STUDY OF TUBE REDUCTION USING DIE-LESS DRAWING

A Thesis Submitted to the College of Engineering of Al-Nahrain University in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering

By

Amjad Mal-Alla Al-Eadany

(B.Sc.2001)

Safar

April

1425

2004

Certification

I certify that this thesis titled" **study of tube reduction using die-less drawing**"was prepared by **Amjad Mal-Alla Al-Edany** under my supervision at Al-Nahraen University, College of Engineering, in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering.

Signature:

Name: Dr. Talal. Y. Al -Naib

Date: / /

Certificate

We certify, as an examining committee, that we have read the thesis titled "study of tube reduction using die-less drawing" and examined the student Amjad Mal-Alla Al-Edany and found that the thesis meets the standard for the degree of Master of Science in Mechanical Engineering.

Signature:	Signature:
Name: Asst. Prof. Dr. Talal. Y. Al-Naib (Supervisor)	Name: (Member)
Date: / /	Date: / /
Signature:	Signature:
Name:	Name:
(Member)	(Chairman)
Date: / /	Date: / /

Approval of the College of Engineering

Signature:

Name: prof. Dr. Fawzi M. Al-Naima (Dean of Engineering College)

Date: / /

ABSTRACT

A study of die-less tube sinking is carried out in which the conventional reduction die is replaced by a die-less stepped bore reduction unit (DRU). The DRU is a single cylinder of two concentric bores with different diameters, where the smallest bore diameter is slightly greater than the outer tube diameter before deformation. The gap between the drawn tube and the DRU is filled with a polymer melt.

The reduction of tube diameter is effected by means of the plastohydrodynamic action of the polymer melt.

Since the smallest bore size of such a device is dimensionally greater than the outer tube diameter, the leading end is not needed, also metal-to-metal contact, and hence wear, is not longer problem. On the other hand a layer of polymer coats the drawn tube, which is important in protecting the tube.

In this study the method of solution is based on the principle of minimum work rate, where the total work rate required for the die-less drawing process is divided into two parts, the first is for shearing the polymer melt (which in this study is considered as non-Newtonian fluid), and the second part of the work rate is for deformation of the tube material.

The profile of deformation is needed to be assumed, the nearest assumed profile to the true profile is that which gives minimum work rate. In this study the assumed profile is chosen to be quadratic equation.

It was found that for a given DRU dimensions and shear stress constant of polymer melt, increasing the drawing speed raises slightly the reduction ratio in the tube diameter, coating thickness, and the drawing stress. The reduction ratio as well as the coating thickness, and the drawing stress are also increased when the gap or/and length ratios of the unit are increased. However increasing the shear stress constant decreases the reduction ratio, coating thickness and the drawing stress.

CONTENTS

AbstractI
ContentsII
NomenclatureV
CHAPTER ONE1
INTRODUCTION AND LITRETUE SURVEY
1.1 Introduction
1.1.1 Conventional drawing process
1.1.2 Die-less drawing process2
1.1.3 Newtonian and non-Newtonian fluids
1.2 literature survey
1.3 Objective
CHAPTER TWO14
THEORETICAL ANALYSIS
2.1 Introduction14
2.2 Analysis of the process with the assumption of no
tube deformation15
2.3 Analysis when tube deformation occurs23
2.3.1 Determination of the point where deformation occurs
2.3.2 Work rate required for shearing the polymer melt
2.3.2.1 Work rate required for shearing the polymer melt in

2.3.2.2 Work rate required for shearing the polymer melt in zone II	
2.3.3 Work rate required for deformation of the tube material	30
2.3.4 Procedure of solution	34
2.3.4.1 Evaluation of the tube reduction	34
2.3.4.1 Evaluation of the drawing stress and coating thickness	
CHAPTER THREE	46
RESULTS AND DISCUSSION	
3.1 Drawing with no tube deformation	46
3.2 Effect of the assumed profile	47
3.3 Reduction in tube diameter and coating thickness	49
3.4 Drawing stress	51
CHAPTER FOUR	65
CONCLUTIONS AND RECOMMENDATIONS	
FOR FUTURE WORK	
4.1 Conclusions	65
4.2 Recommendations for future work	66

References

Appendix A

Appendix B

CHAPTER ONE INTRODUCTION AND LITRETURE SURVEY

1.1 Introduction

Long components of uniform cross section can be produced not only by extrusion but also by drawing. Instead of being pushed, the material is now pulled through a stationary die of gradually decreasing cross section [1].

Wires are largely produced by drawing process, in addition to direct application such as electrical wiring, wire is the starting material for many products including wire frame structure, nails, screws and bolts, rivets, wire fencing, etc.

Seamless tubes are made by a variety of hot working techniques but below a minimum size they must be further reduced cold [1], this can be done by drawing process, such drawn tubes perform important function in hydraulic systems of vehicles, airplanes, industrial machinery, water distribution systems, and in such application as hypodermic needles.

