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Ministry of Higher Education  
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# *Design of an Einzel Lens Using Non-Classical Variation Technique*

**A Thesis**

**Submitted to the College of Science of Al-Nahrain  
University in Partial Fulfillment of the Requirements  
for the Degree of Master of Science in Physics**

**by**

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# بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

( فَأَمَّا الزَّبَدُ جُفَاءً  
وَأَمَّا مَا يَنْفَعُ النَّاسَ فَيَمْكُتُ  
فِي الْأَرْضِ )

صدق العظيم

الآية (17)  
سورة الرعد

*Detection to my  
children*

*(Aubay, Yosef, and Zaid)*

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# Synopsis

A computational investigation has been carried out in the field of charged-particle optics. The work is concerned with the design of einzel (uni potential) lens by using non-classical variation method under different magnification conditions.

The potential field distribution of the einzel lens has been represented by an analytical function. The paraxial ray equation has been solved for finding the short beam trajectory of the charged particles traversing the lens.

The axial potential distributions and its first and second derivatives, the optical properties such as the focal length and spherical and chromatic aberrations are determined by using non-classical variation method. The electrode shape of the einzel lens has been determined in the two dimensions.

The aberrations of the electrostatic lens from our results depend on the beam trajectory of the charged particles, where the aberration is small when the beam trajectory is shorter.

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## List of Symbols

$U=U(z)$	Axial potential distribution
$U', U''$	First and second derivatives with respect to $z$ respectively
$L$	Linear operator
$r$	Trajectory
$m$	Mass of charged particles
$p$	Momentum of charged particles ( Kg. m / s)
$q$	Charge of particles (C)
$f_i$	Focal length of image side
$f_o$	Focal length of the object side
$M$	Linear magnification.
$U_1, U_2$	Electrodes voltage
$C_{ci}, C_{co}$	Chromatic aberration coefficient in the image-and object-side respectively.
$C_{si}, C_{so}$	Spherical aberration coefficient in the image- and object side respectively.
$H_i, H_o$	Principle planes in the image and object side respectively
$z$	Optical axis
$R_p(z)$	Radial height of the electrode shape along the lens axis (mm).
$z_i$	Image plane position (mm)
$z_o$	Object plane position (mm)

# Chapter one

## 1- Introduction

### 1-1 The Electrostatic Lens

Any axially symmetric electrostatic field is in fact a lens. Electrostatic fields are produced by sets of electrodes held at suitable potentials; in other words, any axially symmetric electrodes system constitutes an electrostatic lens. Electrostatic lenses are finding increasing applications in many areas of science and technology, because of their versatility and the rapid development of modern instrumentation. (Szilagyí 1988, Hawkes and Kasper 1989).

### 1-2 Types Of Electrostatic Lenses

Any electrostatic field with axial symmetry (rotational symmetry) acts upon charged-particle beam moving in the near-axis region the same way a light-optical lens acts on a light beam. In other words any rotationally symmetric electrostatic field has imaging properties. In general, an electrostatic charged-particle lens is any region of an axially symmetrical electrostatic field in which there occurs the inequality  $U''(z) \neq 0$ , where  $U_0(z)$  is the axial potential distribution and  $U''(z)$  is the second derivative of  $U_0$  with respect to  $z$ . Depending on how this field region is generated, i.e. depending on the electrode shape, voltages, and the distribution of the electrostatic field in front and beyond the lens (in the object and image regions ) one can classify several kinds of electrostatic lenses. Figure 1.1 depicts how the plot of the potential  $U_0(z)$  behaves along the  $z$ -axis for several kinds of electrostatic lenses. In classical texts of charged-particle optics electrostatic lenses were classified into groups according to the relationships between their electrode potentials. The main groups are:-

- (a) **The immersion lens:** It has two different constant potentials at its sides i.e. the electrostatic field  $E(z)$  is zero at the two terminals. Immersion lenses accelerate or retard the particles while the beam is focused, and may consist of as few as two electrodes [Baranova and Yavor 1984]. Figures (1.1a)-(1.1d).
- (b) **The cathode lens:** It can erroneously be called an immersion objective with a field abruptly terminated on the object side by the source of the charged particles. Figure (1.1e).
- (c) **The unipotential (einzeln) lens:** This kind of lens has the same constant potential at both the object and image sides i.e. the charged-particle energy remains unchanged. Figures (1.1f)-(1.1i).
- (d) **The diaphragm (single-aperture) lens:** In this lens there is a homogeneous field on at least one side i.e. the potential on one side or both is not constant but increases or decreases linearly. This means that there is an electrostatic field of constant intensity in the immediate vicinity of the lens. Figures (1.1j)-(1.1m).
- (e) **The foil lens:** It consists of thin metal films transparent to the particles and possessing discontinuous field distributions [Szilagyi 1988].

Apart from the classification of lenses according to  $\Phi_0(z)$  one can distinguish one, two, and multi-electrode lenses. Moreover, one can also distinguish between strong and weak charged-particle lenses. If the potentials at both sides of the lens are not equal, the lens may work in acceleration or deceleration (retarding) modes. A special case of a retarding lens is the electron or ion mirror, which is formed by strongly retarding lens whose potential changes its sign at a certain axial point causing by that the reversal of the particle direction.

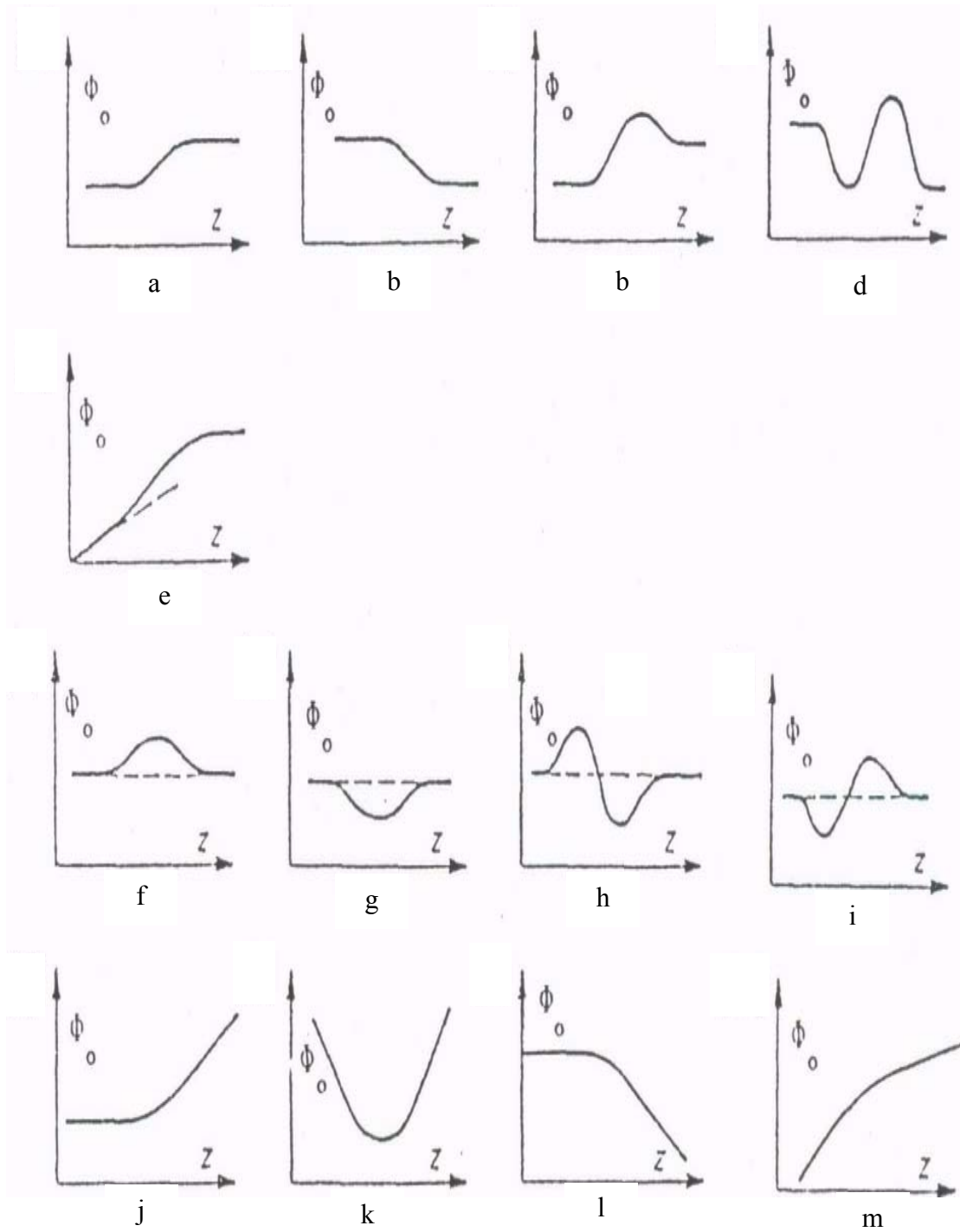


Figure (1-1): Distribution of the axial potential  $U_0(z)$  in electrostatic lenses :  
 (a) to (d) immersion lenses; (e) cathode lens; (f) to (i) unipotential lenses; (j)  
 to (m) diaphragm lenses

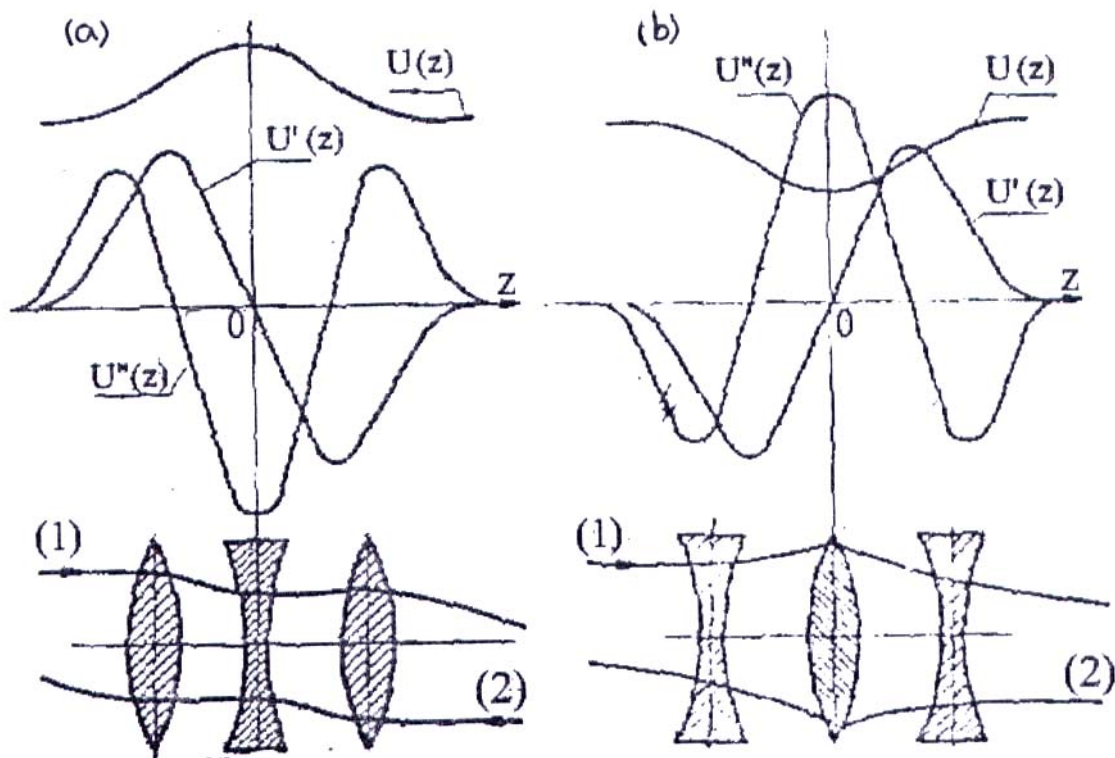
### **1-2-1 Einzel (unipotential) Lens**

The most commonly used einzel (unipotential) lenses have three electrodes. The distinctive feature of einzel lenses is that they have the same constant potential  $U_1$  at both the object and the image side, the central electrode is at a different potential  $U_2$ , therefore, they are used when only focusing is required but the beam energy must be retained.

