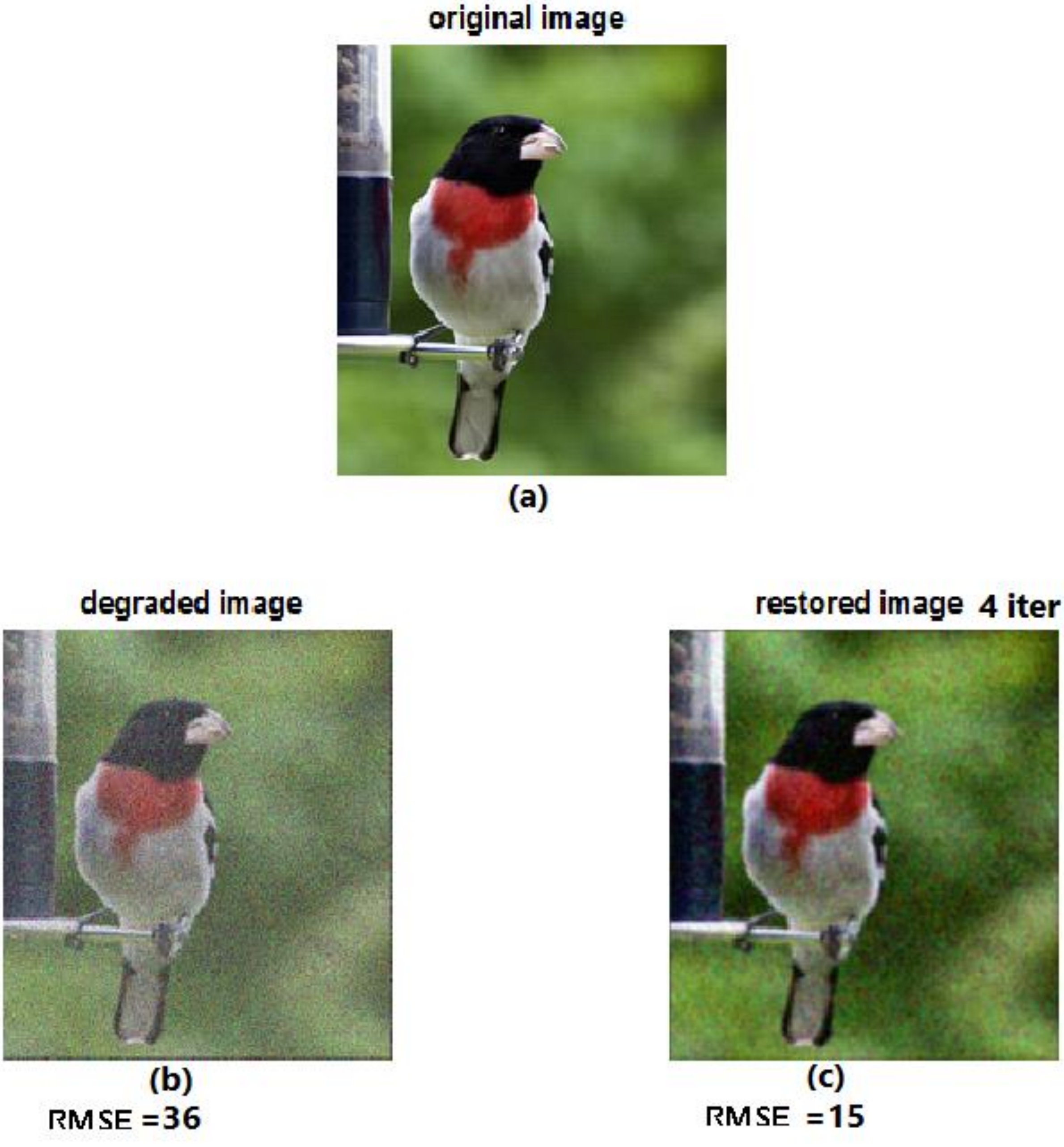


Figure (4-23) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 10. (c) restored image.

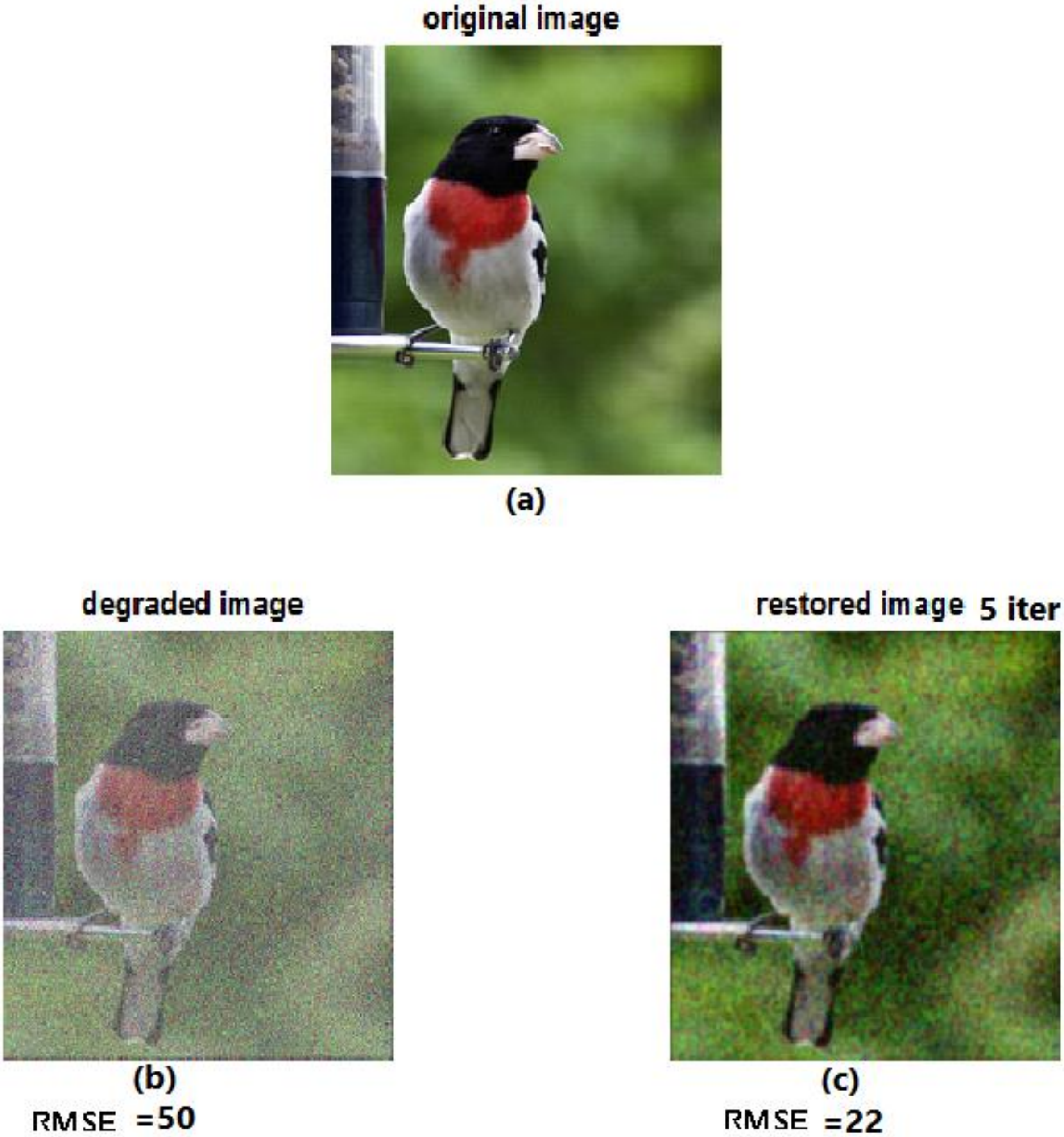


Figure (4-24) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 5. (c) restored image.

II. Wiener filter

Wiener filter improve most the three selected images and the best result of quality image showed by the minimum value of RMSE. And the result of filter shown in the following figures as follow:

- **satellite image**

Wiener filter, use to Restore the degraded satellite image, blurring with Gaussian blurring function for different standard deviation values and distorted with Gaussian noise at different noise ratio as shown in the following figure.

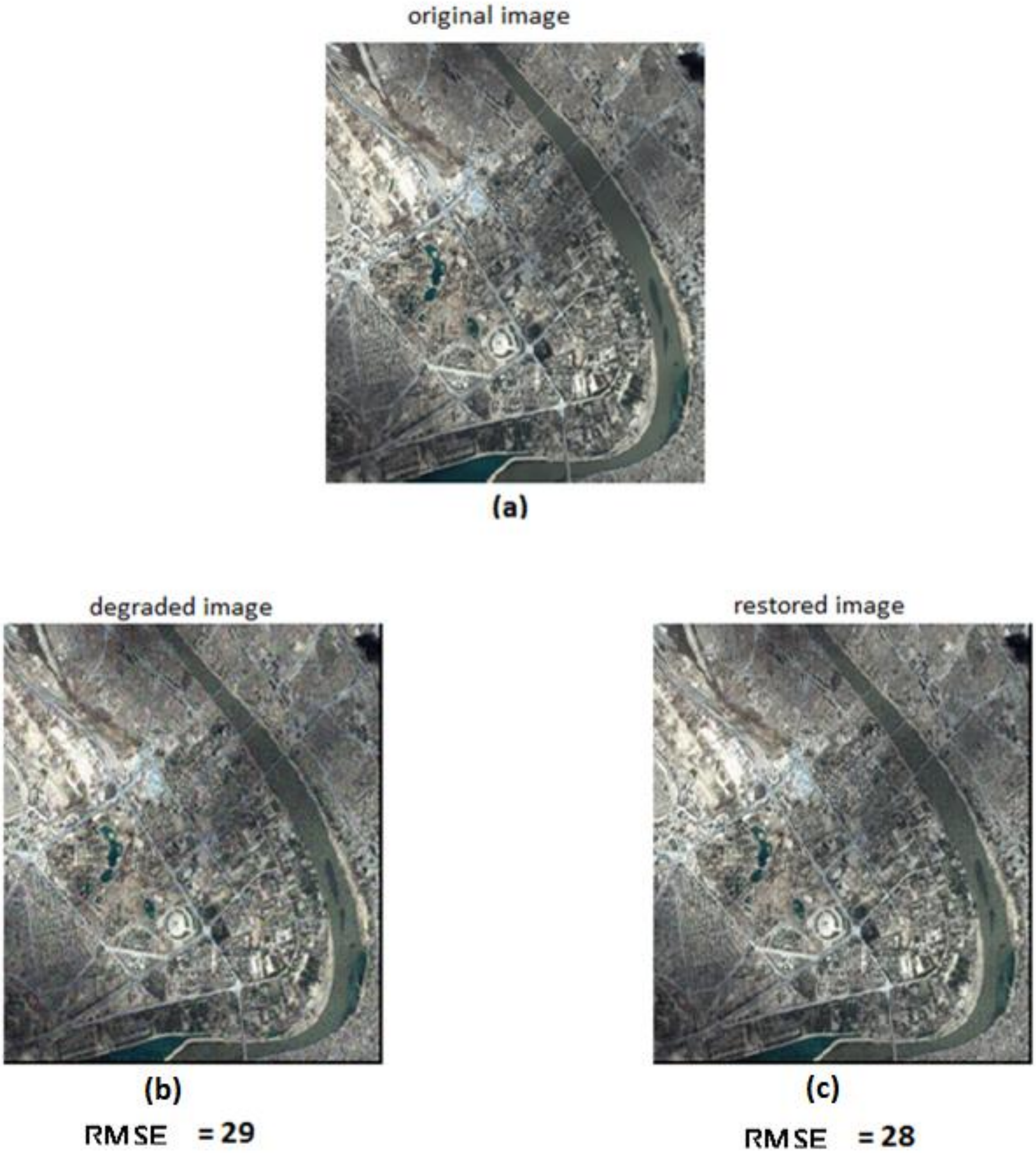


Figure (4-25) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 20. (c) restored image.

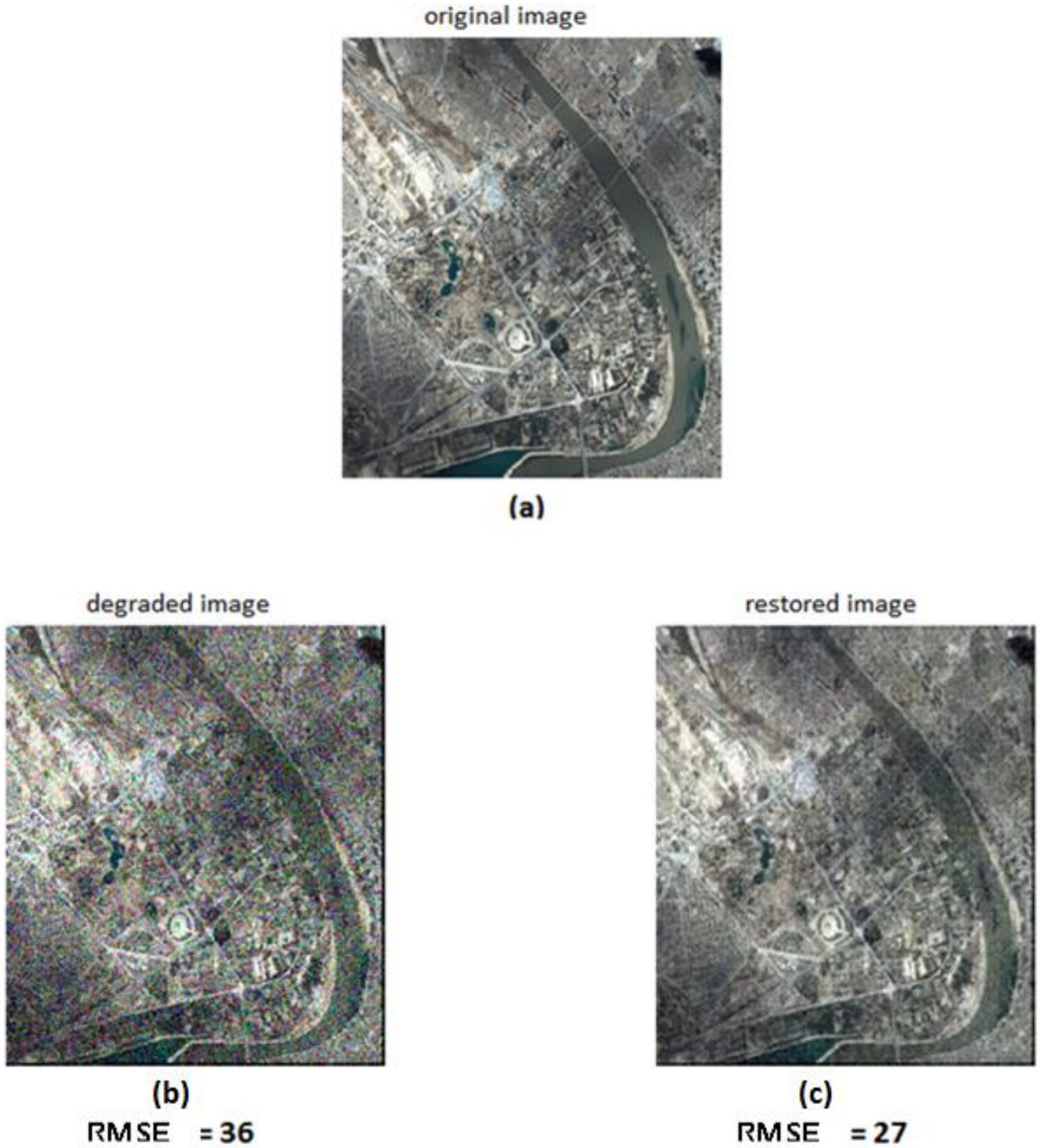


Figure (4-26) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 10. (c) restored image.