Seamless tubes are sometimes drawn simply through draw dies, either to reduce their diameter (tube sinking) or to change their shape (say, from round to square) .If their thickness is to be reduced, an internal die (mandrel) may also be needed such as moving mandrel, stationary mandrel, or plug.

1.1.1Conventional drawing process

In conventional drawing process, the diameter of the product is continually reduced to specific size whilst pulling it through a trumpet shaped tungsten carbide die where very high pressure generates between the die and

1

the product itself. In industrial practice, boundary lubricants are used in order to reduce the drawing load and to prevent die wear during the drawing. With the presence of a boundary lubricant conventional die, many problems are encountered which may be summarized as follows

- 1) Since the reduction die has a smaller bore size than the diameter of the product to be drawn leading end is absolutely necessary.
- 2) Breakage at start up may occur frequently.
- 3) Metal to metal contact takes place resulting in die wear.

1.1.2 Die-less drawing process

In die-less drawing, the tube or the wire is pulled through a tubular orifice of tapered or stepped bore, which is filled with viscous fluid as shown in Fig. (1.1). The most important feature of the process is that the smallest bore size of the orifice is always greater than the diameter of the un-deformed tube or wire hence metal to metal contact never takes place, also leading end is not needed.

The mechanics of the process using either type of unit is based on the plasto-hydrodynamic behavior of the viscous fluid medium.

The pulling action of the product through the viscous fluid generates hydrodynamic pressure and gives rise to drag force. Depending on the type of the fluid and the unit, the combined effect of the pressure and the drag force can be sufficient to cause plastic yielding and to subsequently deform the tube (or the wire) permanently.

Experimental work has shown that mild steel, copper and stainless steel wires can be drawn using either type of the unit, with polymer melt as the hydrodynamic pressure medium, and that products having comparable dimensional and surface qualities can be obtained [2]. It has also been

2

demonstrated [2] that a single unit may be used to produce tubes of different cross-sectional areas by varying either drawing speed or pressure medium.

Tube sinking is a geometrically similar process to wire drawing and hence the technique of dieless wire drawing should be equally applicable to this process.

The polymer melt, in addition to acting as a lubricant, was also found to form a coating on the drawn wire or tube. This coating is useful in protecting the product against corrosion and also acts as a lubricant during any subsequent forming operation e.g., bending or cold heading.

1.1.3 Newtonian and non-Newtonian fluids

Newtonian characteristics of the fluids means that the rate of deformation or the shear rate at a point in the fluid is directly proportional to the local shear stress [3]. Where the viscosity is the constant of the proportionality, which is constant at given temperature and pressure.

The fluids have non Newtonian characteristics when the shear rate at a point in the fluid is not be directly proportional to the shear stress, Fig (1.2) shows the relationship between shear stress and shear rate for Newtonian and different types of non Newtonian fluids.

In industry, fluids are being increasingly encountered which not exhibit Newtonian flow behavior. In many engineering design problems, it is important that this non-ideal behavior be taken into account. Such non-Newtonian fluid should not be regarded as curiosities because many materials which are of industrial importance are highly non Newtonian in character.

Engineering problems involving non-Newtonian fluids are of many diverse. In fact, for all the well-known problems involving Newtonian fluids there is usually non-Newtonian counterpart. These include fluid transport involving pipeline design, pumping and flow measurement, lubrication, mixing operations and heat transfer.

1.2 Literature survey

It is believed that the Sumerians were perhaps the first to draw wire, as early as 4000 B.C. using hard wood and stones with tapered holes as dies. Their skill was, however, limited to the making of gold, silver, and copper wire, which were used for ornamental purposes only.

Limited production, on a commercial scale, was first undertaken in Germany about the 14th century, and in England in the 15th century. The wire drawing industry however did not make any progress until the 19th century with the discovery of electrical power [4]. Later on multi-die machines and Tungsten carbide dies were used, but the operating principle remained the same, the wire is pulled through a tapered reduction die and the material deforms plastically whilst passing through the die. The die in this case acts primarily to reduce the wire diameter to a specific size with acceptable surface finish.

In industrial wire drawing practical lubrication is used to reduce the drawing load and die wear and hence improve the machine life and surface finish of the product.

Usually, boundary lubrication is affected by pre-treating the wire and applying a suitable lubricant (usually dry-soap) along the die and the wire interface, the die wear however is still at a significantly high level .In attempting to reduce the die wear to a lower level an alternative lubrication (hydrodynamic) system was pioneered by Christoferson and Naylore [5] by employing along and narrow tube before the conventional die as shown in Fig (1.3). The lubricants used in this case were oil, which was pumped into the

4

tube by the motion of the wire, which provided true hydrodynamic lubrication in the die. Experimental work show [5] that the friction in the die itself can then be so reduced that even when the drag in the tube is allowed for, the total drag force is less than with the existing method of soap lubrication and tests with steel dies have shown large reduction in wear.