Einzel lenses are symmetrical with respect to the center of the lens for both of its foci. Hence, they are frequently called symmetrical lenses [**Paszowski 1968**]. The symmetrical lens is usually used in cathode-ray tubes, and many other electron optical devices. It is possible, to destroy the symmetry by applying different voltages to the two outer electrodes and still have a practicable lens. The focusing action of the system remains essentially the same as in the symmetrical case, but it is not in general used owing to the obvious advantages of the latter [**Cosslett 1950**].

The einzel lens has two modes of operation relative to the central electrode voltage  $U_2$ . The two modes are called accelerating (accel) mode when  $U_2 > U_1$  and the decelerating (decal) mode when  $U_2 < U_1$ . The accel mode aberration is lower than that of the decal mode, but the accel mode requires substantially high central electrode voltage [Kurihara 1985].

The axial potential distribution of a three-electrode einzel lens has the typical form shown in figure (1-2). It has one maximum or minimum depending on whether the middle electrode's voltage is higher or lower than that of the side electrodes.



**Figure (1-2):** axial potential distribution  $U(z)$  and its first and second derivative  $U'(z)$  and  $U''(z)$  respectively (a) accel mode (b) decal mode [Paszkowski 1986].

Sometimes it is possible to fit the axial distribution of potential in a lens to some simple function, with sufficient accuracy, and then study the optical properties. According to Plass 1942 and Cosslett 1950 it was indicated by Scherzer that in the symmetrical einzel lens the axial field could be represented by an expression of the following form:

$$U(z) = V_0 + A e^{-bz^2} \quad (1-1)$$

where  $V_0$ ,  $A$  and  $b$  are constant. The assumed a weak lens [ $V_0 \gg A$ ] and a symmetrical lens [ $U(-z) = U(z)$ ]. Plass 1942, however, has considered the following simple distribution:

$$U(z) = V_0 (1 + 1/2 e^{-z^2/2}) \quad (1-2)$$

Three-electrode electrostatic lenses were investigated by [Kanaya and Babw 1977] whose axial potential distribution is given by the following equation:

$$U(z/w) = U(z_c) \exp[K_0 \tan^{-1}(z/w)^n] \quad (1-3)$$

where  $U(z_0)$  is the central electrode voltage,  $n$  is an integer representing the degree of sharpness of the potential distribution,  $K_0$  is a constant parameter indicating a positive or negative field depending on the voltage ratio, and  $w$  is the half width of the field (some times called full width at half maximum ,FWHM).

It is aimed in the present work to find a more simple analytic expression that would describe the axial potential distribution of einzel lenses with electron optically acceptable aberration. The following expression is suggested to represent the potential distribution along the optical axis of an einzel lens:

$$U(z) = \exp[-C_1(z-5)^2] + C_2 \quad (1-4)$$

Where  $C_1$  and  $C_2$  are constant.

We chose this equation in this work because the path of the beam trajectory is shorter between the starting and ending points. And  $C_1$  is representing the change in the length of outer electrodes and  $C_2$  the change in the ratio of voltage.

### **1-3 Properties of Electrostatic Lenses**

The most important features of electrostatic lenses are (see for example, [Szilagyi 1988; Hawkes and Kasper 1989] :

(a) For the non-relativistic cases the focusing properties as well as the aberrations are independent of the quotient of charge-to-mass ( $Q/m$ ) of the particles. Therefore, the electrostatic lenses may be used for any system focusing various ions.

(b) Only potential ratios have influence on the lens properties. Therefore, if particles of the opposite sign have to be focused, the signs of all electrode potentials must be reversed to arrive at the same properties. The particles trajectory remains the same if both the sign of the particles charge and the potentials of the electrodes are reversed.

(c) Electrostatic lenses are characterized by their simple electrodes fabrication, alignment, and small size and weight. Furthermore, their low power requirements suggest the need of lighter and more stable power supplies. The major manufacturing problems are electric breakdown and accumulation of charges on the insulating surfaces. Under usual conditions of vacuum the electrodes must be separated from each other so that the maximum field strength does not exceed 15 kV/mm.

(d) The real problem with electrostatic lenses is the difficulty of evaluation of their properties because of the large number of characteristic parameters such as the number of the electrodes, the electrodes voltage ratios, aperture size, the electrodes thicknesses and spacing between them, and their radial and longitudinal dimensions. Therefore, the comparison of the properties of different lenses is very difficult, where the lens properties are published in the form of tables and graphs; as an example the cardinal elements and the aberration coefficients are presented as functions of voltage ratios and geometrical parameters. Hence, unlike magnetic lenses, no universal design curves are available for the electrostatic lenses.



## **1-4 Numerical Determination Of Electrostatic Field**

Determination of electrostatic fields usually requires the solution of a complicated boundary value problem. Most electrostatic field calculations involve charge distribution on the surface of the conductors only i.e. the space charge effect is neglected, therefore, the field equation to be solved will be Laplace's' equation [**Bonjour 1980**]. The solution of Laplace's' equation ( $\nabla^2\phi = 0$ ) with specified boundary conditions makes it possible to determine the potential as a function of coordinates from which the components of the field intensity can be calculated [Szilagyi 1988]. Many computer programs have been developed for solving problems in electrostatic charged particle optics, and energy all of them are based on Finite Difference Method (FDM), Finite Element Method (FEM), and Bounding Element Method (BEM). BEM is also known as charge density Method or boundary charge method [**Cubric, et al 1999**]. In the present work we will use the non-classical variation method in our calculation.

## **1-5 Historical Development**

A property of electrostatic einzel lenses of three apertures was calculated by **Read (1969)**. A least-squares collocation method had been used to calculate the potential distributions of electrostatic einzel lenses consisting of three thin apertures. The focal lengths, principal plane positions and spherical aberration coefficients had been obtained by numerical integration of the ray equations.

The potential distribution inside three equidiameter coaxial cylinders separated by finite distances had been derived and was used to calculate the focal lengths, midfocal lengths and spherical aberration coefficients of some triple cylinder lenses having three different voltages applied to them. The lenses have the length of the central cylinder equal to its diameter or to half its

diameter. Data were presented for such lenses with the middle potential both higher and lower than the outer potentials **A. Adams and F. H. Read (1972)**.

A novel approach to electron/ ion optical synthesis and optimization was presented by **Szilagyi (1985)**. Low-aberration field distributions are sought by dynamic programming or optical control procedures in the form of continuous curves constructed of cubic splines.

**Sze'p and Szilagyi (1988)** presented a novel approach to the synthesis of axially symmetric electrostatic lenses with minimum aberration. The method uses computer optimization to determine the electrode potentials of a multi electrode lens. The same method can be used for designing lenses for many different applications.

The use of the electron beam trajectory as a major parameter for determining the design of electrostatic lens at zero and infinite magnification operational conditions was investigated in detail by **Ahmad (1993)** in the absence of space charge-effect and by **Al-Ani (1996)** in the presence of space charge.

### **1-6 Optimization Methods: Analysis and Synthesis**

Any reasonable design of an electron or an ion optical system must take the aberrations into account. There are two fundamental approaches to optimize the parameters of charged-particle optical system :( i) the analysis and (ii) the synthesis of electron and ion systems [Szilagyi 1985]. The optimization approach looks for such electron and ion optical elements that would provide the required optical properties with minimum aberrations.

The first approach is based on trial and error. The method of analytical is usually trial and error. The designer starts with given simple sets

of electrodes and tries to improve the design by analyzing the optical properties and changing the geometrical dimensions as well as the electric parameters of the system. This process must be repeated until it converges to an acceptable solution. Due to the infinite number of possible configuration, the procedure is extremely slow and tedious.

In the second approach, the word synthesis, which is sometimes called inverse problem of charged-particle optics, i.e. for finding field distribution that would produce given trajectories (**Szilagyi 1988**). This approach is based on the fact that for any imaging field, its optical properties and aberration are totally determined by the axial distribution of the field. Only the axial distributions and their derivatives appear in those expressions. Then, instead of analyzing a hopelessly vast amount of different electrode and pole piece configurations we can take the criteria defining an optimum system as initial conditions and try to find the imaging field distribution ( and hence synthesis the electrodes ) that would produce it. i.e. in synthesis approach, one tries to find the best axial field distribution or the best shapes of this axial distribution that would satisfy the given constraints.

### **1-7 The Aim Of The thesis**

The present work, aims at finding the optimum design of electrostatic einzel lens which given rise to minimum spherical and chromatic aberrations. The synthesis approach of optimization method is used in this work. The paraxial ray equation using non-classical variational technique has been solved for finding the analytical function of the beam trajectory with minimum path between the starting and ending points or between the two terminals of the lens. The optical properties of the einzel lens as function of length.

## Chapter Two

### 2-1 VARIATIONAL FORMULATION OF INITIAL– BOUNDARY VALUE PROBLEMS

The difficulties in solving initial-boundary value problems have led to search for variation problems equivalent to the given Initial-Boundary value problems.

There are two approaches for evaluating the functional, which could be described as the first approach, but after introducing some of the basic definitions and concepts related to the subject:

#### **Definitions:**

1- Let  $U$  and  $V$  be linear space and let  $L$  be an a operator with domain  $D(L) \subseteq U$  and range  $R(L) \subseteq V$ . then  $L$  is called a linear operator if  $D(L)$  is subspace of  $U$  and if:

$$\text{i- } L(u_1 + u_2) = L(u_1) + L(u_2)$$

$$\text{ii- } L(\alpha u_1) = \alpha L(u_1)$$

Where  $\alpha$  is scalar and  $u_1, u_2$  are vectors in  $D(L)$ . One can notice that if  $D(L) = U$ , then  $L$  is a linear operator from  $U$  into  $V$ .