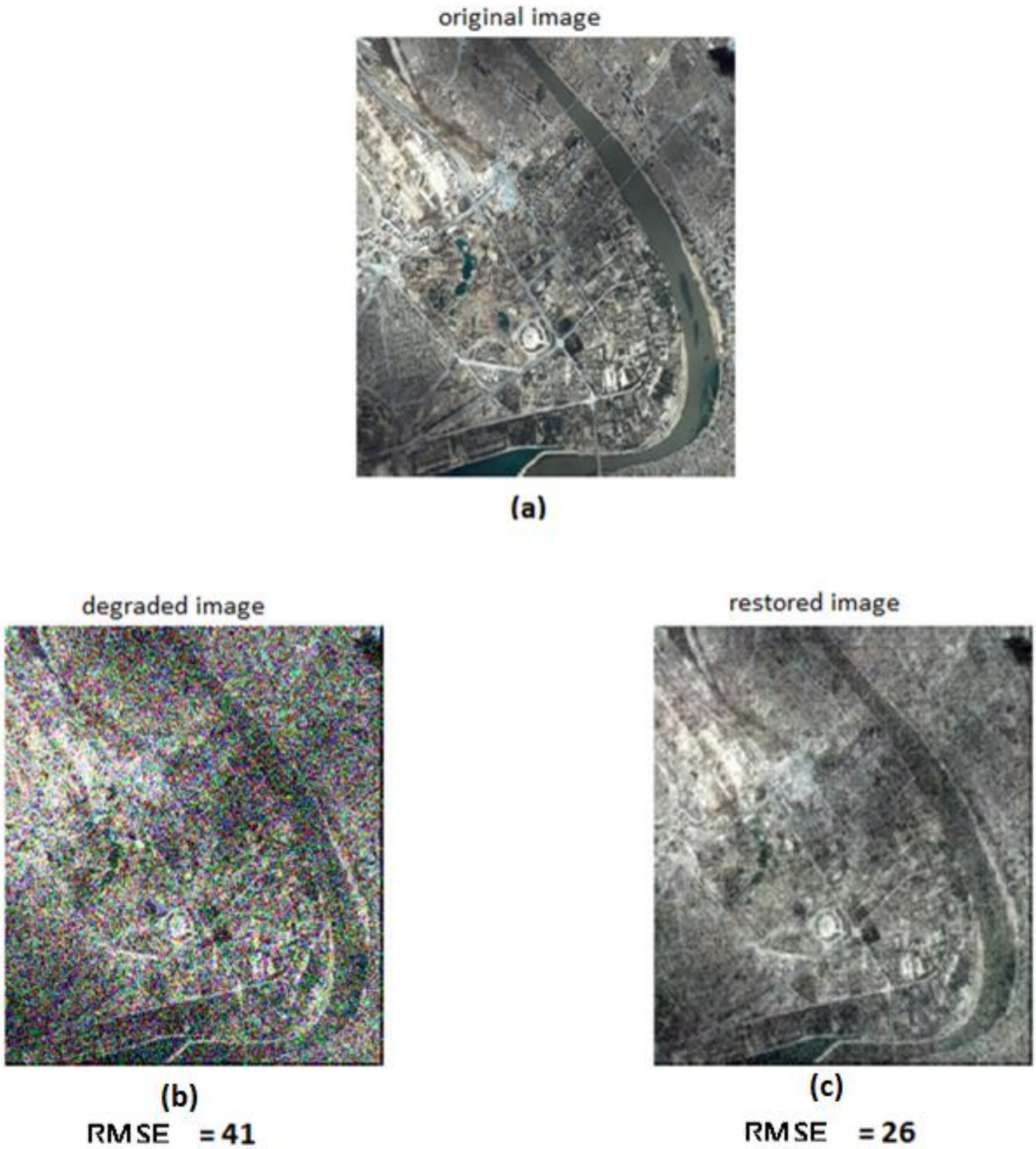


Figure (4-27) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 5. (c) restored image.

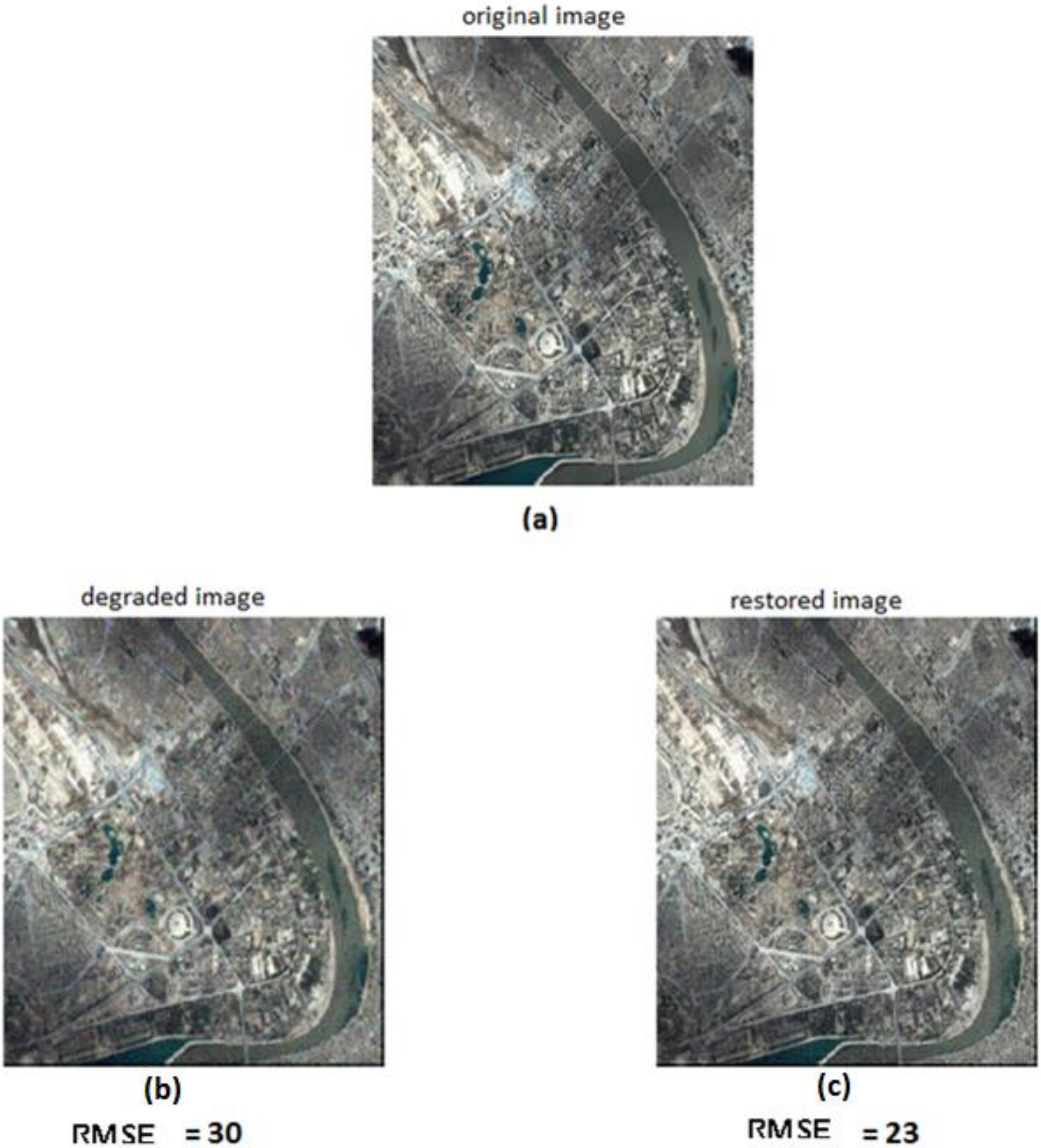


Figure (4-28) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 20. (c) restored image.

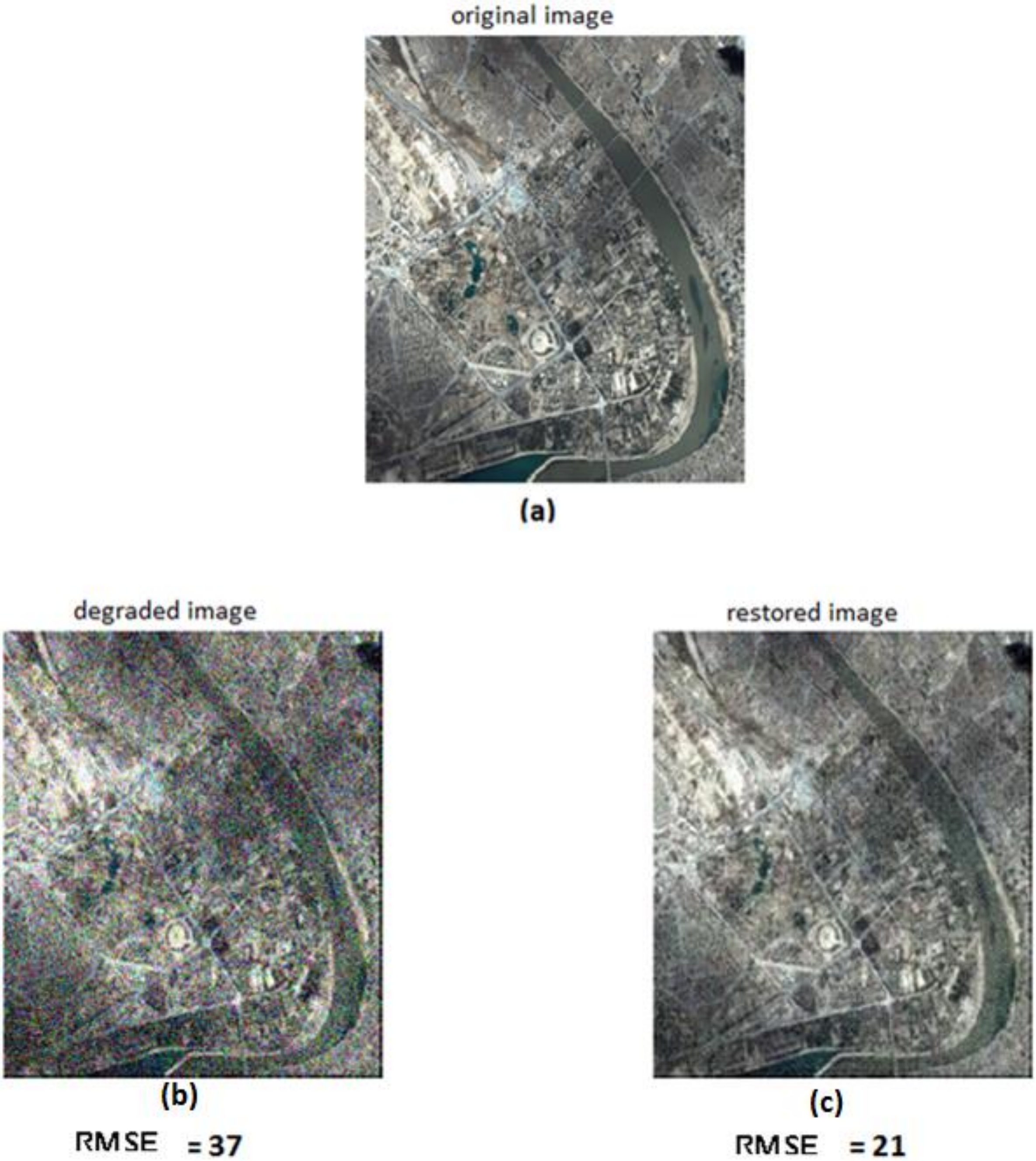


Figure (4-29) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 10. (c) restored image.

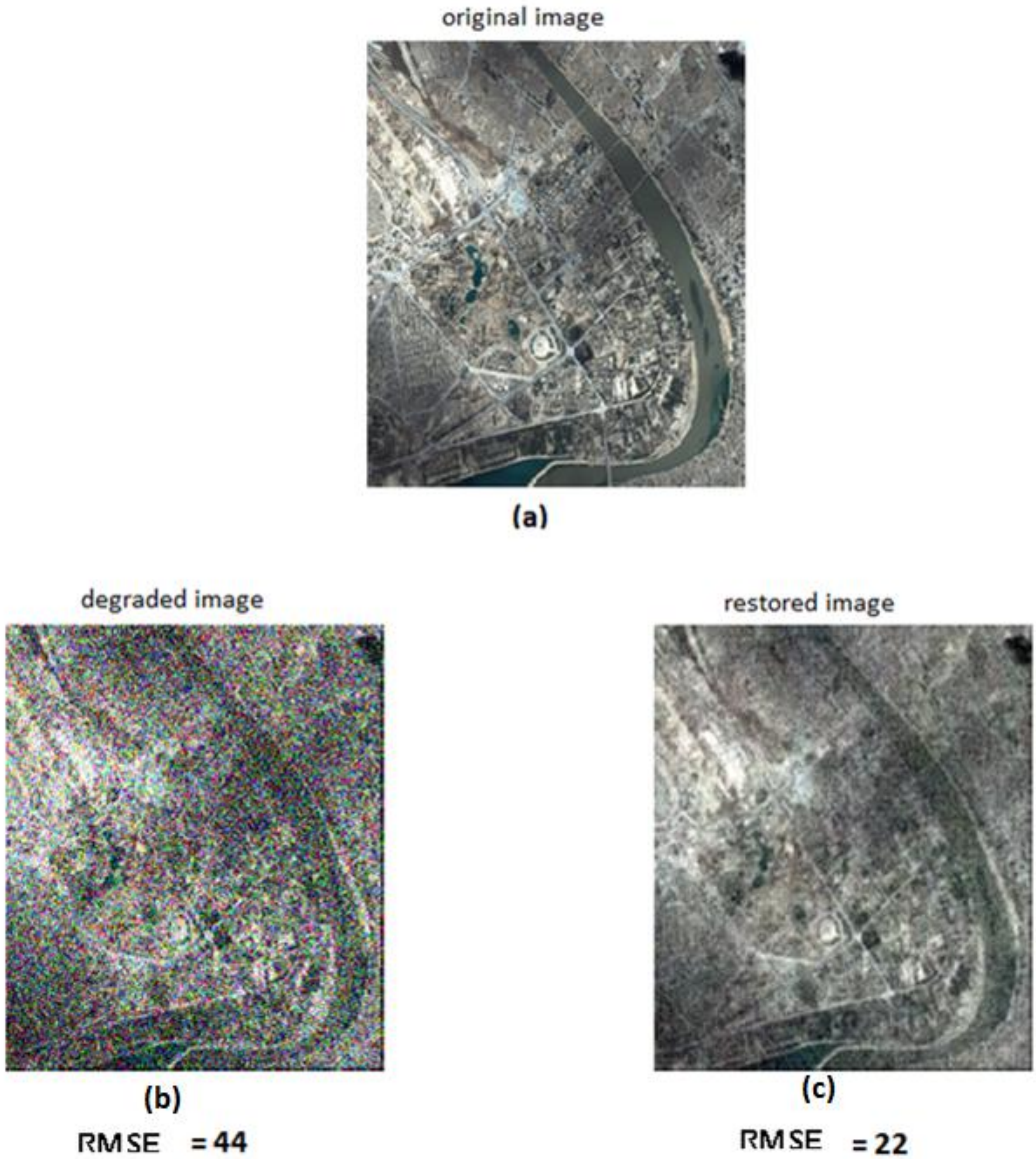


Figure (4-30) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 5. (c) restored image.