Bloor et al. [6] present a comprehensive analysis of the lubrication (based on Newtonian characteristic of the fluid) in plane strain drawing process using a conventional dies (i.e without pressure tube). An elasto-plasto hydrodynamic lubrication analysis was applied where the die region is divided into four regions as shown in Fig (1.4). The influence of elastic deformation of the work material from entry to exit from the die and the die land factors, which in other analysis have generally been neglected, are considered separately but are then integrated with the analysis for the tapered portion of the die to give a continuous analysis through the deformation region.

Thompson and Symmons [7] on the other hand studied a hydrodynamic lubrication of wire drawing using polymer melt based on Newtonian analysis. The unit consisted of a shortened Christopherson tube (pressure tube) preceded by a conventional die. The polymer was first fed into a melt chamber and kept at a constant temperature. The molten polymer was then drawn into the pressure tube by the motion of the wire. The object of their study was to coat the wire during the drawing operation and examine the adhesion between the polymer coat and the wire. It was claimed that the hydrodynamic lubrication of the wire was achieved successfully but the thin wall coat adhered on the wire was unsatisfactory.

A non-Newtonian analysis of the lubrication and coating of the wire was presented by Crampton et al. [8], using a similar unit to that used in [7] which differs from it in that the polymer viscosity has been defined both as shear rate and pressure changed.

Polymer melt lubrication was further studied by Hashmi et al [9], the analysis of the process was based on non Newtonian characteristic of the fluid, also strain rate sensitivity of the wire material and the effect of the pressure on the viscosity were taken into account.

It was shown that the polymer coating deposited on the wire was reduced as the drawing speed of the wire was increased.

Crampton et al. [8], and Hashmi et al. [9] enabled the prediction of the hydrodynamic pressure, drawing stress together with the reduction in area and the thickness of the polymer coat on the wire.

On the basis of experimental evidence [5,7,8,9], it was found that the deformation of the wire commences in the tube itself (pressure tube) before reaching the reduction die, which effectively acts only as a seal. Under these conditions the die geometry becomes of secondary importance and deformation actually takes place as if an effective die of continuously changing die angle is being used. With this observation, it was thought that a unit with its bore size greater than the un-deformed wire diameter might be designed to generate high pressure to reduce the wire diameter.

Hashmi et al [10] were the first to introduce the dieless wire drawing using polymer melt. It was shown [10] that a reduction in cross-sectional area in excess of 20 percent could be obtained in a single pass when a wire was pulled through such a dieless reduction unit filled with polymer melt. In this case the wire was pulled through a tubular orifice of a tapered bore configuration as shown in figure (1.1.a). The smallest bore of the unit being slightly greater than the incoming wire diameter. It was shown [10] that when the drawing velocity was increased the reduction in diameter decreased, and the agreement between the predicted and actual percentage reductions in area was seen to be very poor due to the assumption that the polymer melt was Newtonian.

Hashmi and Symmons [11], studied this process by using a tubular orifice filled with viscous fluid through which a circular cross-sectional solid wire, which was assumed to be rigid non-linearly strain hardening, was being pulled through, and the viscous fluid was assumed to be Newtonian. Finite difference numerical technique was applied to solve the equations for the plasto-hydrodynamic pressure and the resulting axial stress, which in turn enabled prediction of the non-linear deformation profile of the continuum and the reduction in diameter for a given drawing speed. The trend of theoretical results obtained had very poor agreement with that of experimental results.

A plasto-hydrodynamic lubrication of tube sinking was studied by Hashmi[12] using either type of the DRU and a program of exploratory tests was conducted. The results of these initial tests were presented and the implications of the findings towards further development were discussed. An experimental study was applied on copper tubes and (WVG23) polypropylene as the pressure medium. In this study two different types of defects where observed on the product (dimples and depression). Hashmi[12] thought a successful sinking of thick walled tubes can be achieved using the die-less reduction unit and the sinking of thin walled tubes may still be possible, by using long units to increase the back tension. Also in this work it was demonstrated experimentally that the principles of plasto-hydrodynamic die less wire drawing were equally applicable to tube sinking.

A stepped bore dieless reduction unit based on non-Newtonian analysis was used for wire drawing by Parvinmehr et al. [13] using a polymer melt as a pressure medium. They showed [13] in their study that a reduction ratio of about 20 % can be achieved in single pass using steel and copper as the wire material. The only limitation observed in their study was the decrease in the reduction of the wire diameter at higher drawing speeds. Some physical properties of the drawn wires were examined and the result showed that the product was comparable to those wires drawn using conventional methods.

M.I Panhwar et al. [14] has presented a non Newtonian plastohydrodynamic analysis of the tube sinking process for stepped bore reduction in which the polymer melt is used as the pressure medium. The effect of shear rate and pressure on viscosity of the polymer together with the shear stress, are included in the analysis, also the strain hardening and strain rate sensitivity of the tube material were incorporated .An experimental study incorporated too and was carried out with copper tubes, and low density polyethylene(Alkathaline WVG 23) polymer melt was used. The results predicted on the bases of the non Newtonian theoretical analysis were found to be in closer agreement with this obtained experimentally than with those predicted using Newtonian analysis.