2- A linear space  $U$  is said to be normed linear space if it is endowed with a non-negative real valued function  $\| \cdot \|$ , called a norm, which satisfies:

$$\text{i- } \| u \| = 0 \text{ if and only if } u=0$$

$$\text{ii- } \| \alpha u \| = |\alpha| \|u\|, \alpha \text{ is scalar (real or complex)}$$

$$\text{iii- } \|u_1 + u_2\| \leq \|u_1\| + \|u_2\| \text{ (triangle inequality )}$$

for any vectors  $u_1, u_2 \in U$ .

**3-** A function  $F$  on a linear space  $U$  is called linear if:

i-  $F(u_1 + u_2) = F(u_1) + F(u_2)$

ii-  $F(\alpha u_1) = \alpha F(u_1)$

for any  $u_1, u_2 \in U$  and any scalar  $\alpha$ .

**4-** A functional  $F(u, v)$  depending on two elements  $u$  and  $v$  belonging to some normed linear space  $U$ , is said to be bilinear form if it satisfies:

i-  $F(u + w, v) = F(u, v) + F(w, v)$

ii-  $F(\alpha u, v) = \alpha F(u, v)$

and:

iii-  $F(u, w + v) = F(u, v) + F(u, w)$

iv-  $F(u, \alpha v) = \alpha F(u, v)$

For any  $u, v$  and  $w$  belonging to the normed linear space  $U$  and any real number  $\alpha$ .

**5-** The bilinear form  $\langle u, v \rangle$  is called non-degenerate on  $U$  and  $V$  if the following two conditions are satisfied:

i-  $\langle u, \bar{v} \rangle = 0$  then  $\bar{v} = 0$  for every  $u \in U$

ii-  $\langle \bar{u}, v \rangle = 0$  then  $\bar{u} = 0$  for every  $v \in V$

**6-** Linear operator  $L$  is said to be symmetric with respect to the chosen bilinear form  $\langle u, v \rangle$  if  $L$  satisfies the condition:

$$\langle Lu_1, u_2 \rangle = \langle Lu_2, u_1 \rangle$$

for every pair of elements  $u_1$ , and  $u_2$  belong to  $D(L)$  (the domain of  $L$ )

- 7- Let  $U$  and  $V$  be two normed linear space, and let  $\langle u, v \rangle$  be a bilinear form, then  $\langle u, v \rangle$  said to be symmetric if  $\langle u, v \rangle = \langle v, u \rangle$ , for all  $\langle u, v \rangle \in U \times U$ .

**Remark (2.1):**

As it is pointed out by Magri in [Magri, F, 1974], an operator  $L$  is said to be symmetric with respect to the chosen bilinear form  $\langle \cdot, \cdot \rangle$  if  $\langle Lu, v \rangle = \langle Lv, u \rangle$  and when the operator is not symmetric, it could be transformed into symmetric one by letting  $(u, v) = \langle u, Lv \rangle$ . Therefore, it is possible to find a variational formulation for any linear problem or system of linear equations of any kind (differential, partial, integral and any other kind) associated with boundary conditions or initial conditions.

therefore, it is possible to prove that the bilinear form that takes the linear operator  $L$  is symmetric whatever the chosen type of the bilinear form  $\langle u, v \rangle$  since:

$$(Lu_1, u_2) = \langle Lu_1, Lu_2 \rangle = \langle Lu_2, Lu_1 \rangle = \langle Lu_2, Lu_1 \rangle$$

So we can use the bilinear form  $(u, v) = \langle u, Lv \rangle$  to give the variational formulation for any linear equation with symmetric operator.

**theorem (2-1) [Magri, F., 1974]:**

There is a variational problem corresponding to linear equation  $Lu = f$ , if and only if the operator  $L$  is symmetric relative to the bilinear form which is non-degenerate, where the functional is given by:

$$J(u) = \frac{1}{2} \langle Lu, u \rangle - \langle f, u \rangle \dots\dots\dots(2-1)$$

***Proof:***

the problem now is to prove that the critical points for (2-1) are

equivalent to the solution of  $Lu = f$ , since:

$$\begin{aligned}\delta J &= J(u + \delta u) - J(u) \\ &= 1/2 \langle L(u + \delta u), u + \delta u \rangle - \langle f, u + \delta u \rangle - 1/2 \langle Lu, u \rangle + \langle f, u \rangle \Big|_{\text{linear}} \\ &= 1/2 \langle Lu, u \rangle + 1/2 \langle Lu, \delta u \rangle + 1/2 \langle L \delta u, u \rangle + 1/2 \langle L \delta u, \delta u \rangle - \langle f, u \rangle - \\ &\quad \langle f, \delta u \rangle - 1/2 \langle Lu, u \rangle + \langle f, u \rangle\end{aligned}$$

Hence:

$$J(u) = 1/2 \langle Lu, \delta u \rangle + \langle L \delta u, u \rangle - \langle f, \delta u \rangle$$

Since  $\langle \cdot, \cdot \rangle$  symmetric, then:

$$\langle L \delta u, u \rangle = \langle Lu, \delta u \rangle$$

Therefore:

$$\begin{aligned}\delta J(u) &= 1/2 \langle Lu, \delta u \rangle + 1/2 \langle Lu, \delta u \rangle - \langle f, \delta u \rangle \\ &= \langle Lu, \delta u \rangle - \langle f, \delta u \rangle \\ &= \langle Lu - f, \delta u \rangle\end{aligned}$$

if  $u^*$  is a critical point, implies that  $\delta J(u^*) = 0$  which that implies

$\delta J(u^*) = 0$  which that implies  $\langle Lu^* - f, u^* \rangle = 0$ , for all  $\delta u^*$

since  $\langle \cdot, \cdot \rangle$  is non-degenerate, therefore  $Lu^* - f = 0$ , and hence  $Lu^* = f$

Conversely, if  $u$  is a solution to  $Lu=f$ , in another form  $Lu-f=0$ , and upon squaring both sides:

$$(Lu - f)^2 = 0 \dots\dots\dots(2-2)$$

By integrating both sides of (2-2), we get:

$$\int_0^T (Lu - f)^2 dx = 0$$

$$\int_0^T ((Lu)^2 - 2fLu + f^2) dx = 0$$

$$\int_0^T ((Lu)^2 - 2fu) dx + \int_0^T f^2 dx = 0$$

Letting  $\int_0^T f^2 dx = k$ , and since  $\langle u, v \rangle = \int_0^T u \cdot v dx$ . therefore:

$$\begin{aligned} \langle Lu, Lu \rangle &= \int_0^T Lu \cdot Lu dx \\ &= \int_0^T (Lu)^2 dx \end{aligned}$$

And

$$\langle f, Lu \rangle = \int_0^T f Lu dx$$

Hence, we have:

$$\langle Lu, Lu \rangle - 2\langle f, Lu \rangle + k = 0$$

Or

$$\frac{1}{2} \langle Lu, Lu \rangle - \langle f, Lu \rangle + c = 0$$

i.e.,  $J(u) + c = 0$ , which means that  $u$  is a critical point to  $J(u)$ .

After these introductory definitions, we are in a place to describe the

First approaches, which are as follows:



## 2-2 The First Approach (Magri's Approach):

Consider the linear equation in operator form:

$$Lu = 0 \dots\dots\dots (2-3)$$

Where  $u$  denotes a scalar – vector valued function and  $L$  denotes a linear operator with domain  $D(L)$  in a linear space  $U$  and range  $R(L)$  in second linear space  $V$ .

In order to solve equation (2-3) using variation approach one have to se Search for a functional  $J(u)$  defined on the domain of the linear operator  $L$ , whose critical are the solutions of the given equation (2-3). This problem may be called the inverse problem of calculus of variation, while the usual problem of finding the critical points of a pre-assigned functional may be called the direct problem.

Usual examples of non-degenerate bilinear forms are

$$\langle v, u \rangle = \int_0^T v(t)u(t) dt$$

$$\langle v, u \rangle = \int_0^T \sum_h v_h(t)u_h(t) dt$$

$$\langle v, u \rangle = \iiint_v \sum_{h,k} v_{hk}(x)u_{hk}(x) dv$$

### **2-3 Paraxial-Ray Equation in Electrostatic Field:**

The trajectory of an ion or electron beam through an axially symmetric electrostatic lens field, in terms of the axial potential  $U$  and its first and second derivatives  $U'$  and  $U''$  respectively, is given by the following equation (**Szilagyi 1988**):

$$\frac{d^2 r}{d z^2} + \frac{u'}{2 \cdot u} \cdot \frac{d r}{d z} + \frac{u''}{4 \cdot u} \cdot r = 0 \dots\dots\dots(2-4)$$

where  $r$  is the radial displacement of the beam from the optical axis  $z$ , and the primes denote a derivative with respect to  $z$ . It should be noted that three important deductions can be made from equation (2-4):

(a) The quotient of charge to mass ( $q/m$ ) does not appear; indicating that the path is the same for any charged particle, no matter what may be its quotient provided it enters the field with the same constant kinetic energy. The particles with different  $q/m$  come to the same focus, but arrive there at different times, hence an electrostatic lens field alone cannot separate the charges in space, yet, and they may be separated in time.

(b) The equation is homogeneous in  $U$ , so that if the voltage is increased proportionally for all the electrodes, the trajectory remains unaltered.

(c) The equation is homogeneous in  $r$  and  $z$ . Increases in the dimension of the whole system produce corresponding increases in the dimensions of the trajectory, since the equi-potentials, though of the same form, are enlarged. If the object is doubled in size, the image will be doubled in size, the ratio between the two remaining constant.

From this method (non-classical variation method) we can apply equation (2-3) to solve the paraxial ray equation by the: assume the operator L is represent the:

$$\frac{d^2}{dz^2} + \frac{U'}{2 \cdot U} \cdot \frac{d}{dz} + \frac{U''}{4 \cdot U}$$

Where U is the axial potential we can get from the equation (1-4). and the symbol (r) in the equation (2-4) is represent the symbol (u) in equation (2-3). we can assume (in any types of equation) where the satisfied the initial condition. In zero magnification the initial condition is

$$r(0) = 1 \quad r'(0) = 0$$

and the

$$r(Z) = 1 + A_0 Z^2 + A_1 Z^3 + A_2 Z^4 \dots\dots\dots(2-5)$$

Where A<sub>0</sub>, A<sub>1</sub>, A<sub>2</sub> are constant. After that we substitute the equation (2-5) in equation of (2-4) and get the U from equation (1-4) and use these in equation (2-2) [(f) is equal zero] and integrating the equation.

The solutions of this equation are the minimum values for the A<sub>0</sub>, A<sub>1</sub>, A<sub>2</sub> are representing the critical point where make the function is minimum. When get the minimum values of the A<sub>0</sub>, A<sub>1</sub>, and A<sub>2</sub> than the trajectory is minimum path between the starting and ending points and from this result the spherical and chromatic aberration are minimum also.