- **Embryo image**

Wiener filter, use to Restore the degraded Embryo image, blurring with Gaussian blurring function for different standard deviation values and distorted with Gaussian noise at different noise ratio as shown in the following figures.

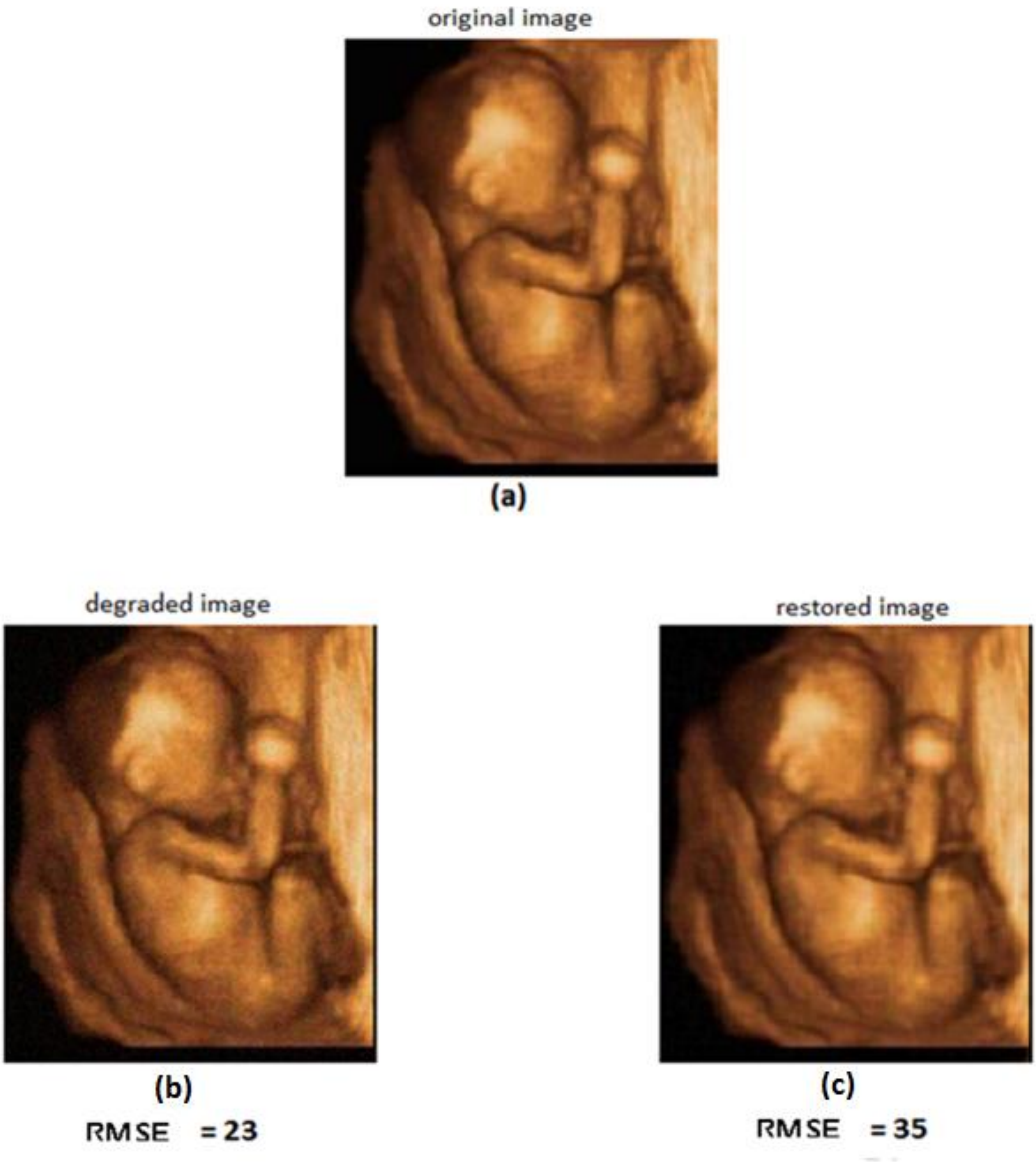


Figure (4-31) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 20. (c) restored image.

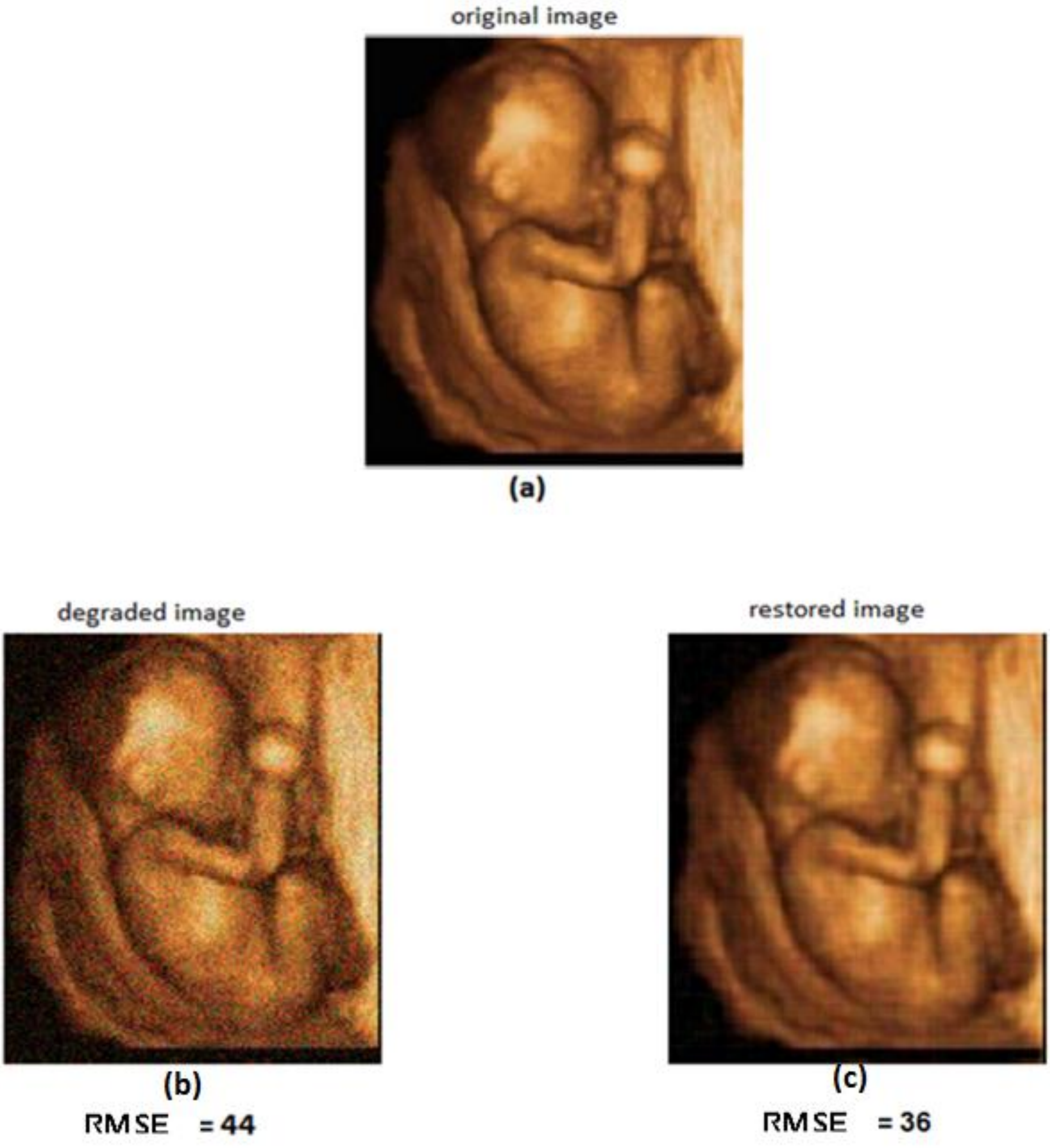


Figure (4-32) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 10. (c) restored image.

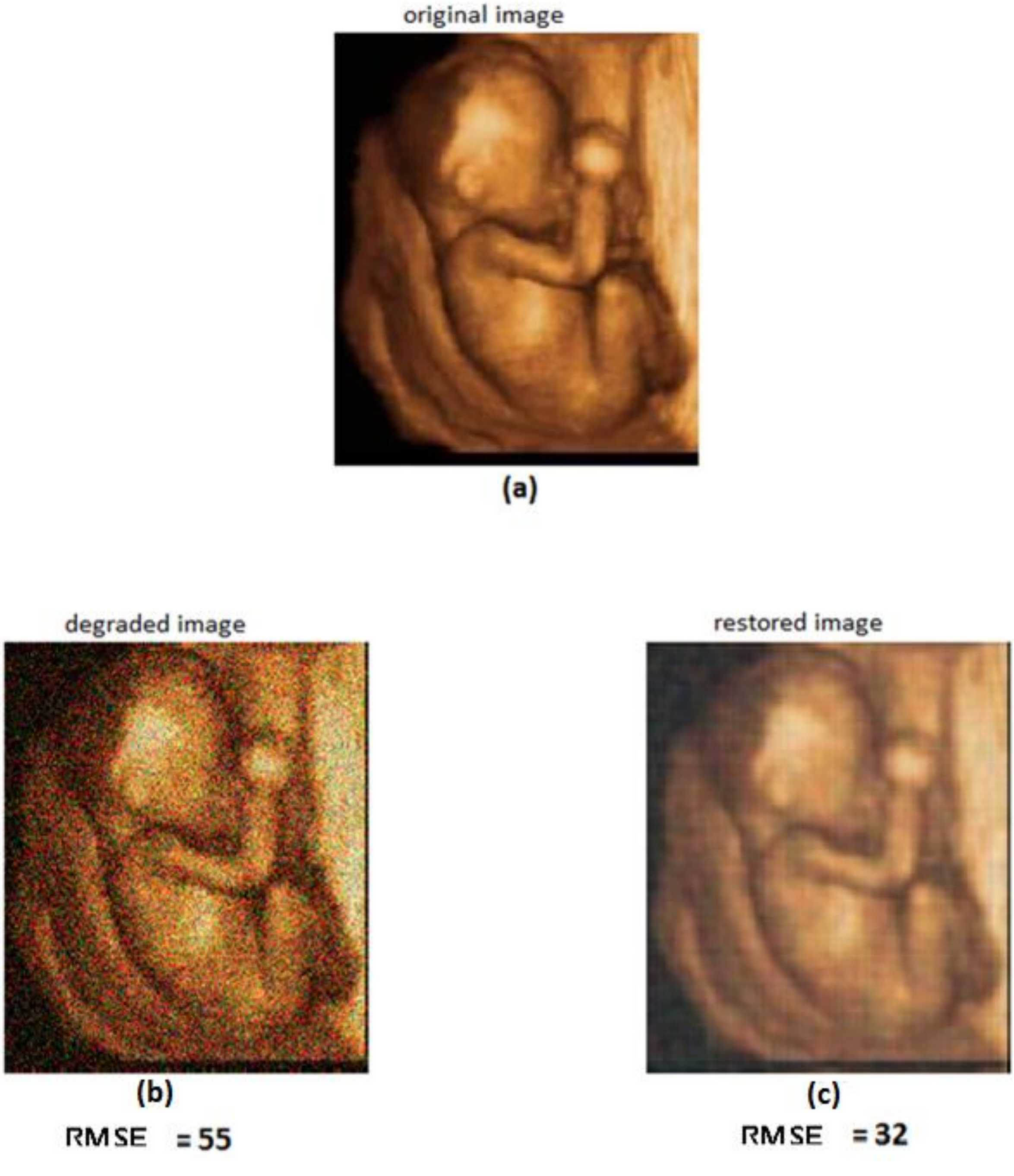


Figure (4-33) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 5 (c) restored image.

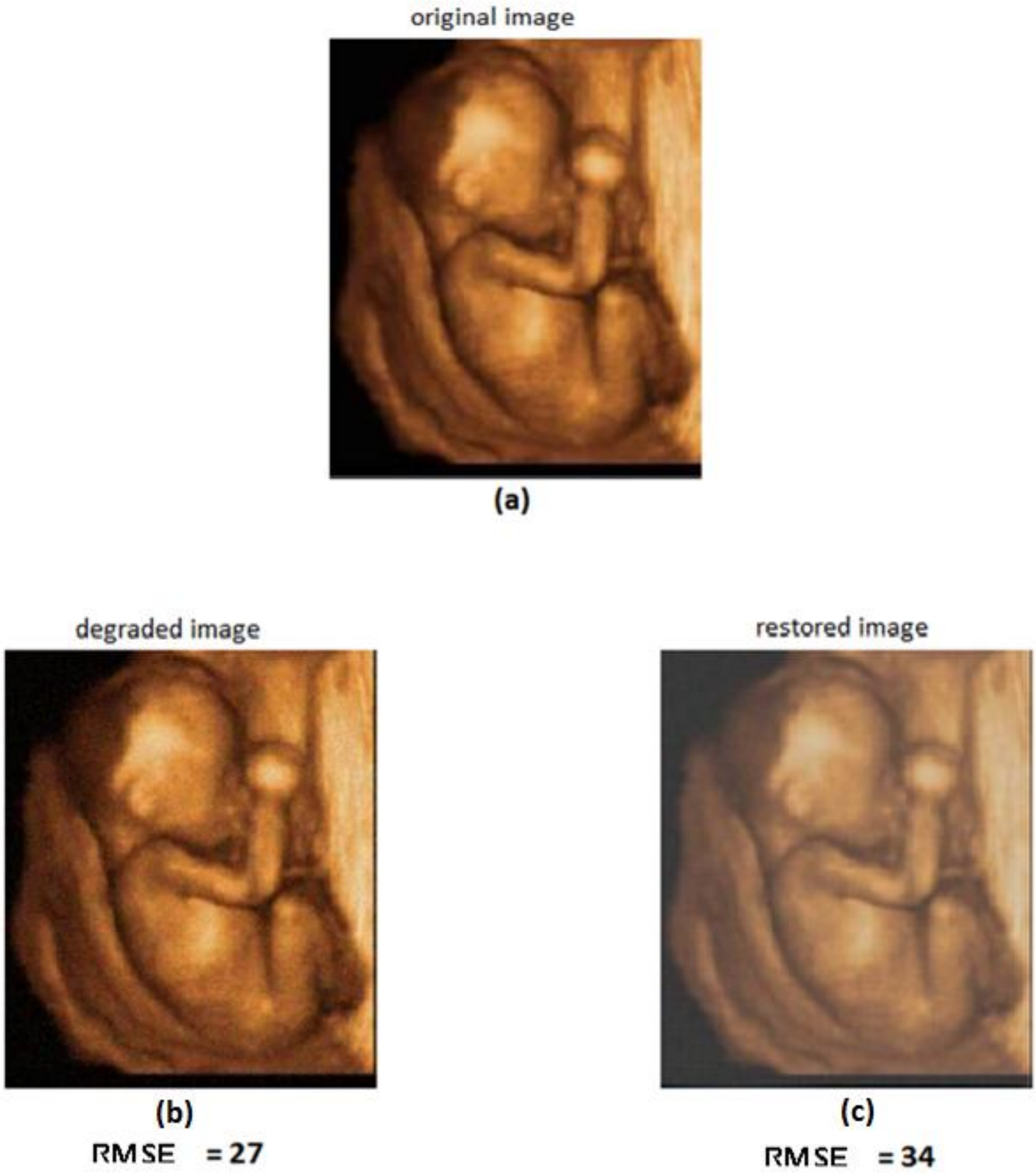


Figure (4-34) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 20. (c) restored image.