Die-less wire drawing was further studied by Al-Rawi [15], for strain rate sensitive material (superplastic tin-lead eutectic). The pressure distribution and axial stress within the stepped bore reduction unit, which is filled with polymer melt (polyethylene WVG23) where predicted. It was shown that the reduction ratios obtained by using this process was found to be higher than that obtained using conventional materials. The maximum reduction in area obtained was about 52 % and the drawing stress was found to be less than that obtained for conventional materials.

The effect of the pressure and shear rate on the viscosity of the polymer melt were taken into account [13,14,15], the same empirical equation represents the viscosity of the polymer were also used.

8

Alsammaray [16] studied the die-less wire drawing using stepped bore reduction unit were the pressure medium was super-plastic material (tin-lead eutectic alloy) as pressure medium. The maximum pressure attainable within the unit was found to be higher than that obtained using polymer melt as a pressure medium, but at the expense of drawing stress. He also concludes that the drawing speed is a factor that affects signifacntly all of the other die-less wire drawing parameters.

It has been shown from experimental and theoretical investigations [12-15] that the coat thickness is reduced both speed and product material strength increase, while increasing the drawing speed causes the product reduction ratio to decrease.

1.3 Objective

The objective of this work is to investigate the die-less tube sinking process based on the minimum work rate principle. The different parameters such as drawing speed, dimensions of the DRU, and the shear stress constant of the polymer melt are taken into account in the analysis.



Figure 1.1 Schematic diagram showing (a) the stepped bore (b) the tapered bore die less reduction units



Figure 1.2 shows the relationship between shear stress and shear rate for Newtonian and different types of non-Newtonian fluids



Figure 1.3 typical arrangement of inlet tube and die [5].

- 1. Elastohydrodynamic inlet region.
- 2. Plasto-hydrodynamic region of strip deformation.
- 3. Plasto-hydrodynamic region in the land.
- 4. Elastohydrodynamic outlet region.

Figure 1.4 regions considered in the die, strip, and lubrication arrangement [6]

CHAPTER TWO THEORETICAL ANALYSIS

2.1 Introduction

In all previous studies the theoretical analysis applied to find the product reductin in the die-less drawing are based on stress analysis, in this study the evaluation of the reduction is based on the minimum work rate principle spent on the system.

The work rate needed for tube reduction is devided into two parts, the first is for shearing the polymer melt and the second part is for deformation of the tube material.

In this analysis the following assumption are considered:

- 1. The flow of polymer melt is laminar, axial, and isothermal.
- 2. The thickness of the polymer melt layer is small compared to the bore of the die-less unit.
- 3. The pressure in the polymer melt is uniform in the thickness direction at any point along the length of the DRU.
- 4. The tube material is deformed isothermally.
- 5. The process is axi-symmetric.
- 6. No slipping occurs between the tube or the unit and the polymer melt.

The theoretical analysis is divided into two main parts, the first is the analysis of the process with the assumption of no tube deformation, and the second part is the analysis of the tube deformation.

2.2 Analysis of the process with the assumption of no tube deformation.

In this section the analysis is applied to the flow of polymer melt assuming no deformation in the drawn tube, Fig. (2-1).

Since the thickness of the polymer melt layer contained in the die-less reduction unit is small as compared with the bore diameter, the analysis of the flow is carried out in the axial direction.

An empirical relation relating the shear stress and the shear rate of the polymer melt based on non-Newtonain analysis [14] is expressed as:

$$\tau + k\tau^{3} = \eta \left(\frac{du}{dy}\right) \tag{2-1}$$

Where

 τ Is the shear stress of the polymer melt in the unit.

k Non- Newtonian factor (shear stress constant).

 η Is the viscosity of the polymer melt.

 \mathcal{U} Is the velocity of the polymer melt at a distance y from the surface of the tube within the gap.

For the first part of the unit (the part before the step) from equilibrium in the axial direction the relation between the pressure and the shear stress for the polymer melt between the outer surface of the tube and the inner surface of the DRU may be expressed as:

$$\left(\frac{dp}{dx}\right)_{1} = \left(\frac{d\tau}{dy}\right)_{1}$$
(2-2)

Where

 $\left(\frac{dp}{dx}\right)_1$ Is the pressure gradient in the first part of the unit.



Is the shear stress gradient in the first part of the unit.

Integrating the above equation with respect to y gives

$$\tau_1 = \left(\frac{dp}{dx}\right)_1 \cdot y + C_1$$

Where C_1 is constant of integration.