The calculus is the same in the infinite magnification condition but Only change when assuming the trajectory because the initial condition is

$$r(10) = 1 \quad r'(10) = 0$$

Therefore; the r(Z) is became

$$r(Z) = 1 + A_0 (Z-10)^2 + A_1 (Z-10)^3 + A_2 (Z-10)^4 \dots\dots\dots(2-6)$$

## 2-4 Definitions And Operating Conditions

Some definitions and operating conditions of charged-particle optical system are given in this section.

*Object side:* The side of lens at which the charged particles enter.

*Image side:* The side of lens at which the charged particles leave.

*The object plane ( $z_0$ ):* The plane at which the physical object is placed as shown in figure (2-1).

*The image plane ( $z_i$ ):* The plane at which the real image of the object plane  $z_0$  is formed on the image side as shown in figure (2-1).

*Focal point ( $f_i$ ):* A focal point is the image of a bundle of ray's incident on a lens parallel to the axis. If these rays arrive at the lens from the object side, then these rays are collected at the image focal point  $f_i$ .

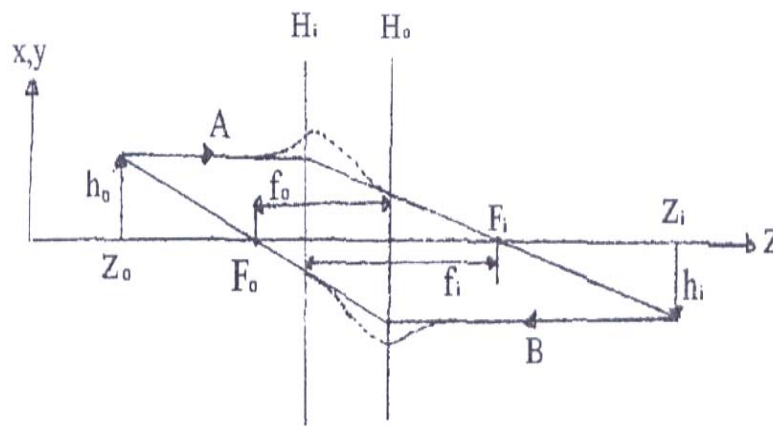


Figure (2-1): The cardinal elements of an axially symmetric electrostatic lens (Baranova and Yavor 1984).

**Magnification ( $M$ ):** In any optical system the ratio between the transverse dimension of the final image and the corresponding dimension of the original object is called the lateral magnification:

$$M = \frac{(\text{image height})}{(\text{object height})}$$

There are three magnification conditions in which a lens can operate, namely:

- a) **Zero magnification condition:** In this case of operating condition  $z_o = -\infty$  as shown in figure (2-2). The final probe-forming lens in a scanning electron microscope (SEM) is usually operated in this condition.

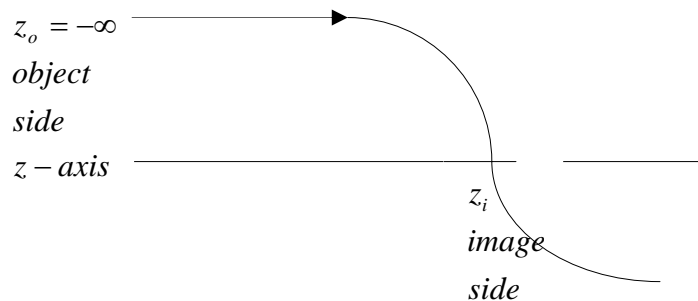


Figure (2-2): Zero magnification condition

- b) **Infinite magnification condition:** The operating condition is such that  $z_i = +\infty$  as shown in figure (2-3). The objective lens in a transmission electron microscope (TEM) is usually operated in this condition.

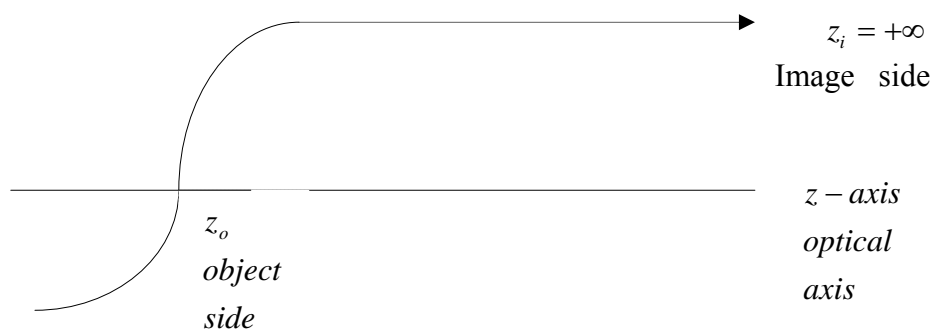


Figure (2-3): Infinite magnification condition

- c) **Finite magnification condition:** the operating condition in this case is that  $z_0$  and  $z_i$  are at finite distance, as shown in figure (2-4). The electrostatic lens in field-emission gun is usually operated in this condition (Munro 1975).

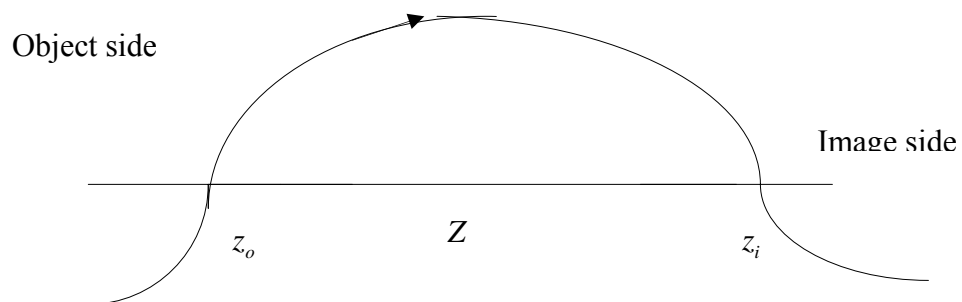


Figure (2-4): Finite magnification condition

## **2-5 Lens Aberrations**

The electron paths, which leave points of the object close to the axis at small inclinations with respect to the axis, intersect the image plane in points forming a geometrically similar pattern. This ideal image is called Gaussian image, and the plane in which the image is formed is called the Gaussian image plane. If an electron leaving an object point a finite distance from the axis with a particular direction and velocity intersects the Gaussian plane at a point displacement from the Gaussian image point, this displacement is defined as the aberration (**Hawkes 1972, Klemperer and Barnett 1971**).

The quality of any electron optical system depends not only upon the wavelength of the electron, but also upon the aberrations from which may suffer. If the accelerating potential and the lens excitations fluctuate about their mean values, chromatic aberration will mar the image.

If the properties of the system are investigated, using a more exact approximation to the refractive index than is employed in the Gaussian approximation, one would find that geometrical aberrations affect both the quality and the fidelity of the Gaussian image (**Grivet 1972, and Renau and Heddle 1986**).

When the properties of the system are analyzed using the non-relativistic approximation, the disparities between the relativistic and non-relativistic results can be conveniently regarded as a relativistic aberration. The most important aberration in an electron-optical system is spherical and chromatic aberration.

### 2-5-1 Spherical aberration

The spherical aberration is one of the most effective geometrical aberrations. It is defined in both light and electron optics, as the change in focal properties of a lens with radial height of the ray. Off-axis electrons spend less time the field than do paraxial electrons because they do not penetrate as far into the field, and as a result the focal distance is longer for electrons whose trajectories make larger angles with the optical axis than for paraxial electrons (**Meisburger and Jacobsen 1982, Szilagyı 1986**).

$$C_{s_i} = \frac{(U_i)^{-1/2}}{16 \cdot (r_i)^4} \int_{z_0}^{z_i} \left[ \frac{5}{4} \cdot \left(\frac{U''}{U}\right)^2 + \frac{5}{24} \cdot \left(\frac{U'}{U}\right)^4 + \frac{14}{3} \left(\frac{U'}{U}\right)^3 \cdot \left(\frac{r'}{r}\right) - \frac{3}{2} \cdot \left(\frac{U'}{U}\right)^2 \cdot \left(\frac{r'^2}{r}\right) \right] \sqrt{U} \cdot r^4 dz \quad (2-7)$$

where  $U = U(z)$  is the axial potential, the axial potential, the primes denote derivative with respect to  $z$ , and  $U_i = U(z_i)$  is the potential at the image where  $z = z_i$ .

### 2-5-2 Chromatic aberration

Chromatic aberration in light optics refers to the change in focal properties with the wavelength of light. In electron optics it refers to the change in focal properties with kinetic energy of the electrons; lower energy electrons, and therefore spend less time in the field and are less strongly converged by the field. Consequently the focal distance is longer for lower energy electrons than for high energy electrons (**Szilagyı 1988**).

$$C_{c_i} = \frac{(U_i)^{1/2}}{(r_i)^2} \int_{z_0}^{z_i} \left( \frac{U'}{2 \cdot U} \cdot r \cdot r' + \frac{U''}{4 \cdot U} \cdot r' \right) \cdot U^{-1/2} \cdot dz \quad (2-8)$$



The spherical and chromatic aberration coefficients are denoted by  $C_s$  and  $C_c$  respectively. In the present investigation the value of  $C_s$  and  $C_c$  are normalized in terms of the image side focal length, (i.e. the relative values of  $C_s/f_i$  and  $C_c/f_i$  are investigated as figures of merit which are dimensionless.

It should be noted that  $C_{s_o}$  and  $C_{c_o}$  in the object plane can be expressed in a similar form of equation (2-7) and (2-8) where  $U_o^{-1/2}$  and  $r_o^{-4}$  respectively. The aberration coefficient and the focal length  $f$  are dependent upon each other [**Johans and wener 1987**].

Thus they have been investigated in terms of  $(f)$ . (i.e. the relative values of the spherical and chromatic aberration coefficient  $C_s/f$  &  $C_c/f$  have been taken in the investigation).

## **2-6 Electrodes Shape**

The final task of optimization of electrostatic field is to reconstruct the electrodes shape of lens that would produce such field. This problem has been solved by using the analytic approach. Where one can apply the technique where were used by [Szilagyı 1984] for constructing the electrodes of an electrostatic lens to reconstruct the electrode shape of electrostatic lens. According to this technique the equation of equi potential surfaces is:

$$R_p(z) = \left[ (U_z - U_p) / U_z'' \right]^{1/2} \quad (2-9)$$

where  $R_p$  is the radial height of electrode shape,  $U_z$  is the axial potential distribution,  $U_z''$  is the second derivative of  $U_z$  with respect to  $z$  and  $U_p$  is the value of the potential at any one of the two electrodes.

## **2.7 Computer Program For Computing The Beam Trajectory And Optical Properties**

A computer program with MathCAD Professional 2001, has been used for determining the trajectory of the electron with the aid of the non-classical variation method. The program has shown high ability of drawing general shapes of the axial flux density distributions and high facility and proficiency of drawing the trajectory of the electron beam traversing rotationally symmetric field. The initial conditions depend on the magnification of the lens.

The optical properties such as the focal length are calculated by integrating the paraxial ray equation (2.4).

The spherical aberration coefficient  $C_s$  and chromatic aberration coefficient  $C_c$  are computed by using the aberration integral formula given in equation (2.7), (2.8). Figure 2.6 illustrates a block diagram of this computer program.