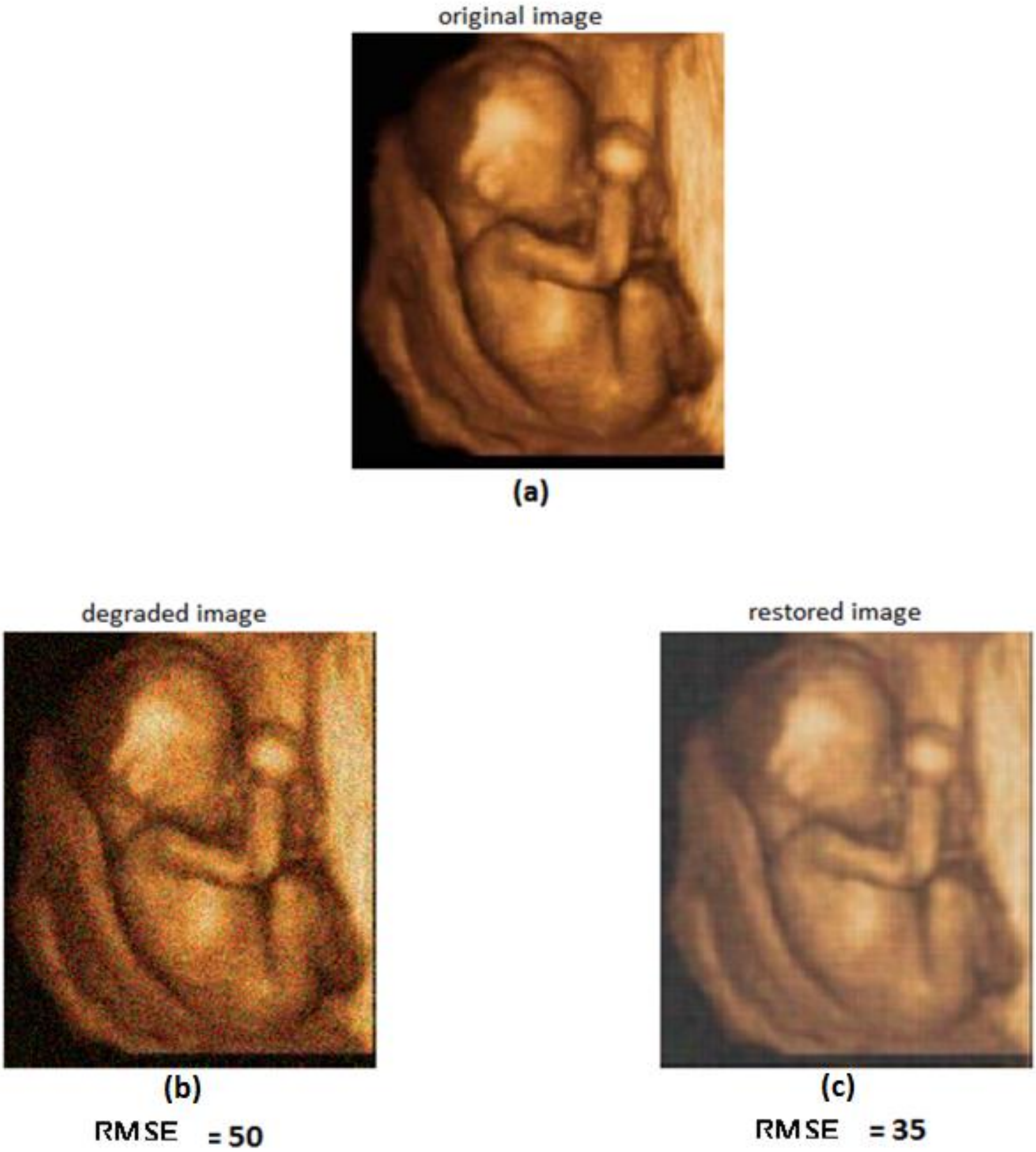


Figure (4-35) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 10 (c) restored image.

(a)

(b)

RMSE = 64

(c)

RMSE = 29

Figure (4-36) Present (a) the original image (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 5 (c) restored image.

- **bird image**

Wiener filter, use to Restore the degraded bird image, blurring with Gaussian blurring function for different standard deviation values and distorted with Gaussian noise at different noise ratio as shown in the following figures.

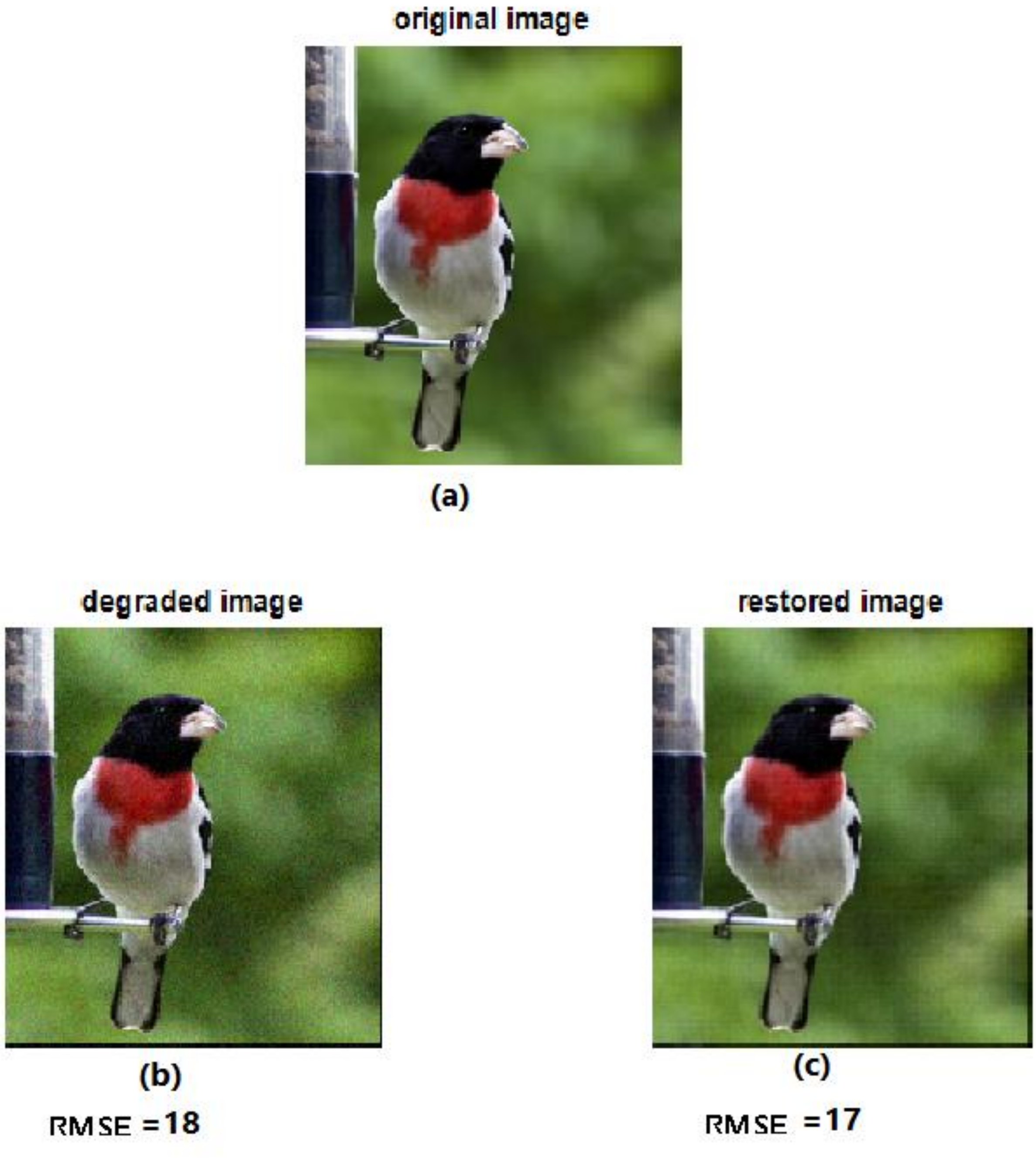


Figure (4-37) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 20. (c) restored image.

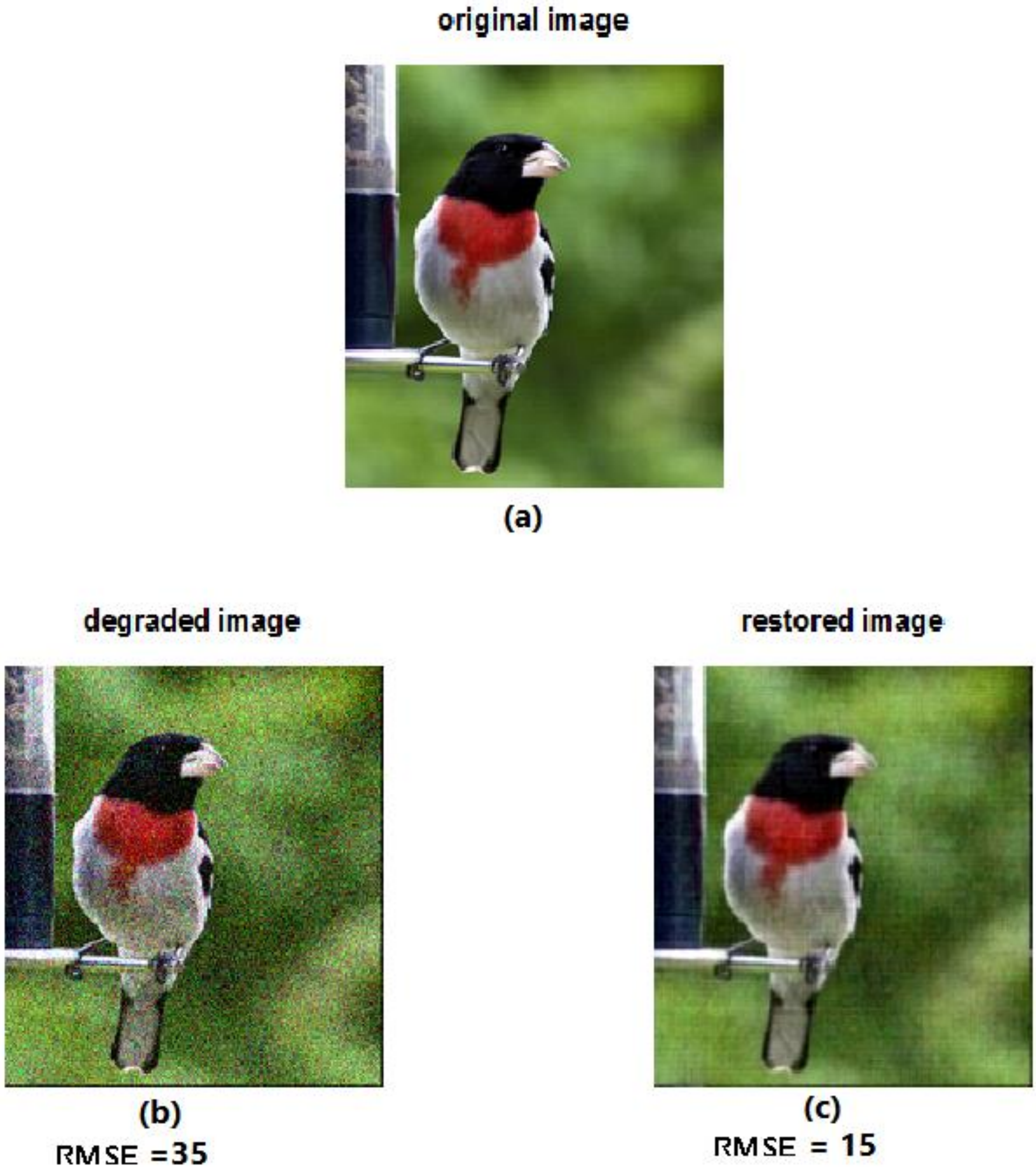


Figure (4-38) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 10. (c) restored image.

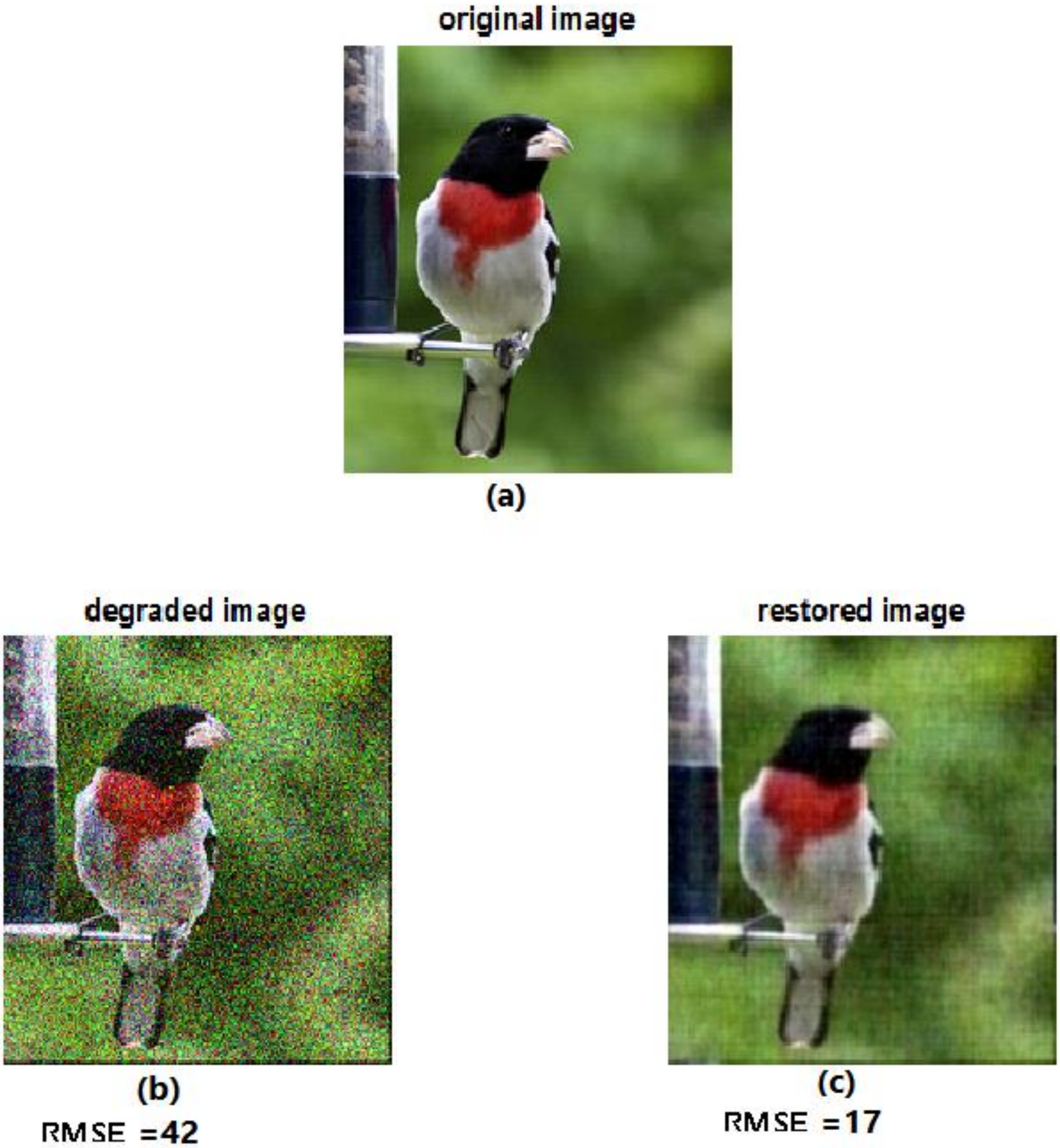


Figure (4-39) Present (a) original image. (b) degraded image, Gaussian function of σ = 1 and Gaussian noise of SNR = 5. (c) restored image.