At the surface of the tube (y=0), the above equation becomes:

$$C_1 = \tau_{C1}$$

Where τ_{C1} is the shear stress at y=0, then

$$\tau_1 = \left(\frac{dp}{dx}\right)_1 y + \tau_{C1}$$

Or

$$\tau_1 = p_1' y + \tau_{C1} \tag{2-3}$$

Where

$$p_1' = \left(\frac{dp}{dx}\right)_1$$

Substituting into equation (2-1) gives

$$\eta \left(\frac{du}{dy}\right)_{1} = p_{1}'y + \tau_{C1} + k\left(p_{1}'^{3}y^{3} + \tau_{C1}^{3} + 3p_{1}'^{2}y^{2}\tau_{C1} + 3\tau_{C1}^{2}p_{1}'y\right)$$

Integrating the above equation with respect to y and noting that (η) also remain constant at all values of y for a given value of x

$$\eta u_{1} = \frac{p_{1}'y^{2}}{2} + \tau_{C1}y + k\left(\frac{1}{4}p_{1}'^{3}y^{4} + \tau_{C1}^{3}y + p_{1}'^{2}y^{3}\tau_{C1} + \frac{3}{2}p_{1}'y^{2}\tau_{C1}^{2}\right) + C_{2}$$

Two boundary conditions can be considered

- i. At the surface of the tube (y=0), $u_1=v_1$.
- ii. At the surface of the unit $(y=h_1)$, $u_1=0$.

Applying boundary condition (i) in the above equation, gives

$$\eta v_1 = C_2$$

Which becomes, after substituting $C_{_2}$

$$\eta u_{1} = \frac{p_{1}' y^{2}}{2} + \tau_{C1} y + k \begin{pmatrix} \frac{1}{4} p_{1}'^{3} y^{4} + \tau_{C1}^{3} y + p_{1}'^{2} y^{3} \tau_{C1} \\ + \frac{3}{2} p_{1}' y^{2} \tau_{C1}^{2} \end{pmatrix} + \eta v_{1} \quad (2-4)$$

Applying boundary condition (ii) in equation (2-4) gives, after simplification and re arrangement

$$\tau_{C1}^{3} + \left(\frac{3}{2}p_{1}'h_{1}\right)\tau_{C1}^{2} + \left(\frac{1}{k}p_{1}'^{2}h_{1}^{2}\right)\tau_{C1} + \left(\frac{\eta v_{1}}{kh_{1}} + \frac{p_{1}'h_{1}}{2k} + \frac{1}{4}p_{1}'^{3}h_{1}^{3}\right) = 0$$
(2-5)

This equation yields two imaginary roots and one real root (details are given in appendix A). The real root is:

$$\tau_{C1} = \left[\frac{-\eta v_1}{2kh_1} + \left(\frac{\eta^2 v_1^2}{4k^2 h_1^2} + \frac{1}{27} \left(\frac{1}{k} + \frac{1}{4} p_1'^2 h_1^2 \right)^3 \right)^{\frac{1}{2}} \right]^{\frac{1}{3}} + \left[\frac{-\eta_1 v_1}{2kh_1} - \left(\frac{\eta^2 v_1^2}{4k^2 h_1^2} + \frac{1}{27} \left(\frac{1}{k} + \frac{1}{4} p_1'^2 h_1^2 \right)^3 \right)^{\frac{1}{2}} \right]^{\frac{1}{3}} - \frac{1}{2} p_1' h_1 \quad (2-6)$$

Equation (2-6) gives the shear stress on the tube surface in the first part of the unit with no tube deformation for known value of p'_1 .

Now the flow rate (Q_I) of the polymer melt in the axial direction within the gap per unit width of the circumferences before the step can be given by:

$$Q_1 = \int_0^{h_1} u_1 dy$$

Where, u_1 is the velocity of the polymer melt in the first part of the unit.

Substituting for u_1 from equation (2-4) into the above equation and integrating gives

$$Q_{1} = \frac{p_{1}'h_{1}^{3}}{6\eta} + \frac{h_{1}^{2}\tau_{C1}}{2\eta} + \frac{k}{\eta} \left(\frac{1}{20}p_{1}'^{3}h_{1}^{5} + \frac{1}{2}h_{1}^{2}\tau_{C1}^{3} + \frac{1}{4}p_{1}'^{2}h_{1}^{4}\tau_{C1} + \frac{1}{2}p_{1}'h_{1}^{3}\tau_{C1}\right) + v_{1}h_{1}$$
(2-7)

On the other hand the continuity equation for the flow of liquid polymer can be written as[14]

$$\frac{d}{dx}(Q_{x1}) + \frac{d}{dy}(Q_{y1}) + \frac{d}{dz}(Q_{z1}) = 0$$

Since the flow in the analysis is assumed axial then

$$\frac{d}{dy}(Q_{y1}) = \frac{d}{dz}(Q_{z1}) = 0$$

Hence

$$\frac{d}{dx}(Q_{x1}) = 0$$

Now after substituting equation (2-6) into equation (2-7) and after differentiating with respect to x, it can be shown that

$$\frac{d}{dx}(p_1') = 0$$

Hence

$$p'_1 = \text{Constant} = p_m / L_1$$

Where

 $p_{\rm m}$ Is the pressure at the step

 L_1 Is the length of the first part of the unit.

Equation (2-7) gives the flow rate in the first part of the unit for known values of $p_{\rm m}$.