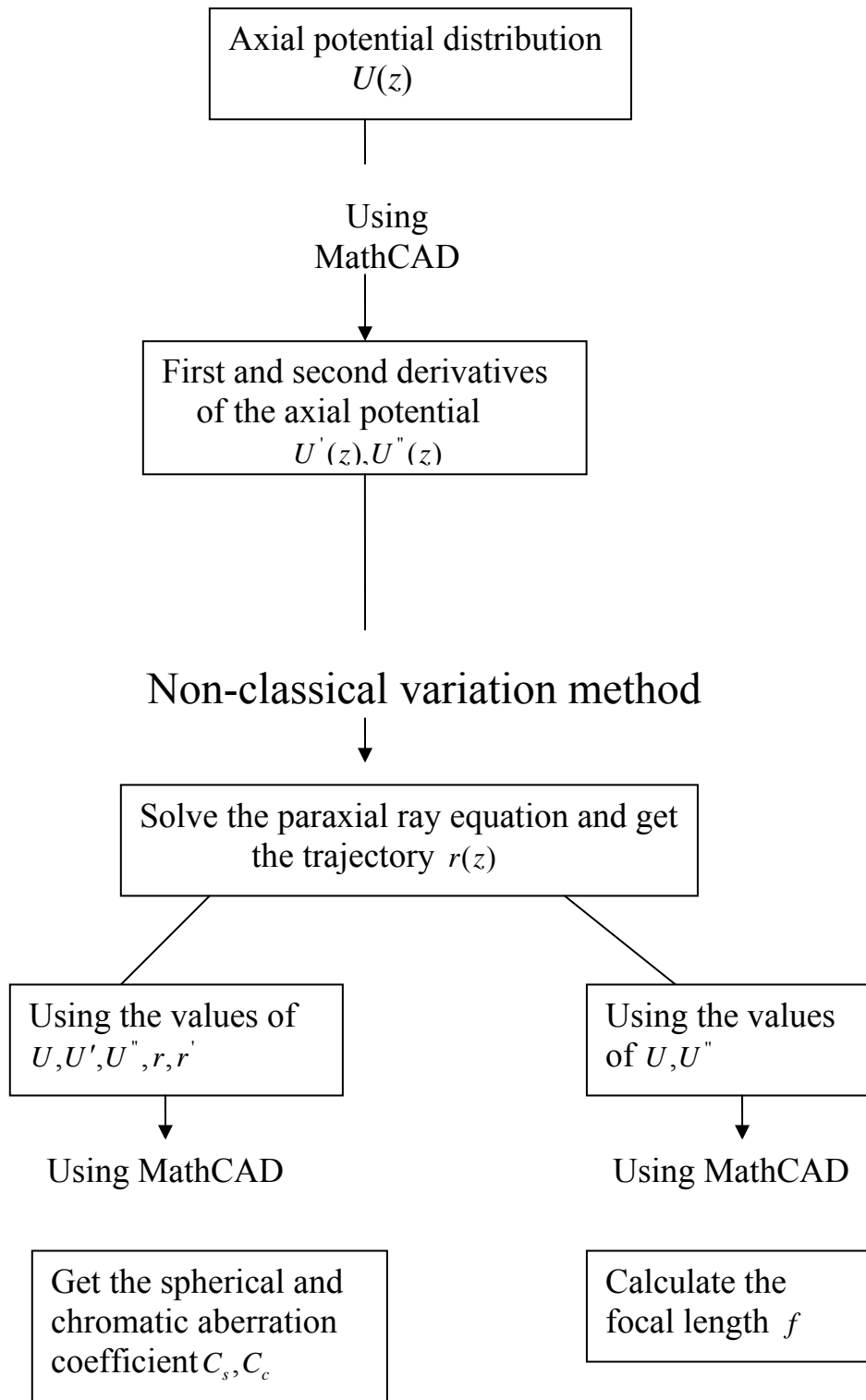


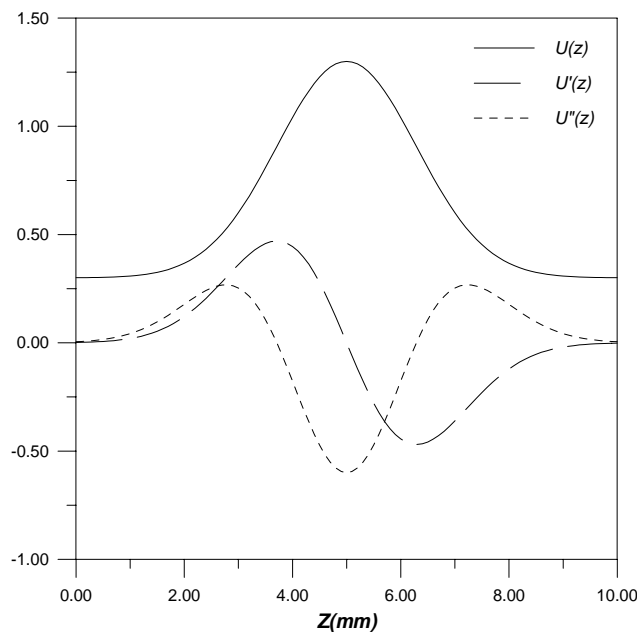
Figure (2.6) A block diagram of the MathCAD program for computing the trajectory and optical properties for einzel lens

## Chapter Three

### 3- RESULTS AND DISCUSSION

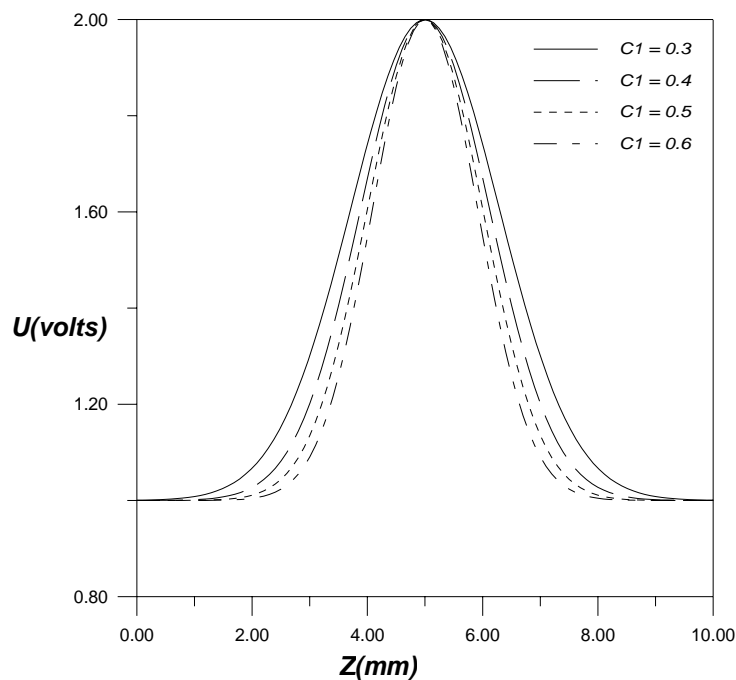
#### 3-1 Axial Potential Distribution of an Einzel Lens

The axial field distribution given in equation (1-4) for an einzel lens is shown in figure (3-1) with its first and second derivatives. This field has been used for determining the trajectory and the aberration coefficients of the lens. Figure (3-1) shows the axial field distribution of an einzel lens whose central electrode is at higher voltage than the two outside electrodes. Since the potential distribution  $U(z)$  is constant at the boundaries, then its first derivative  $U'(z)$  is zero. This indicates that there is no electric field outside the lens i.e. there is a field-free region away from the lens terminals where the trajectory of the charged particles beam is a straight line due to the absence of any force acting on it. The second derivative of the potential;  $U''(z)$  has two inflection points hence the lens has three electrodes.



**Figure (3-1):** The axial potential distribution  $U(z)$  and its first and second derivative  $U'(z)$  and  $U''(z)$  of an einzel lens.

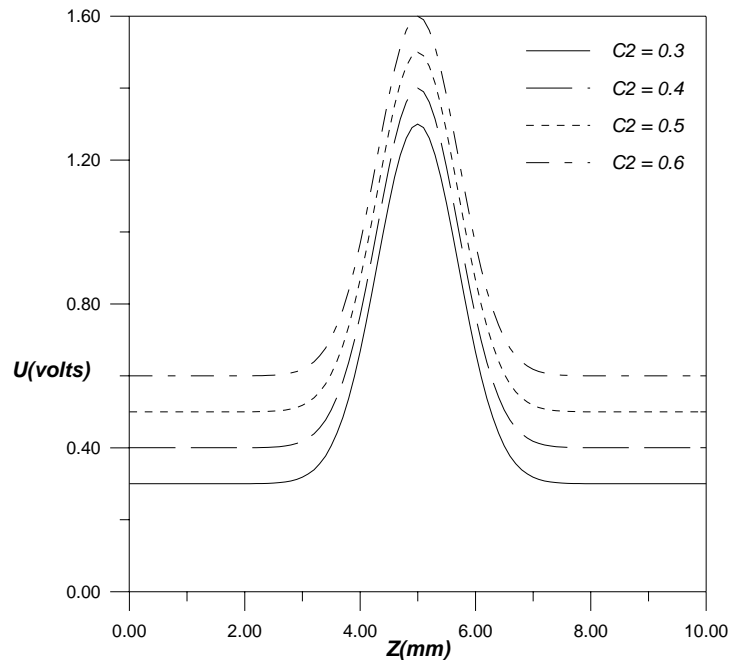
Figure (3-2) shows the axial potential field distribution  $U(z)$  at various values of the constant  $C_1$  and at constant value for  $C_2 = 1$  (see equation (1-4)). It is seen that the change in  $C_1$  don't influent to the peak of the curve but only in the width of the curve, that means decreasing  $C_1$  will increase the separation distances between the central electrode and the outer electrodes. Therefore, studying the optical properties with changing  $C_1$  means the effect of the separation distance on the optical properties.



**Figure (3-2):** The axial potential  $U(z)$  as a function of  $z$  at various values of  $C_1$  and constant value of  $C_2 = 1$ .

Figure (3-3) shows the axial potential field  $U(z)$  as a function of the optical axis  $z$  for different values of  $C_2$  and keeping  $C_1$  constant ( $C_1 = 1$ ). In figure (3-3) the peaks of the curves increase when increasing  $C_2$  with same behaviors for all the curves. Therefore, changing  $C_2$  will change the voltage ratio  $U_i/U_o$  where  $U_i$  the voltage of the central electrode and  $U_o$  is the voltage of the outer electrode. Studying the

change of  $C_2$  on the optical properties actually means the effect of changing the voltage ratio on the optical properties of the einzel lens.