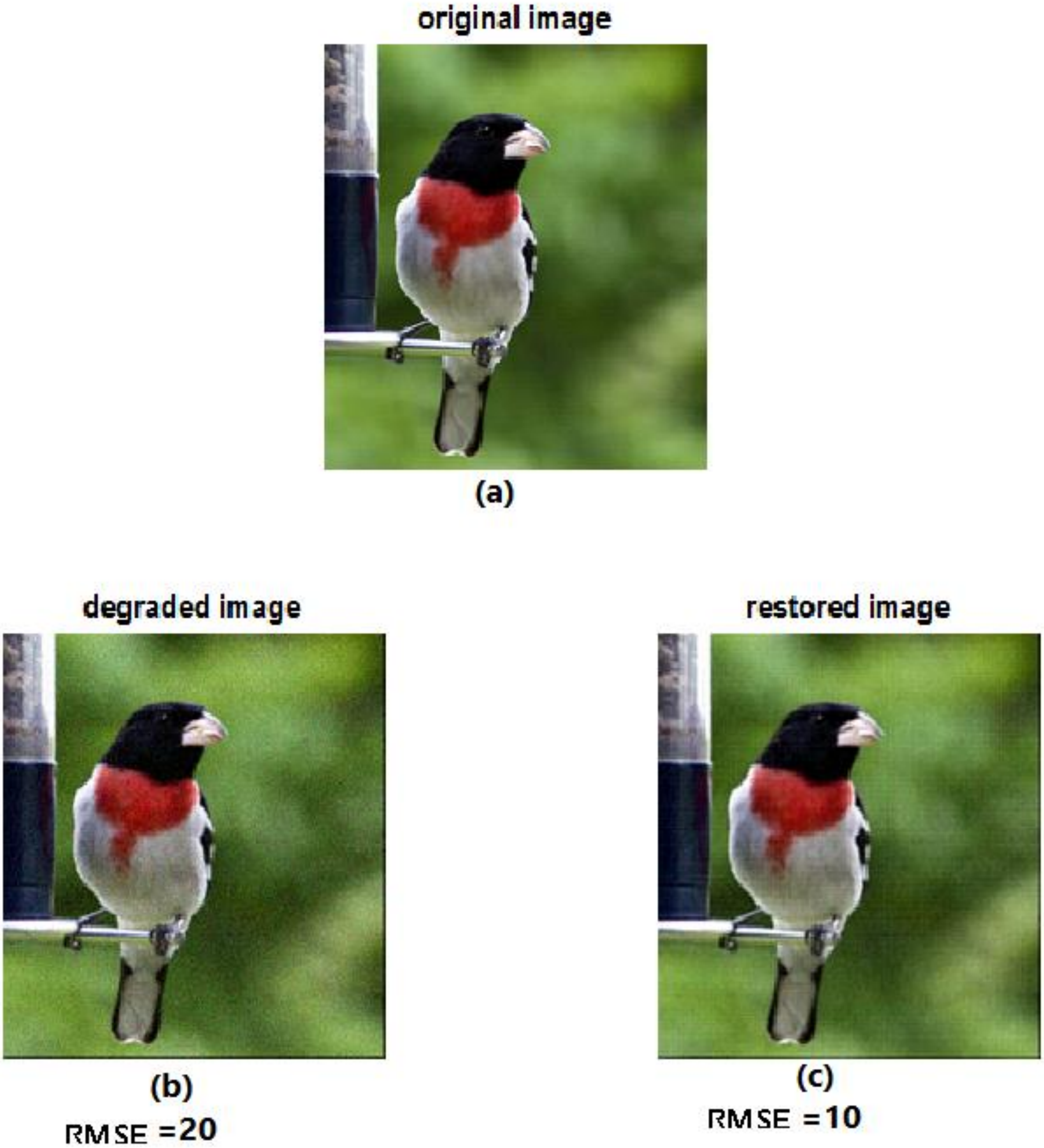


Figure (4-40) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 20. (c) restored image.

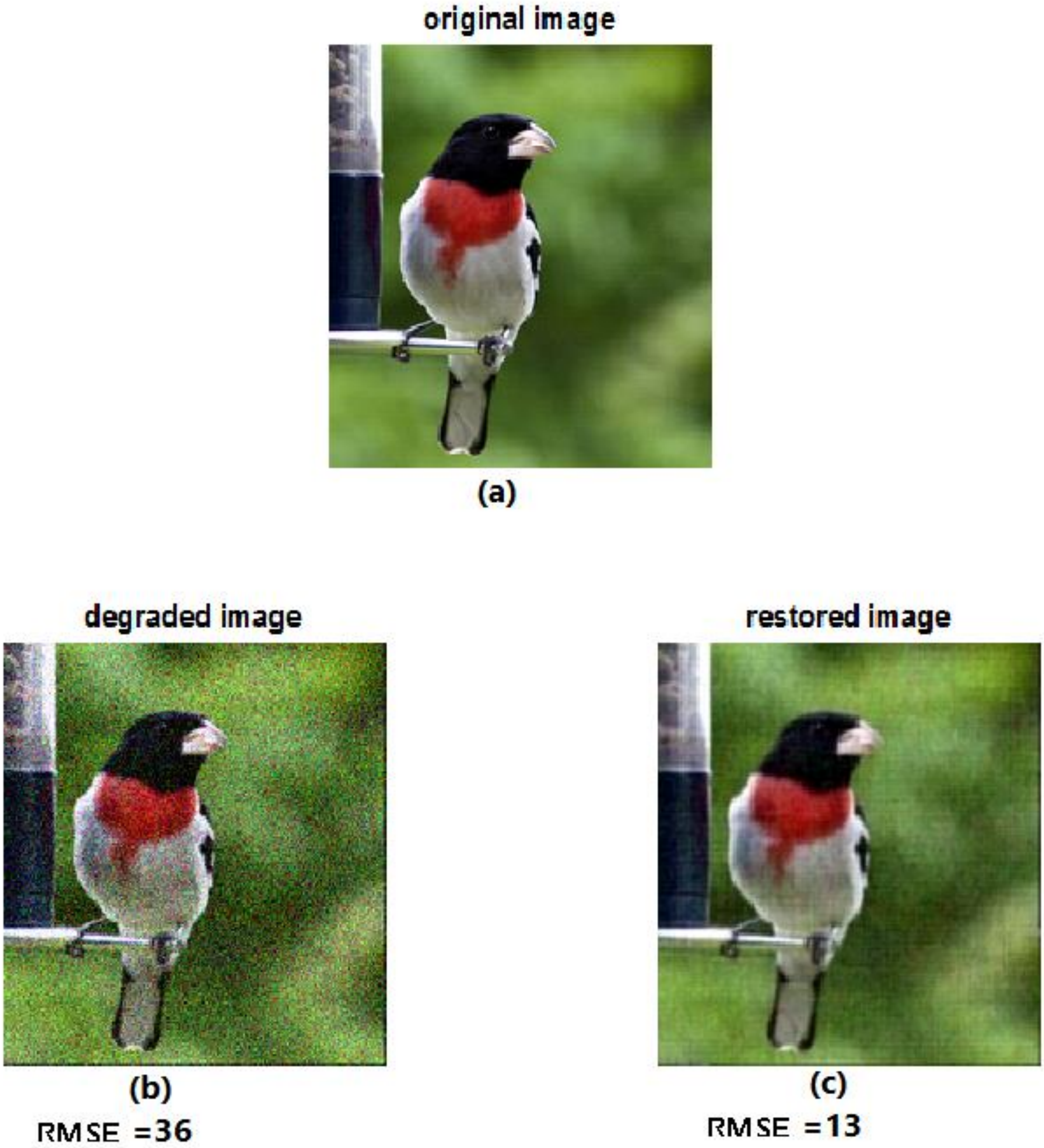


Figure (4-41) Present (a) original image. (b) degraded image, Gaussian function of $\sigma = 2$ and Gaussian noise of SNR = 10. (c) restored image.

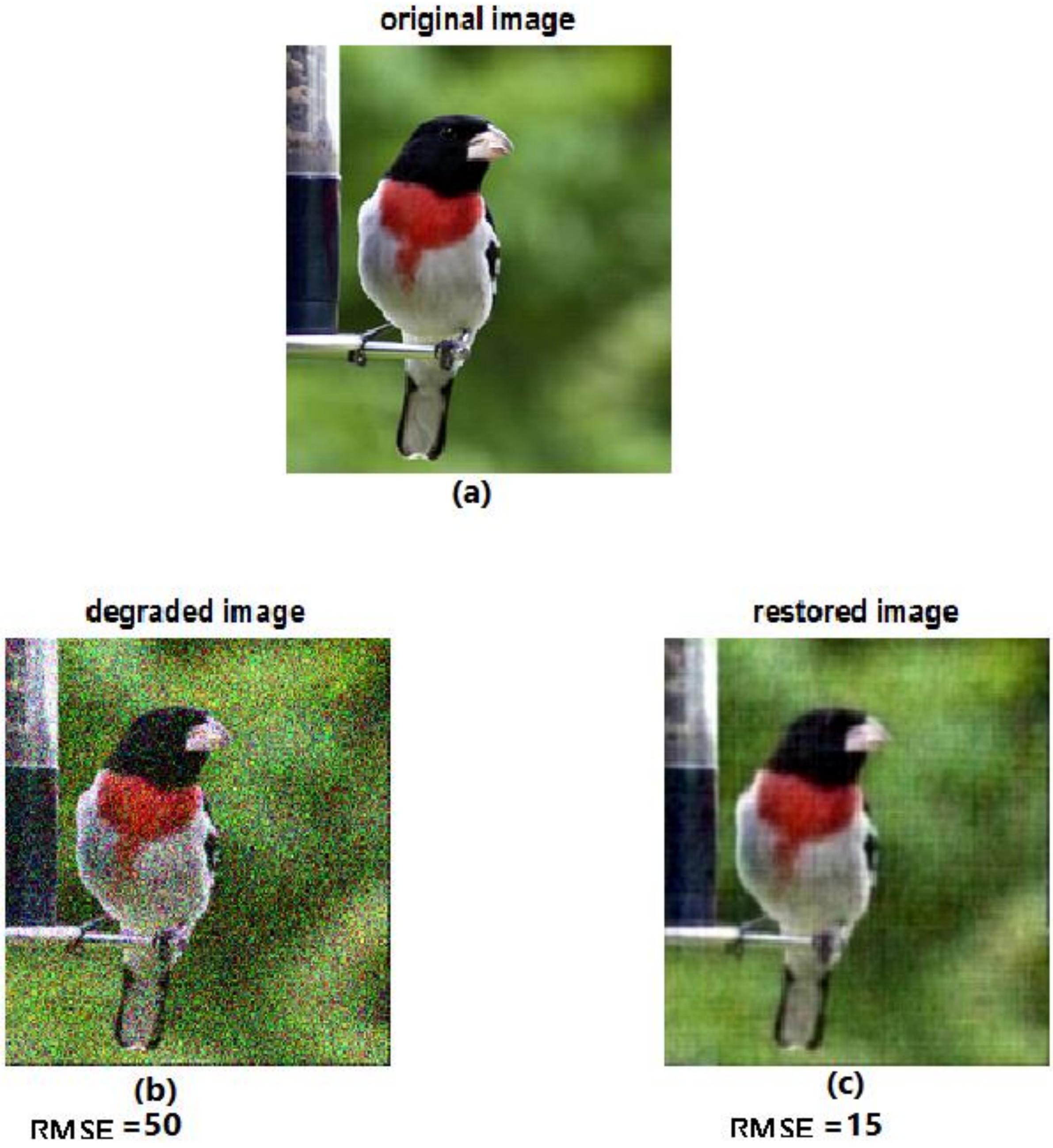


Figure (4-42) Present (a) original image. (b) degraded image, Gaussian function of σ = 2 and Gaussian noise of SNR = 5. (c) restored image.

4.2.3 Relation between parameters

Root mean square error has been used as a criterion to check the effectiveness of image restoration filter. Plotting the relation between Root Mean Square Error and many parameters " SNR and Diff. No. of iteration" to discuss the results as shown in the following figures.

Figure (4-43) show that the RMSE of three selected degraded images increase with increment standard deviation values " σ = 1,2" of Gaussian blurring function and decrease with increasing SNR.

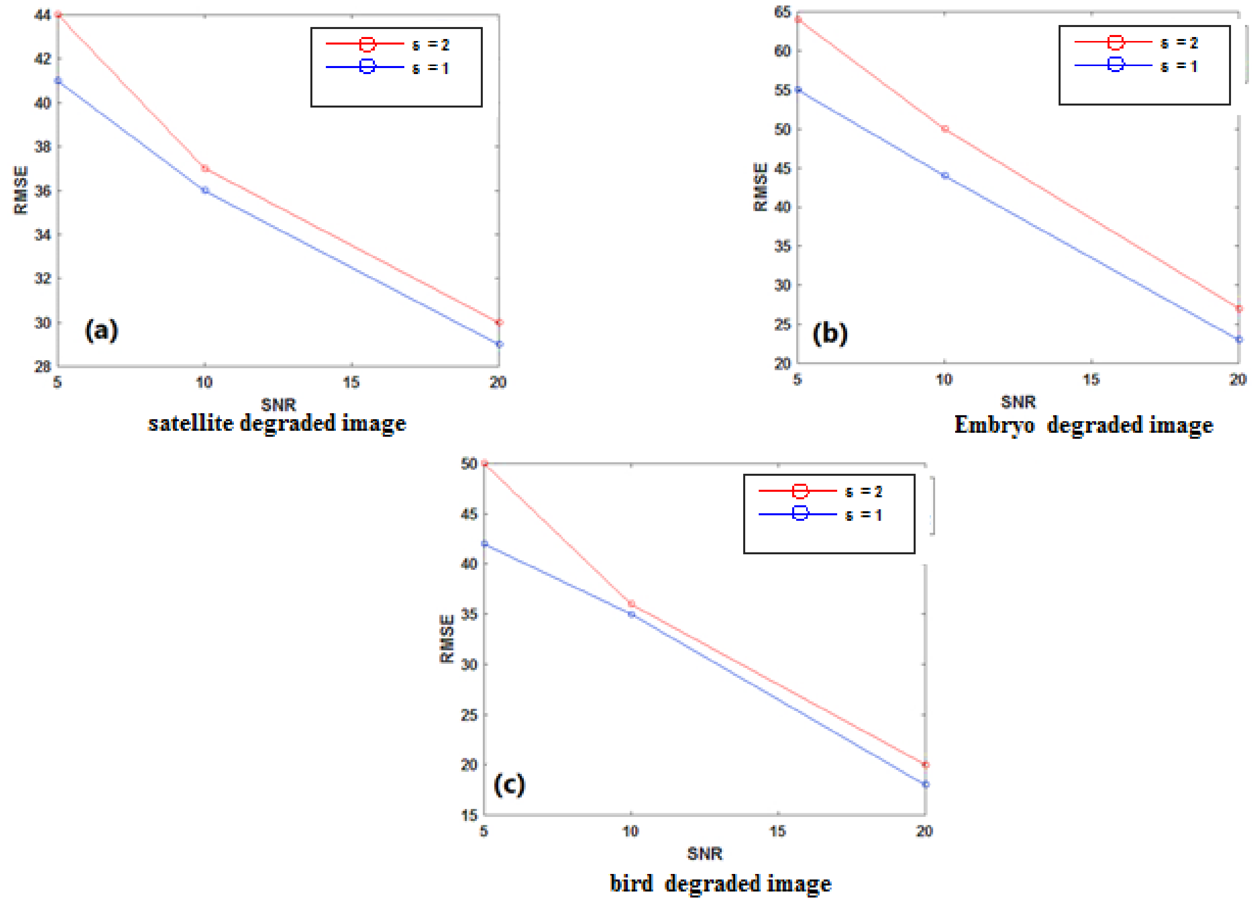


Figure (4-43) shows (a) RMSE Versus SNR of satellite degraded image (b) RMSE Versus SNR of Embryo degraded image (c) RMSE Versus SNR of bird degraded image