To verify the continuity of the polymer melt $(Q_1=Q_2)$, the flow rate in the second part of the unit is evaluated in a similar analysis as that applied at the first part. The relationship between pressure and shear stress in the second part of the unit (after the step) is given by

$$\left(\frac{dp}{dx}\right)_2 = -\left(\frac{d\tau}{dy}\right)_2 \tag{2-8}$$

The minus sign refers to that the pressure gradient is negative in the second part of the unit

Where

$$\left(\frac{dp}{dx}\right)_2$$
 Is the Pressure gradient in the second part of the unit.
 $\left(\frac{d\tau}{dy}\right)_2$ Is the shear stress gradient in the second part of the unit.

Integrating equation (2-8) with respect to y and noting that the pressure gradient is constant for different values of y for any given value of x, gives

$$\tau_2 = -p_2'y + C_3$$

Where

$$p_2' = \left(\frac{dp}{dx}\right)_2$$

 C_3 Is the constant of integration.

At the surface of the tube (y=0), the above equation gives

$$C_3 = \tau_{C2}$$

Where τ_{C2} is the shear stress at y=0,then

$$\tau_2 = -p'_2 y + \tau_{C2} \tag{2-9}$$

Substituting equation (2-9) into equation (2-1) and integrating gives

$$\eta u_{2} = \frac{-p_{2}'y^{2}}{2} + \tau_{C2}y + k \begin{pmatrix} -\frac{1}{4}p_{2}'^{3}y^{4} + \tau_{C2}^{3}y \\ + p_{2}'^{2}y^{3}\tau_{C2} - \frac{3}{2}p_{2}'^{2}y^{2}\tau_{C2}^{2} \end{pmatrix} + C_{4}$$
(2-10)

Where C_4 is the constant of integration

Applying the boundary condition that at y=0, $u_2=v_1$ Hence

 $C_4 = \eta v_1$, substituting in equation (2-10),

$$\eta u_{2} = \frac{-p_{2}'y^{2}}{2} + \tau_{C2}y + k \begin{pmatrix} -\frac{1}{4}p_{2}'^{3}y^{4} + \tau_{C2}^{3}y \\ + p_{2}'^{2}y^{3}\tau_{C2} - \frac{3}{2}p_{2}'^{2}y^{2}\tau_{C2}^{2} \end{pmatrix} + \eta v_{1}$$

Applying the second boundary condition that is, at y=h2, $u_2 = 0$, and after simplification and re-arrangement, the above equation becomes

$$\tau_{C2}^{3} + \left(-\frac{3}{2}p'_{2}h_{2}\right)\tau_{C2}^{2} + \left(\frac{1}{k} + p'_{2}^{2}h_{2}^{2}\right)\tau_{C2} + \left(\frac{\eta v}{kh_{2}} - \frac{1}{4}p'_{2}^{3}h_{2}^{3} - \frac{p'_{2}h_{2}}{2k}\right) = 0$$
(2-11)

This equation has two imaginary roots and one real root (details are given in appendix A). The real root is

$$\tau_{C2} = \left[\frac{-\eta v_1}{2kh_2} + \left(\frac{\eta^2 v_1^2}{4k^2 h_2^2} + \frac{1}{27} \left(\frac{1}{k} + \frac{1}{4} p_2'^2 h_2^2\right)^3\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} + \left[\frac{-\eta v_1}{2kh_2} - \left(\frac{\eta^2 v_1^2}{4k^2 h_2^2} + \frac{1}{27} \left(\frac{1}{k} + \frac{1}{4} p_2'^2 h_2^2\right)^3\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} + \frac{1}{2} p_2' h_2$$
(2-12)

Now the flow of polymer in the second part of the unit per unit width of circumference is

$$Q_2 = \int_0^{h_2} u_2 dy$$

Where u_2 is the velocity of the polymer melt in the second part of the unit.

Substituting for u_2 from equation (2-10) into the above equation and integrating gives

$$Q_{2} = -\frac{p_{2}'h_{2}^{3}}{6\eta} + \frac{h_{2}^{2}\tau_{C2}}{2\eta} + \frac{k}{\eta} \left(-\frac{1}{20}p_{2}'^{3}h_{2}^{5} + \frac{1}{2}h_{2}^{2}\tau_{C2}^{3} + \frac{1}{2}h_{2}^{2}\tau_{C2}^{3} + \frac{1}{4}p_{2}'^{2}h_{2}^{4}\tau_{C2} - \frac{1}{2}p_{2}'h_{2}^{3}\tau_{C2}^{2} \right) + v_{1}h_{2}$$
(2-13)

As mentioned in the second part of the unit, the flow in y and z direction is zero then

$$\frac{d}{dx}(Q_2) = 0$$

Substituting for (τ_{c2}) from equation (2-12) into equation (2-13) and differentiating with respect to x, it may be shown that

$$\frac{d}{dx}(p_2')=0$$

Then

 $p_2' = \text{Constant} = p_m / L_2$

Thus, the pressure gradient in the second part of the unit is also shown to be linear.

It is known that if the pressure increase, the viscosity of polymer increases also. For better analysis this effect should be included.