**Figure (3-3):** The axial potential  $U(Z)$  as a function of  $Z$  at various value  $C_2$  and constant value of  $C_1=1$

Before the compute the optical properties of the einzel lens in the zero and infinite magnification condition we must be chose the values  $C_1$  and the values of  $C_2$  (where the values of  $C_2$  are constant at limited numbers) for the values of  $C_1$ .

where I chose the values of  $C_1 = 0.1, 0.2, 0.3, 0.4, 0.5,$  and  $0.6$  and from the results can I see the best results of the trajectory and spherical and chromatic aberration when the  $C_1$  equal  $0.3$  and when  $C_1$  less than  $0.3$  this method doesn't work in these region where the focal length became negative sign.

**3-2 Optical Properties Under Zero Magnification**

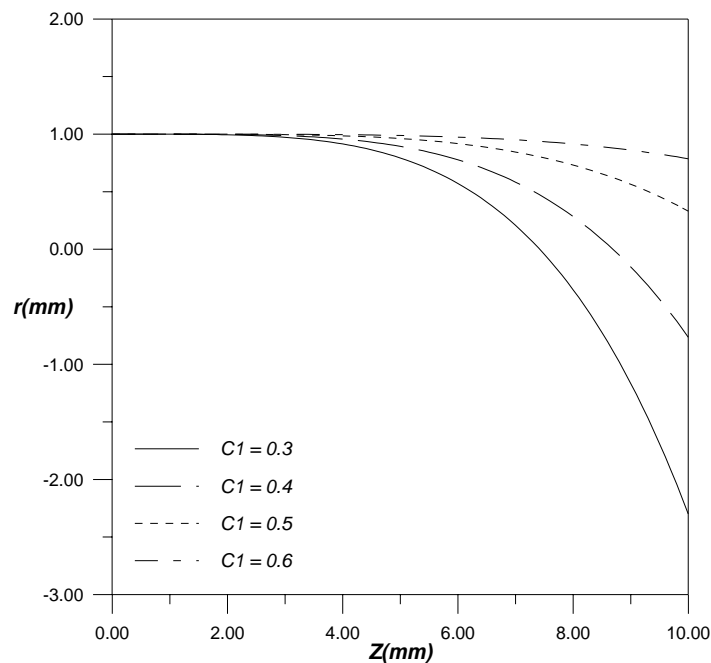
**Condition**

**3-2-1 The trajectory of electron beam**

The electron beam path along the electrostatic einzel lens field under zero magnification condition is shown in figure (3-4) at various values of  $C_1$ . These trajectories have been computed with the aid of solution of the trajectory equation:

$$r(z) = 1 + A_0z^2 + A_1z^3 + A_2z^4 \dots\dots\dots (3-1)$$

Taking into account that  $C_2$  is constant, the Figure shows that the gradient of the electron beam trajectory decreases with increasing the value of  $C_1$ . At  $C_1 = 0.3$  the electron beam intersect the optical axis at the lens while for the other electron beam trajectories (different values of  $C_1$ ) the intersections shifted to the right.

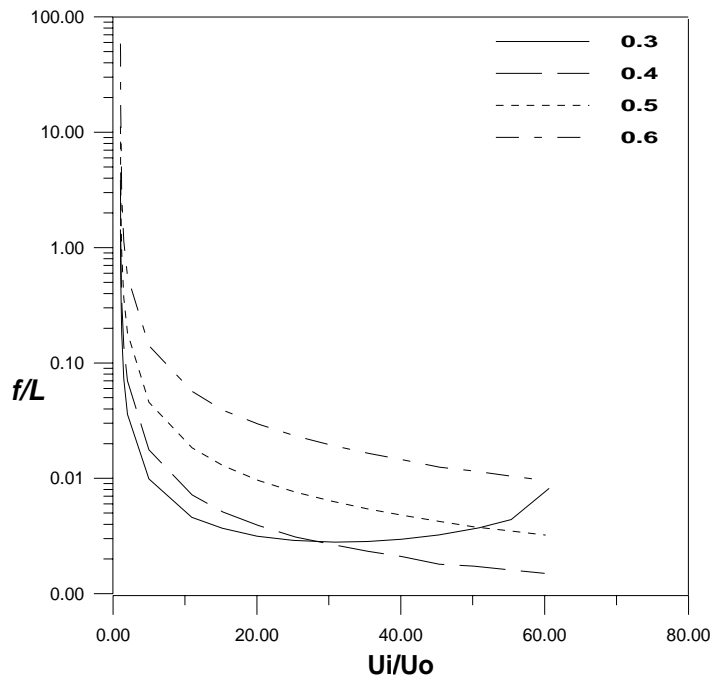


**Figure (3-4):** The electron beam trajectory in einzel electrostatic lens under zero magnification condition at various values of  $C_1$  and Constant value  $C_2= 1$

### 3-2-2 The image-side relative focal length

Under zero magnification condition, the relative image-side focal length  $f_i/L$  (the lens length  $L=10$  mm) of the einzel lens was drawn as a function of the voltage ratio  $U_i/U_o$  is shown in figure (3-5), at various values of  $C_1=0.3,0.4,0.5$  and  $0.6$  at constant value of  $C_2$ . It is seen that  $f_i/L$  decrease with increasing voltage ratio  $U_i/U_o$  except for  $C_1=0.3$  there is a minimum value = 0.0028 at  $U_i/U_o=30.787$  after that  $f_i/L$  increases with increasing  $U_i/U_o$ .

In general the relative focal length decreases as  $C_1$  decreases at any value of the voltage ratio except for  $C_1=0.3$  at  $U_i/U_o > 30$  the value of  $f_i/L$  will be greater than that for  $C_1=0.4$  and  $0.5$ .



**Figure (3-5):** The relative focal length as function of  $U_i/U_o$  under zero magnification condition at various values of  $C_1$  and constant  $C_2$  and  $L=10$  mm

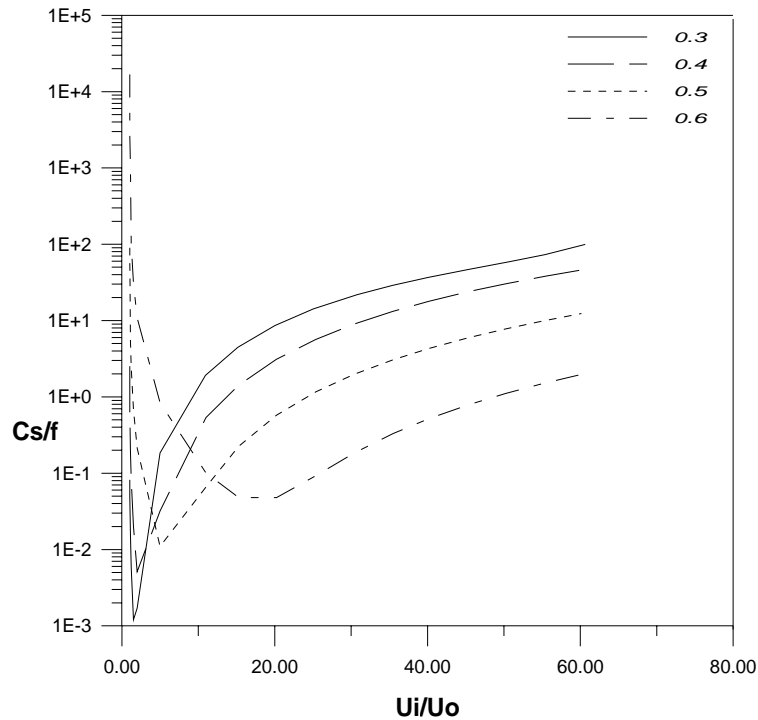


### **3-2-3 The image-side relative aberration coefficients**

The spherical and chromatic aberration coefficients have been given considerable attention in the present work since they are the two most important aberrations in electron optical systems. The present investigation has been focused at their effect on the image side and has been normalized in terms of the image side focal length, i.e. the relative values of  $C_s/f_i$  and  $C_c/f_i$  are investigated as figure of merit which is dimensionless.

#### **3-2-3-1 relative spherical aberration coefficient**

Figure (3-6) shows the image-side relative spherical aberration coefficient  $C_s/f_i$  of the electrostatic einzel lens operated under zero magnification condition as a function of the voltage ratio  $U_i/U_o$  at various values of  $C_1$ . The trajectories which are shown in figure (3-4) have been used for computing the relative spherical aberration coefficients as a function of  $U_i/U_o$  at the value of  $C_1 = 0.3, 0.4, 0.5$  and  $0.6$ . The relative spherical aberration coefficients decreases as the ratio  $U_i/U_o$  increases up to limited values of  $U_i/U_o$ , where all the curves have the minimum values as is shown in table (3-1).



**Figure (3-6):** The relative spherical aberration coefficient as a function of  $U_i/U_o$  at various  $C_1$  under zero magnification condition at  $C_2$  constant.

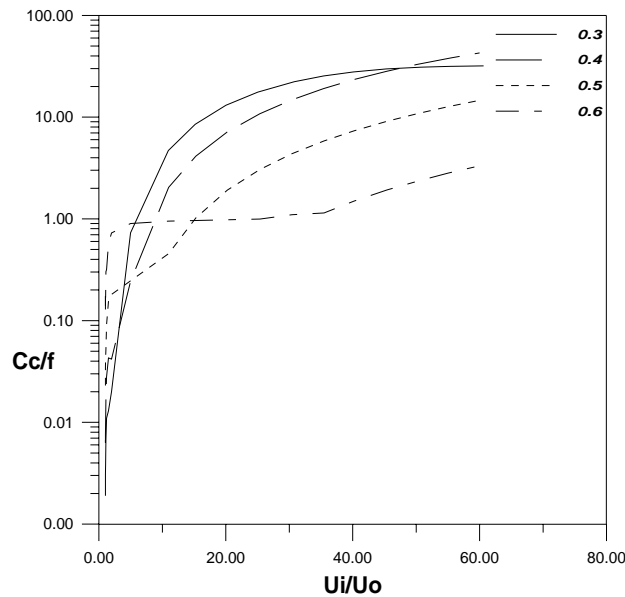
It is seen that  $C_s/f_i$  has a minimum value at  $U_i/U_o = 1.5$  which refer to the value of  $C_1 = 0.3$ . The calculation show that the  $C_1 = 0.6$  gives the optimum value of the relative spherical aberration coefficient for  $U_i/U_o > 20$  and the value of this coefficient in this region still less than unity up to  $U_i/U_o = 50$ .

**Table (3-1).** The minimum values of the image-side relative spherical aberration coefficients for different values of  $C_1$ .

$C_1$	$U_i/U_o$	$(C_s/f_i)_{\min}$
0.3	1.5	0.0012
0.4	2	0.0051
0.5	5	0.011
0.6	15.286	0.048

### 3-2-3-2 relative chromatic aberration coefficient

Within the trajectories which are shown in figure (3-4), the image-side relative chromatic aberration coefficient  $C_c/f_i$  had been computed as a function of the voltage ratio  $U_i/U_o$  at various values of  $C_1$ . Figure (3-7) shows that  $C_c/f_i$  increases with increasing  $U_i/U_o$ . Low values of  $C_c/f_i$  are achieved at high values of  $C_1$ .