Figure (4-44) show (a) Root mean square error of satellite restored image using Wiener filter decrease with increase standard deviation values " σ " of Gaussian blurring function and in general increase for σ =1,2 with increment SNR. (b) Root Mean Square Error of Embryo restored image using Wiener filter decrease with increase standard deviation values " σ " of Gaussian blurring function and in general increasing for σ =1,2 with increase SNR. (c) Root mean square error of bird restored image using Wiener filter decrease with increase standard deviation values " σ " of Gaussian blurring function and in general decreasing for σ =1,2 with increase SNR

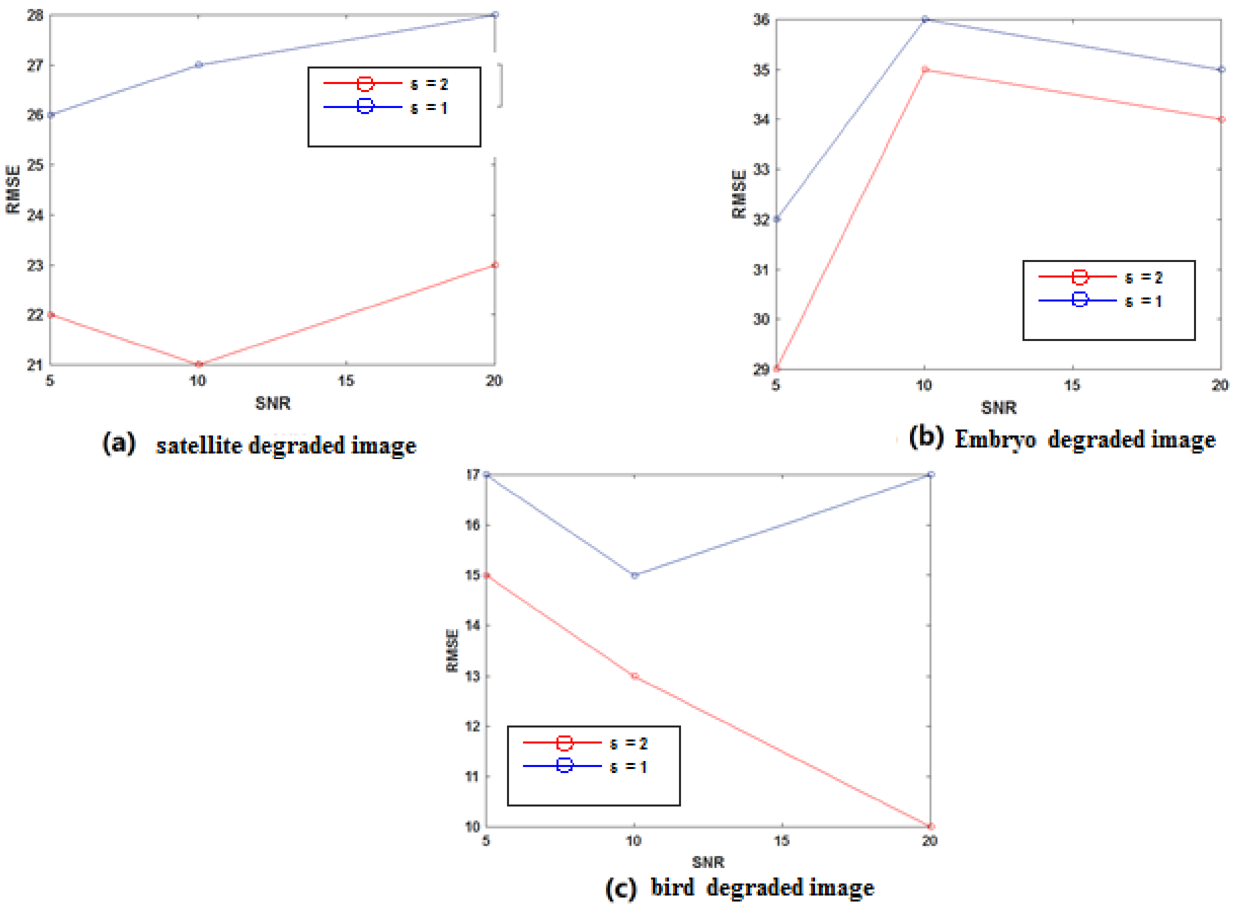
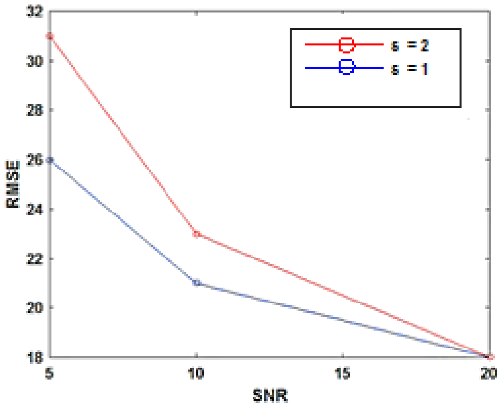


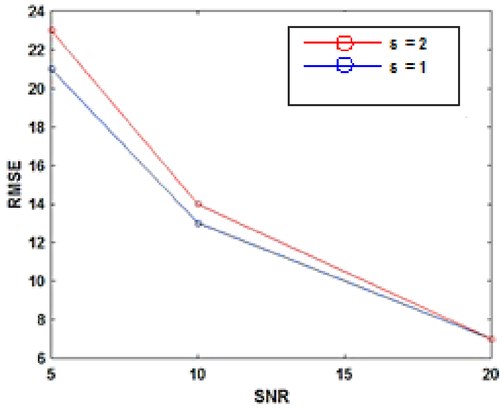
Figure (4-44) shows (a) RMSE Versus SNR of satellite restored image using Wiener filter (b) RMSE Versus SNR of Embryo restored image using Wiener filter (c) RMSE Versus SNR of bird restored image using Wiener filter.

Figure (4-45) Root mean square error of three restored images using Iterative Tikhonov-Miller filter " $\alpha = 1$ for satellite and bird images and $\alpha = 2$ for

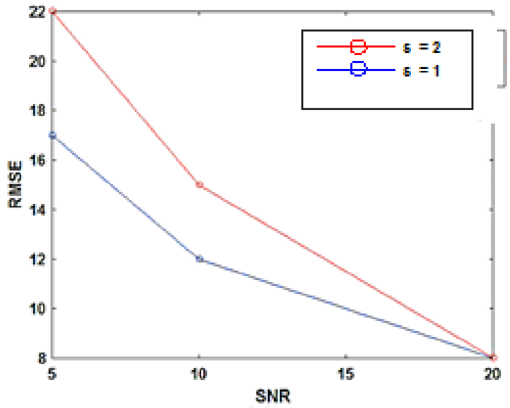
Embryo image" increase with increase standard deviation values " σ " of Gaussian blurring function and decrease with increasing SNR.



(a) satellite degraded image



(b) Embryo degraded image



(c) bird degraded image

Figure (4-45) shows (a) RMSE Versus SNR of satellite restored image using Iterative Tikhonov-Miller filter (b) RMSE Versus of SNR of Embryo restored image using Iterative Tikhonov-Miller filter (c) RMSE Versus SNR of bird restored image using Iterative Tikhonov-Miller filter.

From figure (4-46) (a) it combinational, the relationship between RMSE of satellite restored images using Iterative Tikhonov-Miller filter " α = 1" for different SNR decrease with increase the number of iteration. But when SNR= 5 after 22 iter , SNR= 10 after 16 iter and SNR= 20 after 14 iter, RMSE start to increase with increase the number of iteration. (b) it combinational, the relationship between RMSE of Embryo restored images using Iterative Tikhonov-Miller filter " α = 2" for different SNR, in general decrease with increase the number of iteration. But when SNR= 5 after 21 iter and SNR= 10 after 22 iter, SNR= 20 after 17 iter, RMSE start to increase with increment the number of iteration. (c) it combinational, the relationship between RMSE of Bird restored images using Iterative Tikhonov-Miller filter " α = 1" in general decrease with increase the number of iteration. But when SNR= 5 after 24 iter and SNR= 10 after 19 iter, SNR= 20 after 14 iter, RMSE start to increase with increment the number of iteration, so we stopping the iteration.

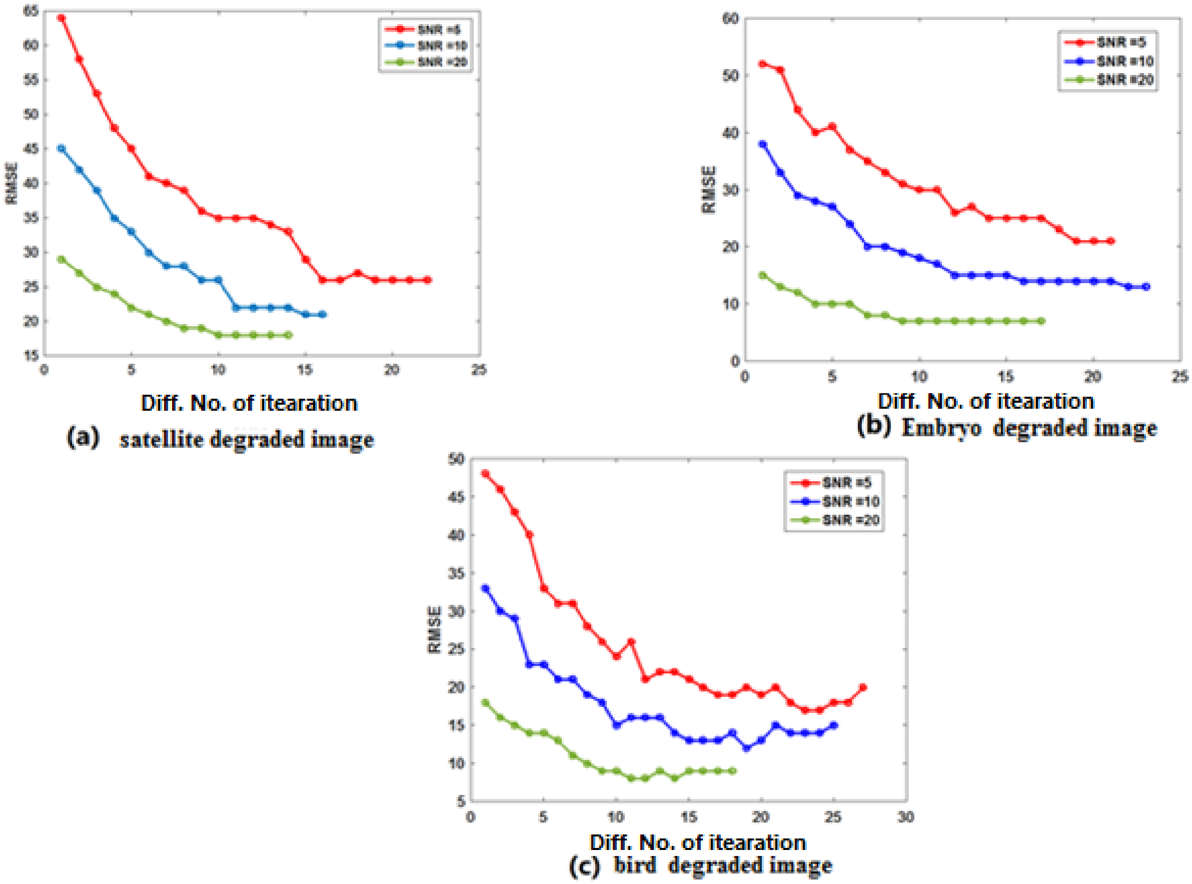


Figure (4-46) shows (a) RMSE Versus no. of iterations of satellite restored image with Tikhonov Filter, and blur with σ =1 (b) RMSE Versus no. of iterations of Embryo restored image with Iterative Tikhonov-Miller filter, and blur with σ =1(c) RMSE Versus no. of iterations of bird restored image with Iterative Tikhonov-Miller filter, and blur with σ =1

Figure (4-47) show RMSE of satellite restored image using Iterative Tikhonov-Miller filter " α = 1" decrease with increase the number of iteration for SNR= 5,10 and 20, but after two iteration when the SNR = 10 and 20 it start increased. (b) RMSE of Embryo restored image using Iterative Tikhonov-Miller filter " α = 2" decrease with increase the number of iteration for SNR= 5,10 and 20, but after three iteration when the SNR = 10 and 20 it start increased.

(c) RMSE of Bird restored image using Iterative Tikhonov-Miller filter " α = 1" decrease with increase the number of iteration for SNR= 5,10 and 20. But when SNR = 5 after 5 iter, SNR= 10 after 4 iter and SNR= 20 after 3 iter RMSE start to increased with number of iteration increment.