A generalized equation relating the viscosity and pressure is used [14] of the form.

$$\eta = \eta_0 + \frac{\left(a + bp_m^2\right)}{\dot{\gamma}} \tag{2-14}$$

Where

 $\eta_{\scriptscriptstyle 0}~$ Is the initial viscosity of the polymer melt

 $\dot{\gamma}$ Is the shear rate

(a,b) are constants depending on polymer characteristics.

Assuming shear rate ($\dot{\gamma}$) equals (v_1 / h_1) gives

$$\eta = \eta_0 + \frac{h_1}{v_1} \left(a + b p_m^2 \right)$$
(2-15)

In equation (2-15) the only unknown parameter is (p_m) .

Numerical values of (p_m) and hence (p'_1) and (p'_2) may be substituted into equations (2-6), (2-7), (2-12), and (2-13) and by using an iteration technique until the continuity equation of flow $Q_1=Q_2$ is satisfied. Therefore, the pressure in the unit, and the viscosity of the polymer melt are evaluated.

2.3 Analysis when tube deformation occurs

In this section the unit is divided into three zones as shown in Fig. (2.2) and the work rate spent in each zone is evaluated.

2.3.1 determination of the point where deformation commences

Consider the axial force equilibrium of an element of the tube within the die less unit, zone I, as shown in Fig. (2.3)

$$\left(\sigma_{x}+d\sigma_{x}-\sigma_{x}\right)\left(\pi D_{1}T\right)-\tau_{c1}\left(\pi D_{1}dx\right)=0$$

Where

 σ_x Is the axial stress in the drawn tube

T is the thickness of the tube wall Simplifying and integrating

$$\sigma_x = \tau_{c1} x / T + C$$

At x=0, $\sigma_x = 0$ and hence C = 0 and

$$\sigma_x = \tau_{c1} x / T \tag{2-16}$$

Now equilibrium of radial forces gives

$$p\frac{D_1}{2}d\theta\,dx = 2\sigma_\theta\,dxT\sin(d\theta/2)$$

Where

 $\sigma_{_{\theta}}~$ Is the hoop stress in the drawn tube

p Is the pressure of the polymer

Which for small angle ($d\theta$), becomes

$$\sigma_{\theta} = pD_1 / 2T \tag{2-17}$$

For plastic deformation to commence, the yield criterion according to Tresca gives (assuming that the stress in the thickness direction for thin walled tube is equal to zero)

$$\sigma_x + \sigma_\theta = Y_o \tag{2-18}$$

Where Y_{o} is the initial yield stress of the tube material.

Let x_o be the distance from the entry to the point where plastic deformation commences. Now according to equation (2-16) and (2-17)

$$\sigma_{xo} = \tau_{c1} x_o / T$$

$$\sigma_{\theta o} = p_o D_1 / 2T$$

Where $\sigma_{xo}, \sigma_{\theta_o}, p_o$ are the axial stress, hoop stress, and the pressure respectively, at $x = x_o$

Since the pressure gradient (p'_1) is constant then

$$\frac{p_o}{x_o} = \frac{p_m}{L_1}$$

Equation (2-18) thus becomes

$$\tau_{c1} x_o / T + p_m x_o D_1 / 2TL_1 = Y_o$$

So that

$$x_{o} = Y_{o}T \left(\tau_{c1} + \frac{p_{m}D_{1}}{2L_{1}} \right)$$
(2-19)

Then σ_{x_0} can be written as

$$\sigma_{xo} = \tau_{c1} Y_o / \left(\tau_{c1} + \frac{p_m D_1}{2L_1} \right)$$
(2-20)

Also the pressure at $x = x_o$ becomes

$$p_o = p_m Y_o \left(\frac{\tau_{c1} L_1}{T} + \frac{p_m D_1}{2T} \right)$$

Simplifying

$$p_o = 2T p_m Y_o / (2\tau_{c1}L_1 + p_m D_1)$$
(2-21)

2.3.2 work rate required for shearing the polymer melt

Some of the work rate spent on the drawing system is due to the shearing of the polymer melt in the DRU. Drawing the tube is effected by a force, which is a combination of shear force and pressure force (in the deformation zone), thus the work rate is the product of the drag force at a point on the drawn tube by the velocity of the tube at that point.

2.3.2.1 Work rate required for shearing the polymer melt in zones I, and III

Consider an element of the tube in zone I as shown in Fig. (2.4.a) Equilibrium of forces in the x direction

$$F+dF-F-\tau_{c1}(\pi D_1 dx)=0$$

Noting that there is no pressure force acting in the x direction. Simplifying the above equation

$$dF = \pi D_1 \tau_{c1} dx \tag{2-22}$$

Now the work rate that spent by this element on the deformation of the polymer melt, is

$$d\dot{w}_{p1} = v_1 dF \tag{2-23}$$

Substituting for dF from equation (2-22) into equation (2-23)

$$d\dot{w}_{p1} = \pi v_1 D_1 \tau_{c1} dx \tag{2-24}$$

All variables in the right hand side of equation (2-24) are constant for $0 < x < x_o$, the total work rate for zone I is therefore

$$\dot{w}_{p1} = \int_{0}^{x_0} \pi v_1 D_1 \tau_{c1} dx$$

Integrating gives

$$\dot{w}_{p1} = \pi v_1 D_1 \tau_{c1} x_o \tag{2-25}$$

Equation (2-25) gives the work rate spent on polymer melt for zone I. Similarly the work spent on the polymer melt for zone III is.

$$\dot{w}_{p3} = \pi v_{2} D_{2} \tau_{c2}^{*} L_{2}$$
(2-26)

Where

- v_2 Is the velocity of the deformed tube in the second part of the unit.
- D_2 Is the diameter of the deformed tube

 τ_{c2}^{*} Is the shear stress on the surface of the deformed tube in the second part of the unit, which is evaluated later.