*Figure(3-7):The relative chromatic aberration coefficient as a function of  $U_i/U_o$  under zero magnification condition at various values  $C_1$  and  $C_2$  &  $L$  constant*

**Table (3-2).** *The minimum values of the image-side relative chromatic aberration coefficients for different values of  $C_1$ .*

$C_1$	$U_i/U_o$	$(C_c/f_i)_{\min}$
0.3	1.025	0.0019
0.4	1.025	0.0067
0.5	1.025	0.023
0.6	1.025	1.29

### 3-3 Optical properties Under Infinite Magnification

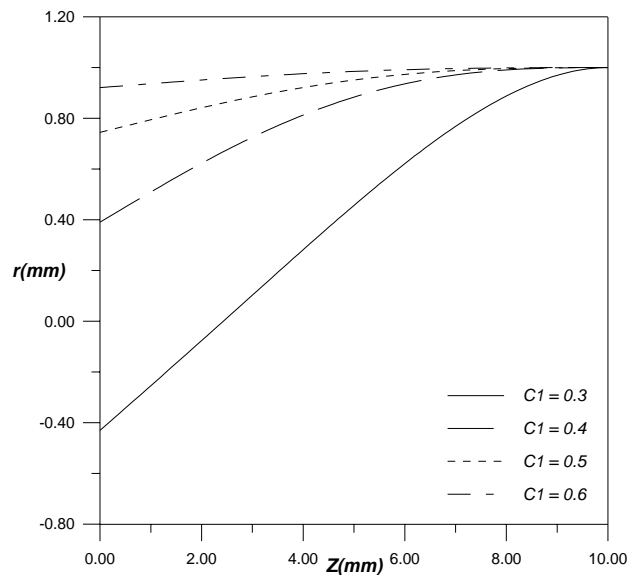
#### Condition

#### 3-3-1 The trajectory of electron beam

The electron beam trajectory along the electrostatic einzel lens field under infinite magnification condition of operation has been computed. Figure (3-8) shows the trajectories of electron beams traversing the electrostatic einzel lens field at various values of  $C_1$ . These trajectories have been computed with the aid of solution of the paraxial ray equation:

$$r(z) = 1 + A_0(z-10)^2 + A_1(z-10)^3 + A_2(z-10)^4 \dots\dots\dots (3-3)$$

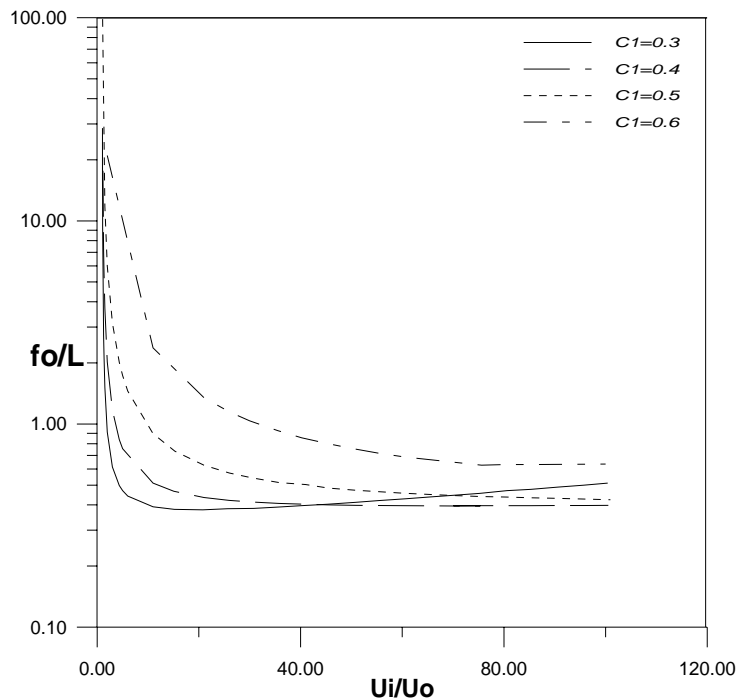
where  $A_0$ ,  $A_1$ ,  $A_2$  are constants and their values are computed using non-classical variational technique with the aid of the boundary conditions of the problem. These trajectories are similar in their general form. The gradient of the electron beam increases with decreasing the value of  $C_1$ . For  $C_1=0.3$  the electron beam intersect the optical axis inside the lens.



**Figure (3-8):** The electron beam trajectory in an electrostatic lens under infinite magnification condition at various values of  $C_1$  and of constant value of  $C_2$  and  $L=10$

### 3-3-2 The object-side relative focal length

Under infinite magnification condition, the relative object-side focal length  $f_o/L$  of the electrostatic einzel lens is shown in figure (3-9) as a function of the voltage ratio  $U_i/U_o$  for various values of  $C_1$  and at constant (limited numbers) value of  $C_2$  and lens length  $L=10$  mm. It is seen that  $f_o/L$  decreases with increasing voltage ratio  $U_i/U_o$  and with decreasing  $C_1$ . Figure (3-9) shows that  $f_o/L$  has a minimum value at which the lens will have its maximum refractive power. The minimum value for  $f_o/L=0.381$  occur when  $C_1=0.3$  and  $U_i/U_o=25.052$ .



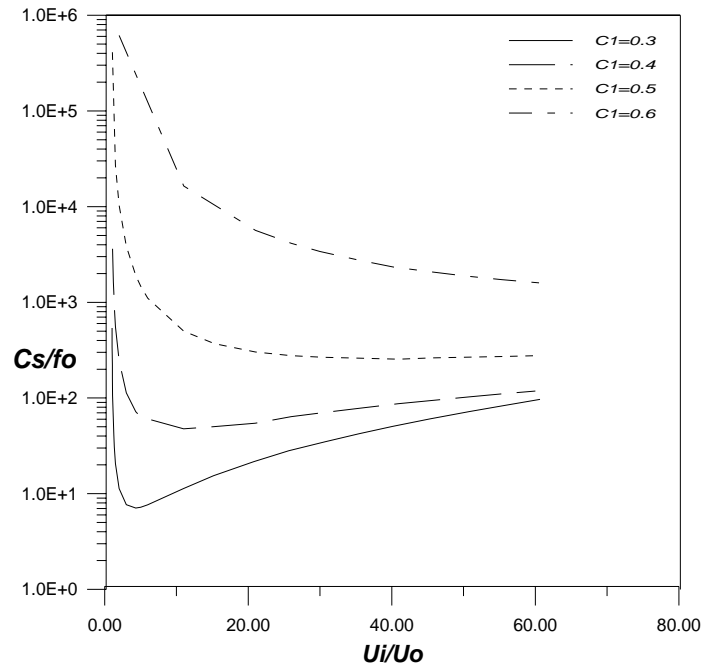
**Figure (3-9):** The relative focal length coefficient as a function of  $U_i/U_o$  under infinite magnification condition at various values  $C_1$  and constant  $C_2$  and  $L=10$

### **3-3-3 The object-side relative aberration coefficients**

The aberration coefficients of the electrostatic einzel lens operated under infinite magnification condition have been computed with the aid of the corresponding trajectory of the electron beam shown in figure (3-8). The spherical and chromatic aberration coefficients  $C_s$  and  $C_c$  respectively, have been given considerable attention since they are the two most important aberrations in electron optical systems.

#### **3-3-3-1 relative spherical aberration coefficients**

The object-side relative spherical aberration coefficient  $C_s/f_0$  of the electrostatic einzel lens operated under infinite magnification condition has been computed as a function of the voltage ratio  $U_i/U_0$  for various values of  $C_1$ . The trajectories in figure (3-8) have been used for computing the relative spherical aberration coefficient at values of  $C_1=0.3, 0.4, 0.5$  and  $0.6$ . Figure (3-10) shows the variation of  $C_s/f_0$  on a logarithmic scale with the voltage ratio  $U_i/U_0$ . It seen that  $C_s/f_0$  has a minimum value for each value of  $C_1=0.3, 0.4,$  and  $0.5$ . The minimum values of  $C_s/f_0$  decreases with decreasing  $C_1$  as clear in Table (3-3). For each value of  $C_1$  the object-side relative spherical aberration coefficient  $C_s/f_0$  decreases with increasing the voltage ratio  $U_i/U_0$ . Therefore, it is preferable to chose low values for both the voltage ratio  $U_i/U_0$  and  $C_1$  with electron-optically acceptable value for  $C_s/f_0$ .



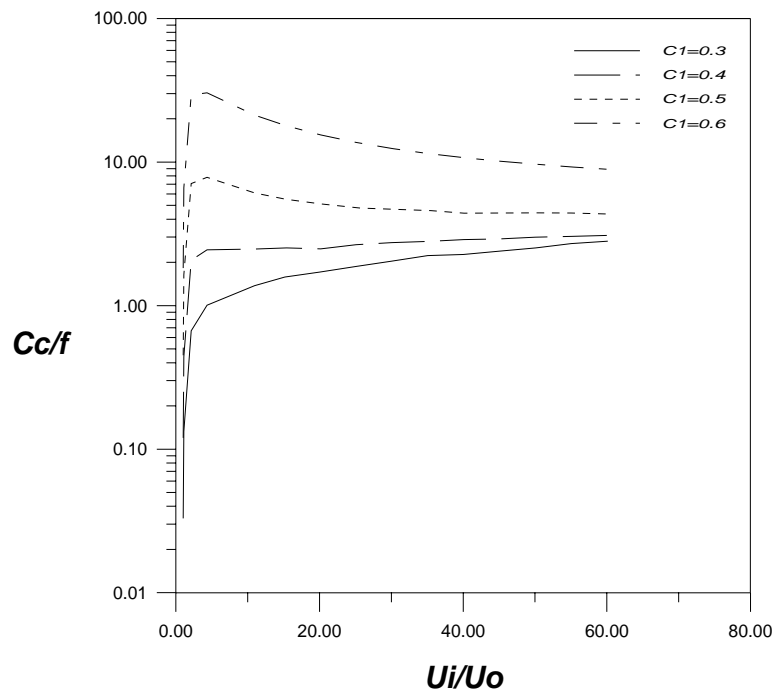
**Figure (3-10):** The relative spherical aberration coefficient as a function of the voltage ratio  $U_i/U_o$  for various values of  $C_1$  and constant value of  $C_2$  and  $L=10$

**Table (3-3).** The minimum values of the objective-side relative spherical aberration coefficients for different values of  $C_1$  (i.e. different values of  $U_i/U_o$ ).

$(C_s/f_o)_{\min}$	$U_i/U_o$	$C_1$
7.048	4.333	0.3
47.57	11.001	0.4
146.601	30.41	0.5
1612	60.031	0.6

### 3-3-3-2 relative chromatic aberration coefficients

With the aid of the trajectories which are shown in figure (3-8) the object-side relative chromatic aberration coefficient  $C_c/f_o$  has been computed as a function of the voltage ratio  $U_i/U_o$  at various values of  $C_1$  and  $L$  is kept constant at 10 mm. Figure (3-11) shows that  $C_c/f_o$  initially increases with increasing the voltage ratio  $U_i/U_o$ , then the object-side relative chromatic aberration coefficient  $C_c/f_o$  of  $C_1 = 0.5$  and  $0.6$  decreases with increasing the voltage ratio  $U_i/U_o$ .