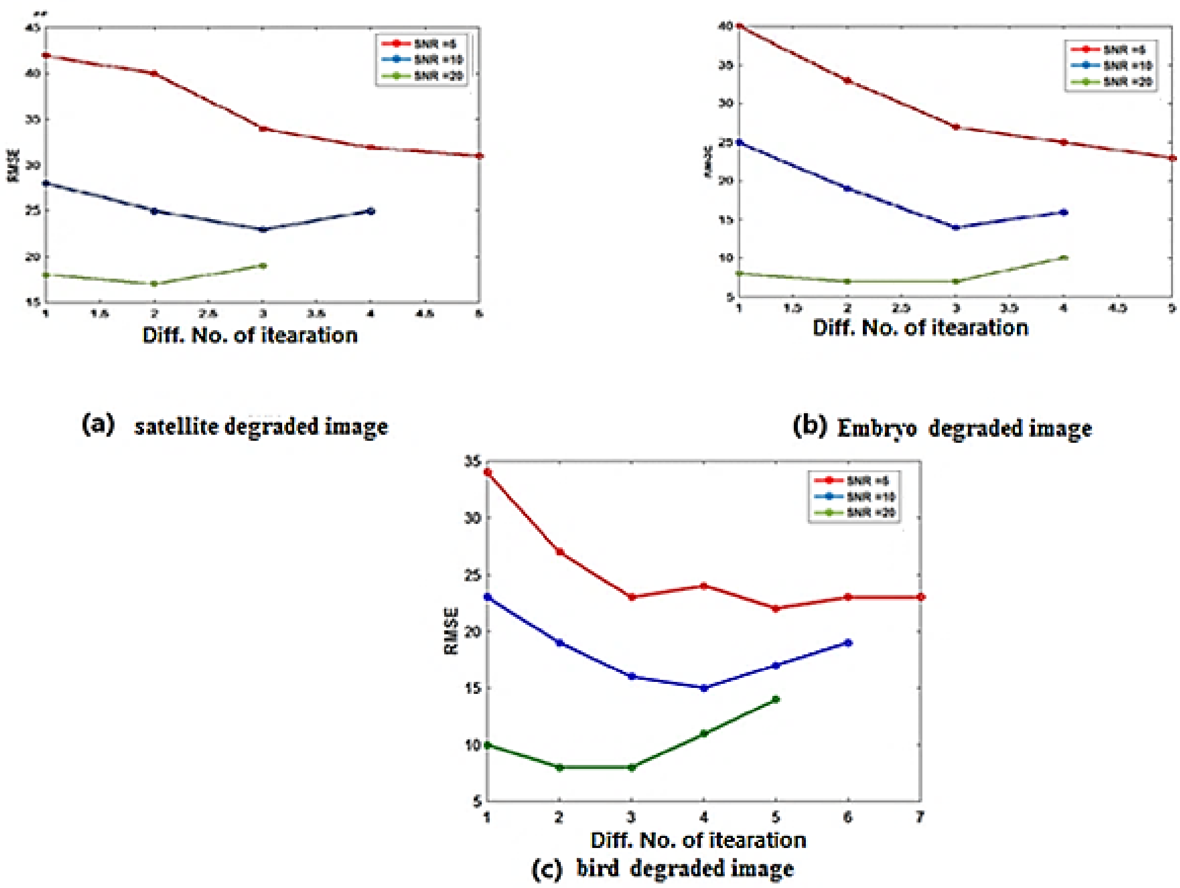


Figure (4-47) shows (a) RMSE Versus no. of iterations of satellite restored image with Tikhonov Filter, and blur with σ =2 (b) RMSE Versus no. of iterations of Embryo restored image with Iterative Tikhonov-Miller filter, and blur with σ =2 (c) RMSE Versus no. of iterations of bird restored image with Iterative Tikhonov-Miller filter, and blur with σ =2

Figure (4-48) show the better result value of RMSE of restored satellite images using Iterative Tikhonov-Miller filter , when the regularization parameter of iterative Tikhonov-Miller filter have value equal to 1.

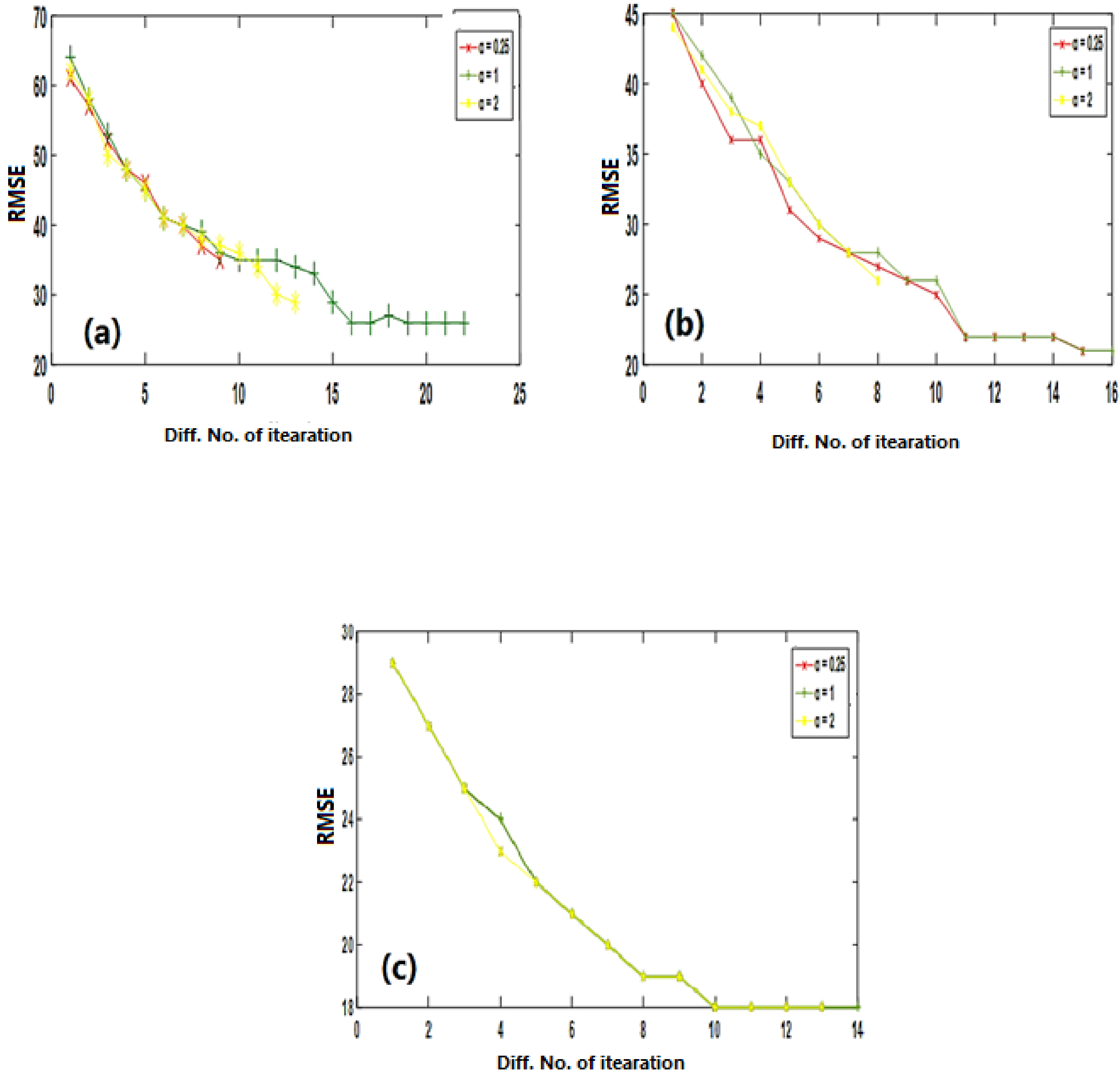


Figure (4-48) shows RMSE Versus number of iteration of satellite restored image using Iterative Tikhonov-Miller filter, blur with σ =1, SNR = 5, 10 and 20 respectively

Figure (4-49) show the better result value of RMSE restored satellite images using Iterative Tikhonov-Miller filter , when the regularization parameter of iterative Tikhonov-Miller filter have value equal to 1.

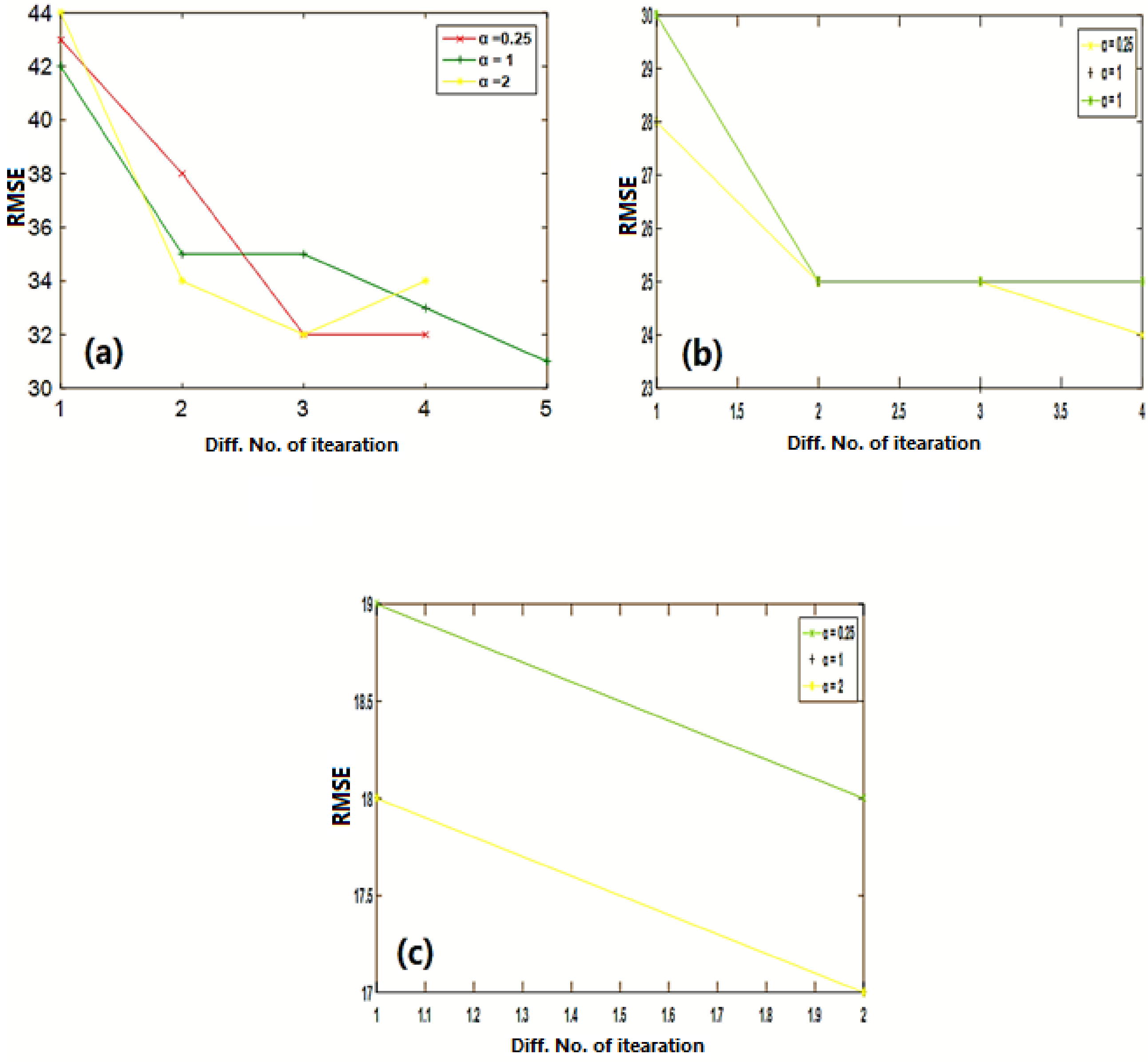


Figure (4-49) shows RMSE Versus number of iteration of satellite restored image using Iterative Tikhonov-Miller filter, blur with σ =2, SNR = 5, 10 and 20 respectively

Figure (4-50) show the better result value of RMSE of restored Embryo images using Iterative Tikhonov-Miller filter, when the regularization parameter of iterative Tikhonov-Miller filter have value equal to 2.

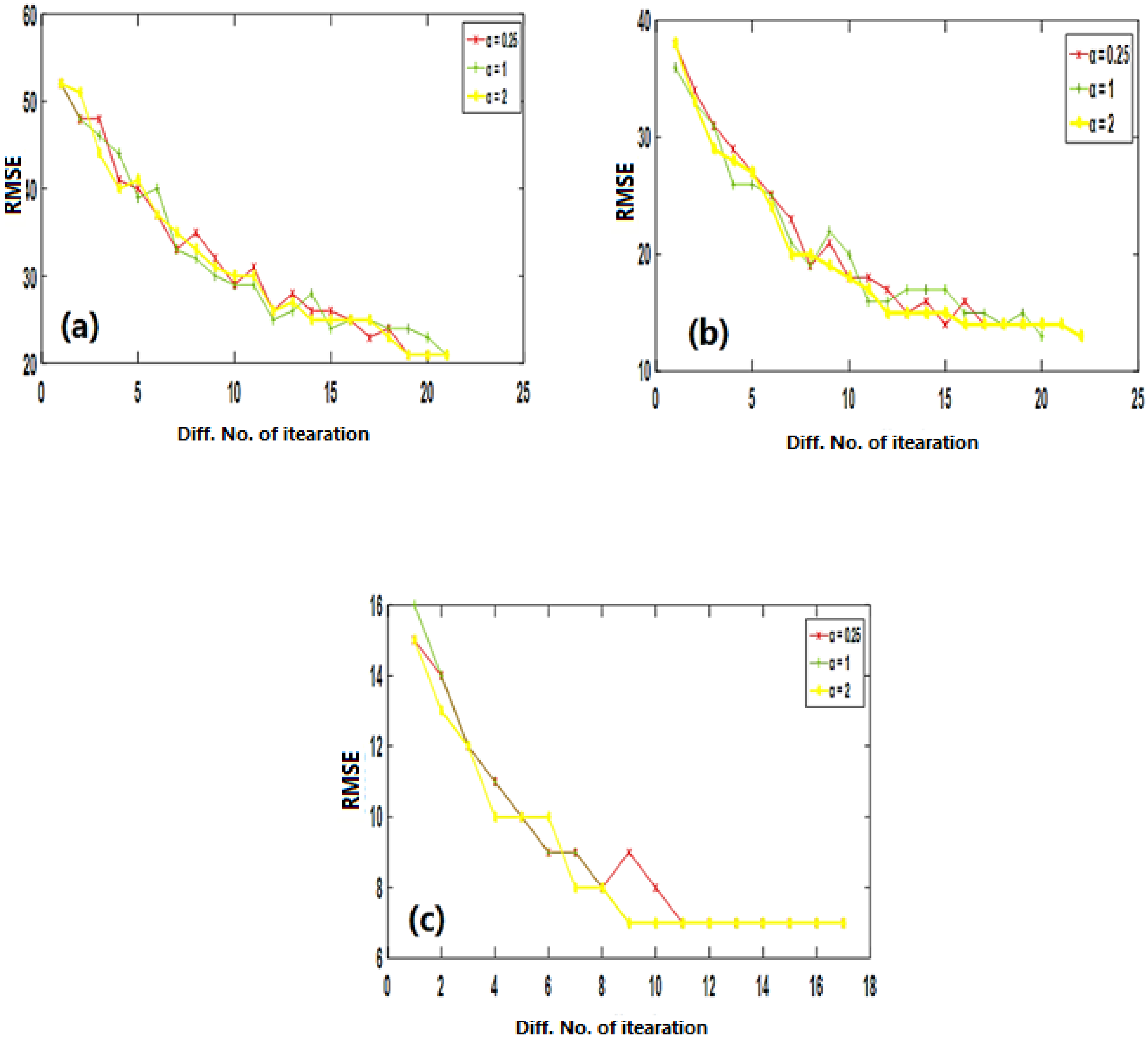


Figure (2-50) shows RMSE Versus number of iteration of Embryo restored image using Iterative Tikhonov-Miller filter, blur with σ = 1, SNR= 5, 10 and 20 respectively

Figure (4-51) show the better result value of RMSE of restored Embryo images using Iterative Tikhonov-Miller filter , when the regularization parameter of iterative Tikhonov-Miller filter have value equal to 2.

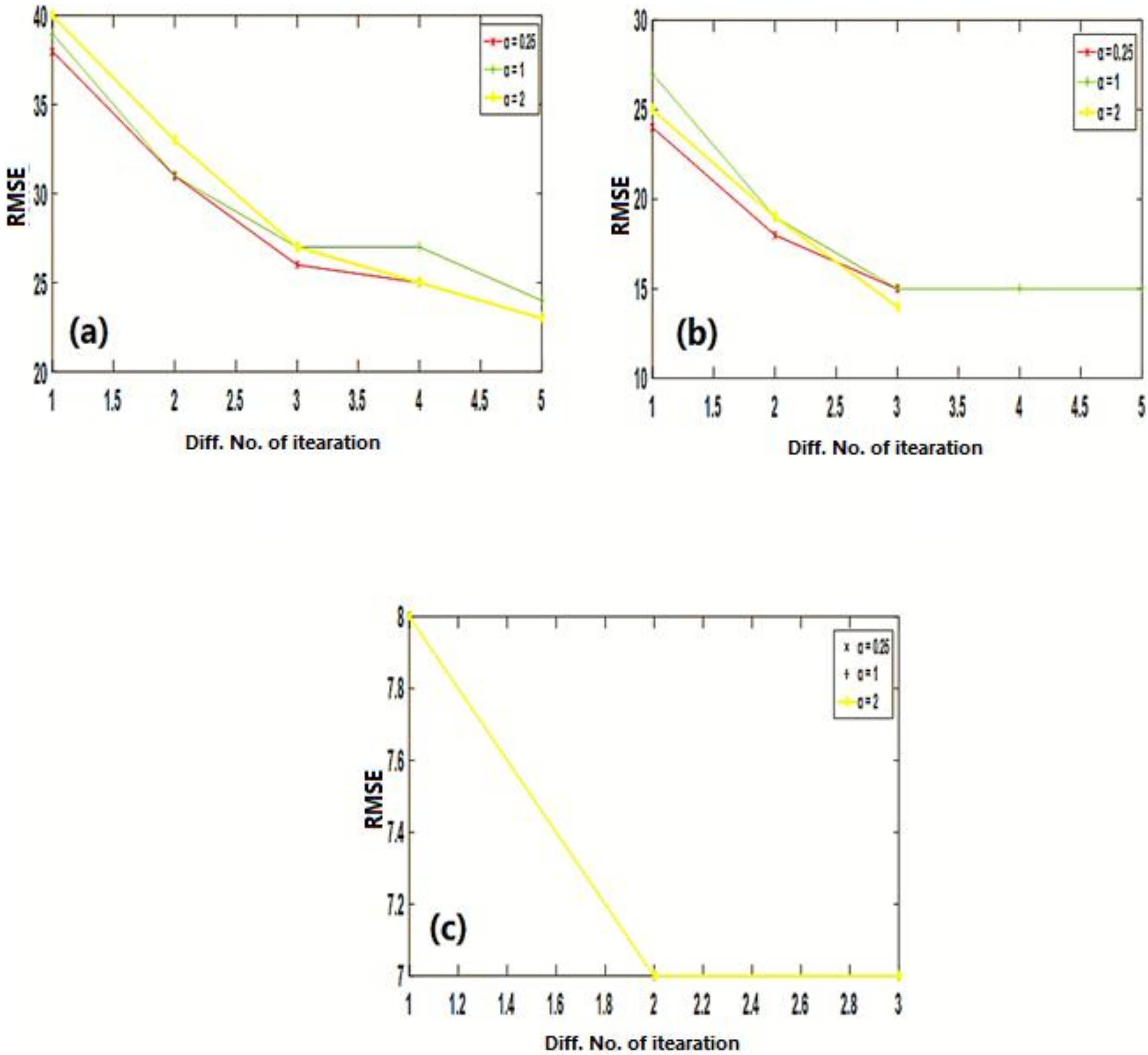


Figure (4-51) shows RMSE Versus number of iteration of Embryo restored image using Iterative Tikhonov-Miller filter , blur with σ = 2, SNR = 5, 10 and 20 respectively

Figure (4-52) show the better result value of RMSE of restored bird images using Iterative Tikhonov-Miller filter , when, the regularization parameter of iterative Tikhonov-Miller filter have value equal to 1.

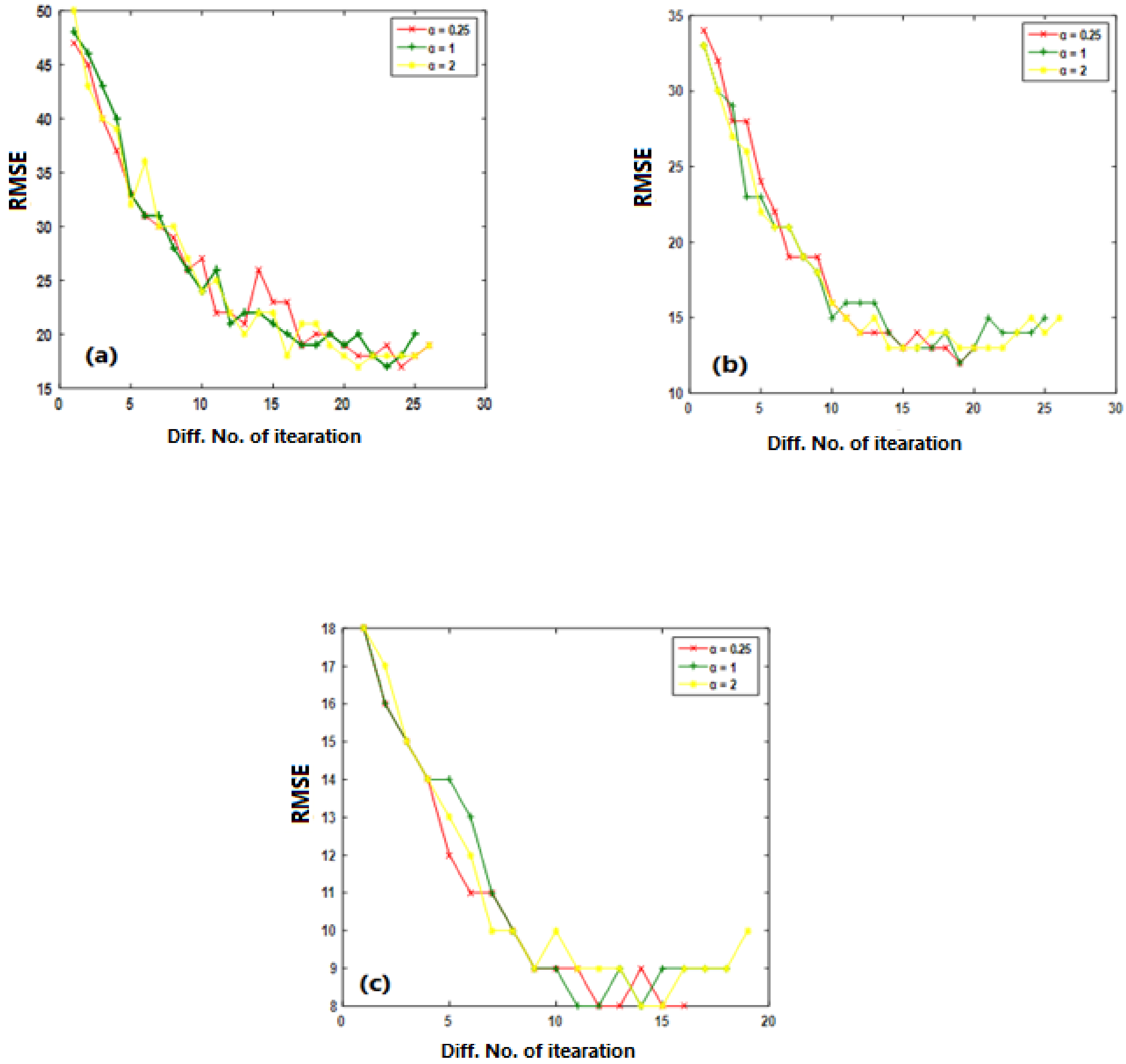


Figure (4-52) shows RMSE Versus number of iteration of bird restored image using Iterative Tikhonov-Miller filter, blur with σ =1, SNR = 5, 10 and 20 respectively

Figure (4-53) show the better result value of RMSE restored bird images using Iterative Tikhonov-Miller filter , when the regularization parameter of iterative Tikhonov-Miller filter have value equal to 1.

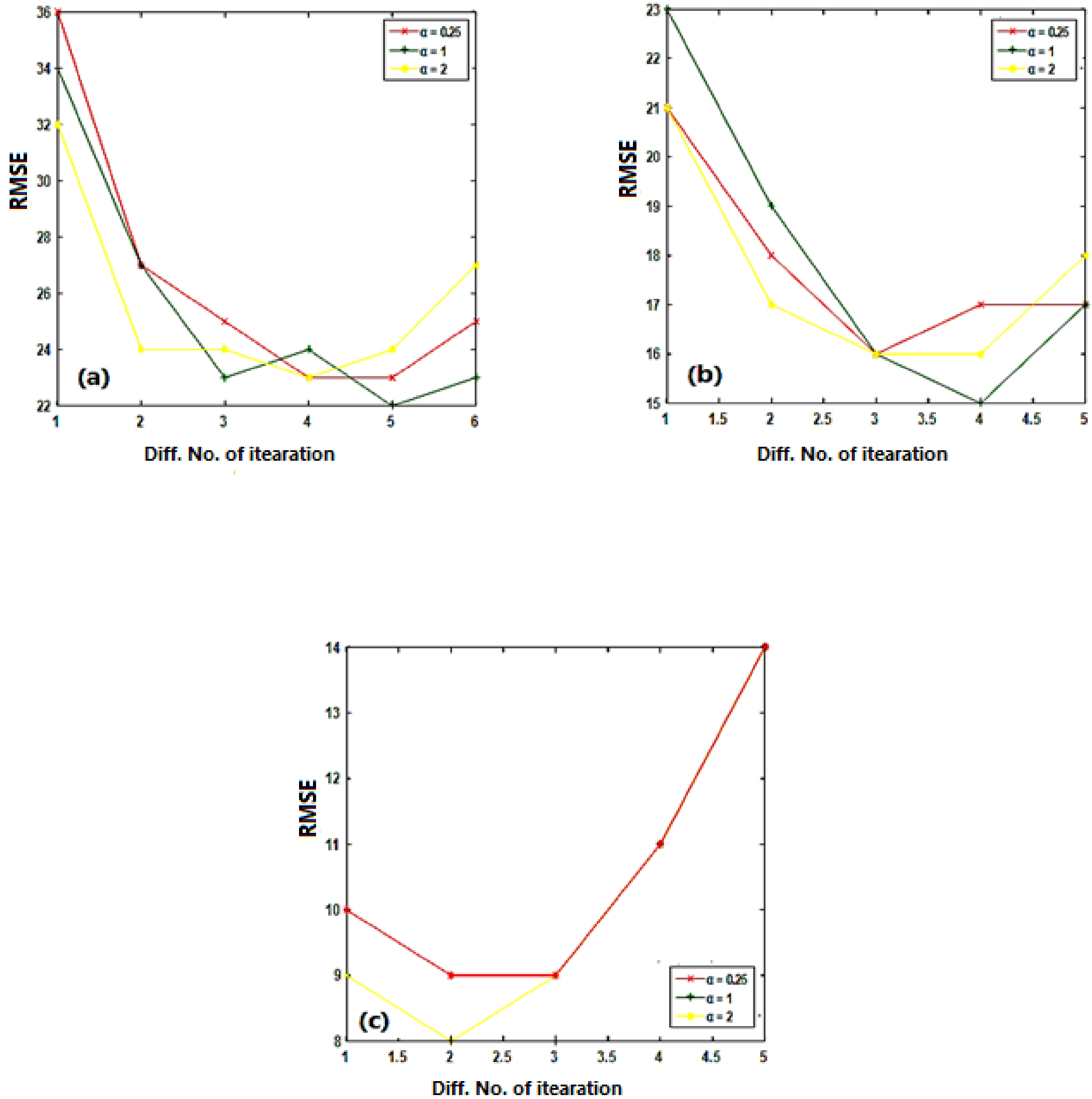


Figure (4-53) shows RMS error Versus number of iteration of bird restored image using Iterative Tikhonov-Miller filter, blur with σ =2, SNR = 5, 10 and 20 respectively

Chapter five

Conclusions and Suggest For Future Work

5.1 Conclusions

The effectiveness of image restoration techniques studied by using root mean square error as follow

- Wiener filter has better performance for more degradation parameters, with low SNR.
- Iterative Tikhonov-Miller filter has better performance when increasing the number of iteration till it divergence, the performance will be low, also has better performance for less degradation parameters, with high SNR.
- Choose the regularize parameter of Iterative Tikhonov-Miller filter of depend on image type.

5.2 Suggestions for Future Work

From the work of this research, some notes can be suggested as
future work:

1- Use a nonlinear technique of image restoration to restore the images.

2- Using nonlinear iterative restoration and compare the results with another kind of iterative image restoration. Such as maximum entropy method, POC methods.

3- Using nonlinear iterative restoration with different regularization parameter.

4. Using a blind deconvolution to restore images.

6- Using another programs to apply restoration algorithm such as C#.

7- Using another type of blurring and noise function such as motion blur.

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الخلاصة:

لقد تم تطبيق المعالجات الصورية في العديد من المجالات العلمية كعلوم الفيزياء والحاسوب والهندسة والكيمياء والبيولوجيا والطب. من المعروف ان عملية الحصول على الصورة باستخدام الطرق البصرية او الالكترونية والتي عادة ما تكون معرضة للتشويه من قبل بيئة المتحسس. مجال ترميم الصورة هو احدى مجالات معالجة الصورة الرقمية الذي يهتم بتحسين الصورة المشوهة. ترميم الصورة ممكن ان يكون خطي او غير خطي واعمى او غير اعمى. تم التركيز في هذا البحث على ترميم الصور الخطي والغير اعمى وفرض ان موديل تشوية الصورة هو عبارة عن التفاف داله الغشاوة بالصورة الاصلية واتلفت باضافة ضوضاء جمعية وان خوارزمية ترميم الصورة هي محاولة للتخلص من داله الغشاوة والضوضاء من الصورة المشوهة بفك التفاف دالة التشويه بالصورة المشوهة وتقليل الضوضاء لانتاج صورة مقدرة قريبة من الصورة الاصلية . الصور المستخدمة شوهت بواسطة دالة غشاوة كاوسية مختلفة الانحراف المعياري σ =١,٢ وضوضاء كاوسية جمعية مختلفة بنسبة الاشارة الى الضوضاء SNR= ١٠، ٥,١٠، ٢٠. التشوية استخدم لعدة أنواع من الصور وهي الصورة الرمادية (صورة الستلايت)وصورة السونار (صورة الجنين) والصورة الملونة (صورة الطائر). وينر فلتر وفلتر ميلر- تكنهوف التكراري استخدمت لترميم الصور المشوهة. بواسطة مقياس جذر معدل مربع الخطاء لقد استنتجنا ان افضل اداء لوينر فلتر هو عندما يكون التشويه عالي اي ان نسبة الاشارة الى الضوضاء قليلة وافضل اداء لميلر- تكنهوف التكراري هو عندما يكون التشويه قليل اي ان نسبة الاشارة الى الضوضاء عالية.

جمهورية العراق
وزارة التعليم العالي والبحث العلمي
جامعة النهرين
كلية العلوم
قسم الفيزياء

ترميم الصور الرقمية باستخدام خوارزمية المرشح التكراري

رسالة

مقدمة الى كلية العلوم / جامعة النهرين
كجزء من متطلبات نيل درجة الماجستير في علوم الفيزياء

من قبل

هاله كاظم حسن

بكلوريوس فيزياء /كلية العلوم للبنات / جامعة بغداد
(٢٠١٣)

اشراف
الاستاذ الدكتور اياد عبد العزيز العاني

صفر ١٤٣٨ هـ
تشرين الثاني ٢٠١٦ م