*Figure (3-11) the relative chromatic aberration coefficient as a function of  $U_i/U_o$  for various values of the  $C_1$  under infinite magnification condition and  $C_2$  is constant and  $L=10$*

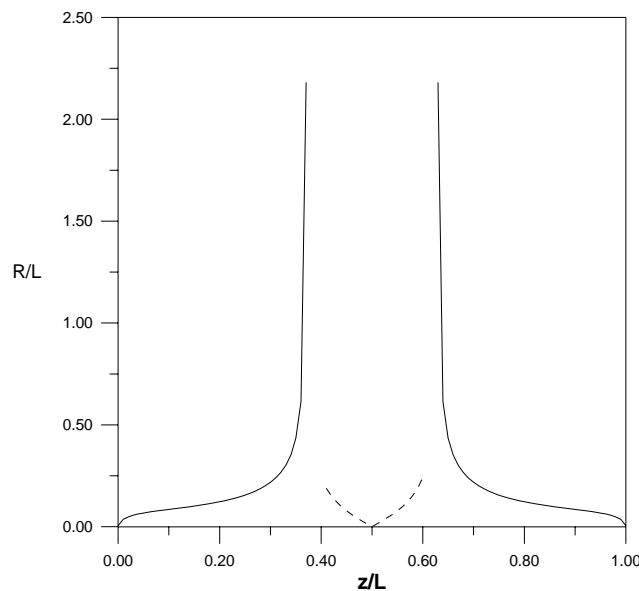


**Table (3-4).** The minimum values of the objective-side relative chromatic aberration coefficients for different values of  $C_1$ .

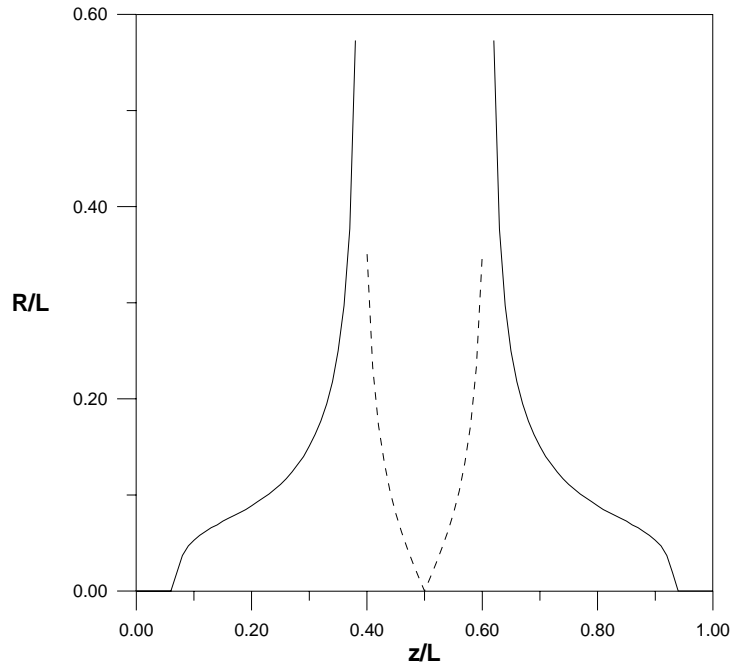
$C_1$	$U_i/U_o$	$(C_c/f_o)_{\min}$
0.3	1.025	0.033
0.4	1.025	0.12
0.5	1.025	0.451
0.5	1.025	1.826

### 3-4 Electrodes shape

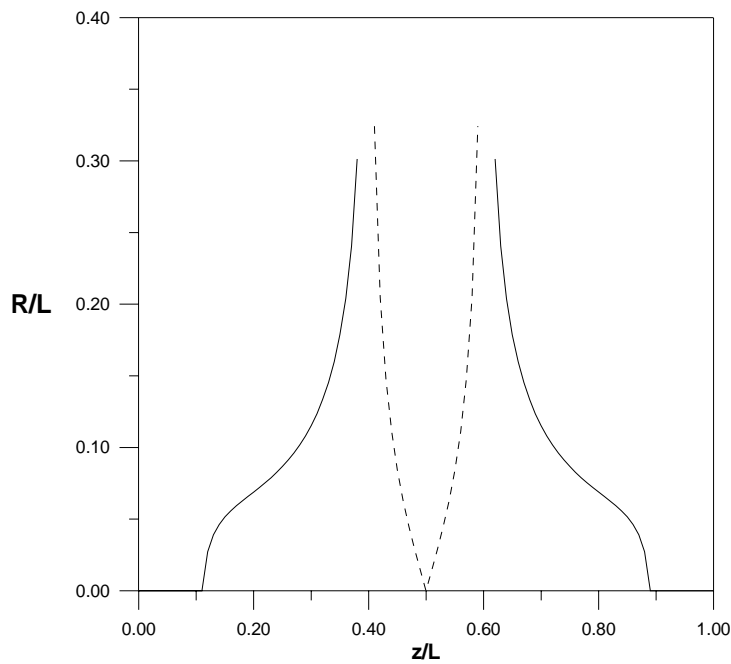
The electrodes shapes are found by using synthesis approach where the electrodes shapes which are corresponding to the axial field distribution of various values of  $C_1 = 0.3, 0.4, 0.5$  and  $0.6$  found with the aid of equation (2-9) and the electrodes shapes are shown in figures (3-12) to (3-15), respectively.



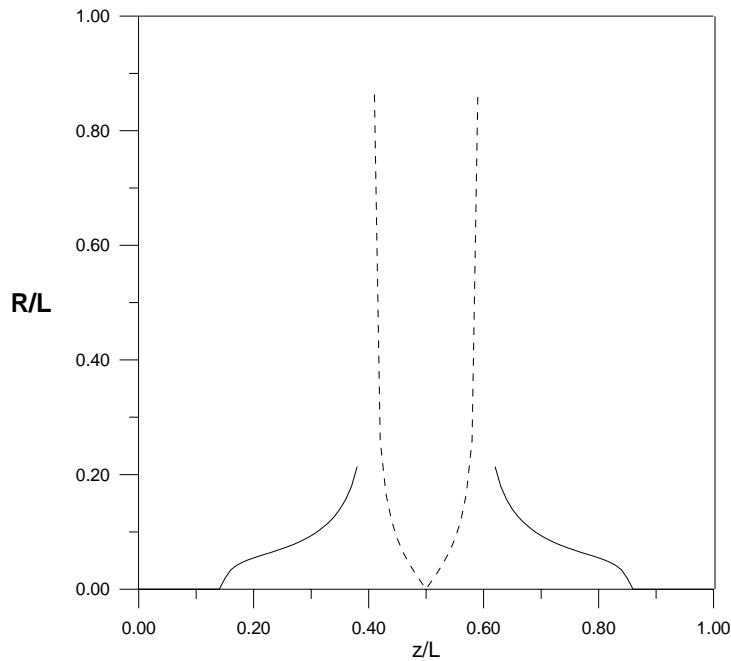
**Figure (3-12):** The three-electrodes of einzel lens which is corresponding to the axial distribution of the constant  $C_1 = 0.3$ ,  $C_2 = 0.07$  and  $L = 10$ .



**Figure (3-13):** The three-electrodes of einzel lens which is corresponding to the axial distribution of the constant  $C_1 = 0.4$ ,  $C_2 = 0.07$  and  $L = 10$ .



**Figure (3-14):** The three-electrodes of einzel lens which is corresponding to the axial distribution of the constant  $C_1 = 0.5$ ,  $C_2 = 0.07$  and  $L = 10$ .



**Figure (3-15):** *The three-electrodes of einzel lens which is corresponding to the axial distribution of the constant  $C_1 = 0.6$ ,  $C_2 = 0.07$  and  $L = 10$ .*

When show the shapes of electrodes we observing when change the values of  $C_1$  the height of electrodes are changing but the reality the height of electrodes don't influential on the optical properties.

The another note from the shapes of electrodes when change the value of  $C_1$  the length of lenses is change where when the  $C_1 = 0.3$  and the  $R/L = 1$  and this is correct but when the  $C_1$  increase the length of the lens is decrease where  $C_1$  is changing length the outer electrode consequently length of lens is changing. For avoid this problem we use the normalizations (where we dividing on the real length of the lens).

## Chapter four

### 4. Conclusion And Suggestion For Future Work

#### 4-1 Conclusion

It appears from the present investigation that it is possible to design various types of electrostatic lenses with small aberrations operated under different potential ratios and magnifications condition. The potential that use in this work is einzel lens potential.

It has been found that it is possible to a design an einzel lens with small aberration and with minimum path of trajectory by used the non-classical variation method where used to solve the paraxial ray equation of charged particles beam.

It has been shown from the results and the graphing found when the decrease of  $C_1$  result the aberration coefficient is decrease also. That is namely we get the best result when the  $C_1$  is equal 0.3.

It has been shown the effect of the change  $C_1$  on the length of lens where when  $C_1$  increase the length of lens decrease and from this result we can calculate the optical property of the einzel lens for different values of  $C_1$  ( $C_2$  is constant ).

## **4-2 Future Work**

- (a) We can study another type's from electrostatic lenses as immersion, cathode and diaphragm lens by using the non-classical variation technique to find the optical properties of these lenses.
- (b) We can use another axial field distribution models to find the optimum properties of einzel lens with the aid of non-classical variation technique
- (c) We can calculate the optical property for any electrostatic lenses when change the  $C_2$  and the  $C_1$  is constant.

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## المستخلص

اجرى بحث حاسوبي في مجال بصريات الجسيمات المشحونة. يهتم البحث بتصميم عدسة احادية الجهد بوساطه استخدام طريقة التغاير غير التقليدي تحت ظروف تكبير مختلفة. ان توزيع مجال الجهد لعدسة احادية الجهد تم تمثيله بدالة تحليلية. تم حل معادلة الاشعة المحورية للمجال المقترح عن طريق ايجاد اقصر مسار للجسيمات المشحونة المارة في العدسة . ومن توزيع الجهد المحوري ومشتقاتيه الاولى والثانية. تم حساب الخواص البصرية كالبعد البؤري والزيغين الكروي واللوني. كذلك تم ايجاد شكل اقطاب عدسة كهروسكونية ببعدين. ويعتمد الزيوغ في العدسات على مسار الجسيمات المشحونة حيث يكون الزيوغ قليل عندما يكون مسار الجسيمات قصير وهذا وجدنها في هذه الدراسة وذلك باستخدام طريقة التغاير غير التقليدي.



جمهورية العراق  
وزارة التعليم العالي والبحث العلمي  
جامعة النهرين  
كلية العلوم  
قسم الفيزياء

# تصميم عدسة احادية الجهد باستخدام طريقة التغاير غير التقليدي

رسالة

مقدمة إلى كلية العلوم في جامعة النهرين  
وهي جزء من متطلبات نيل درجة ماجستير علوم في  
الفيزياء

من قبل

ماهر محمود عبد علي العاني  
(بكلوريوس 1996)

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