2.3.2.2 Work rate required for shearing the polymer melt in zones II

Consider the element of the tube as shown in Fig. (2.4.b), the equilibrium of forces in the x direction gives

$$F + dF - F - \tau_c (\pi D dx) - P(\pi D dD) = 0$$

Where

D Is the diameter of the tube element at the deformation zone

 τ_{c} Is the shear stress at the surface of the element of the tube in the

deformation zone

Simplifying the above equation

$$dF = \pi D(\tau_c dx + P dD) \tag{2-27}$$

Now

$$\tan \alpha = \frac{dD}{2dx}$$

Simplifying

$$dD = 2\tan \alpha dx$$

Substitute for dD in equation (2-27) and simplifying

$$dF = \pi D(\tau_c + 2p \tan \alpha) dx \qquad (2-28)$$

As in zone III, and I the work rate spent by the polymer melt due to motion of the element in zone II is

$$d\dot{w}_{n2} = vdF$$

Where v is the velocity of the element in the x direction in the deformation zone

Substitute for dF from equation (2-28) gives

$$d\dot{w}_{p2} = \pi D v (\tau_c + 2P \tan \alpha) dx \qquad (2-29)$$

Applying the continuity equation for the tube material, and assuming that the thickness (T) is constant for the deformed tube

$$\pi D_1 v_1 T = \pi D v T = \pi D_2 v_2 T$$

Simplifying

$$D_1 v_1 = D v = D_2 v_2 \tag{2-30}$$

Substituting equation (2-30) in the work rate equation, (2-29) then the work rate spent on shearing the polymer melt in zone II is

$$\dot{w}_{p2} = \int_{x_0}^{L_1} \pi D_1 v_1 (\tau_c + 2P \tan \alpha) dx$$
(2-31)

The values of (τ_c, p, α) will found later (see section 2.3.4.1).

Noting that the work rate spent on the shearing of the polymer melt due to radial deformation of the tube is neglected because the radial velocity of deformation of the tube is very small as compared with the axial velocity of drawing.
2.3.3 Work required for deformation of the tube material

The work rate required due to plastic deformation of the drawn tube can be found by taking an element of the tube in the deformation zone as shown in Fig. (2.5).

$$d\dot{w}_m = V\overline{\sigma} \frac{d\overline{\varepsilon}}{dt}$$
(2-32)

Where

- V Volume of the element
- $\overline{\sigma}$ Representative stress
- $\overline{\varepsilon}$ Representative strain

$$V = \pi DT dx \tag{2-33}$$

T is the thickness of the tube wall

The assumed mechanical characteristics of the tube material is

$$\overline{\sigma} = \overline{\sigma}_o + k_o \overline{\varepsilon}^n \tag{2-34}$$

Where

 $\overline{\sigma}_{o}$ The representative stress at the point when yielding commences

 k_{o} , *n* Constants relates to the tube material properties

The representative stress at the commencement of yielding is

$$\overline{\sigma_{o}} = \sqrt{\frac{1}{2} \left[\left(\sigma_{\theta o} - \sigma_{xo} \right)^{2} + \left(\sigma_{xo} - \sigma_{To} \right)^{2} + \left(\sigma_{To} - \sigma_{\theta o} \right)^{2} \right]}$$
(2-35)

Where

 $\sigma_{\theta o}$ Is the hoop stress at the point when yielding commence σ_{xo} Is the longitudinal stress at the point when yielding commence σ_{To} Is the thickness stress at the point when yielding commence For thin walled tube assume $\sigma_{To} = 0$ Now according to Tresca yield criteria

$$\sigma_{\theta o} + \sigma_{xo} = Y_o$$

Simplifying

$$\sigma_{\theta o} = Y_o - \sigma_{xo} \tag{2-36}$$

Substituting equation (3-36) in equation (2-35) gives

$$\overline{\sigma_{o}} = \sqrt{\frac{1}{2}} \left\{ \left[\sigma_{xo} - (Y_{o} - \sigma_{xo}) \right]^{2} + (Y_{o} - \sigma_{xo})^{2} + \sigma_{xo}^{2} \right\}$$

After simplification

$$\overline{\sigma}_{o} = \sqrt{Y_{o}^{2} - 3Y_{o}\sigma_{xo} + 3\sigma_{xo}^{2}}$$
(2-37)

 σ_{xo} can be evaluated from equation (2-20)

The representative strain is evaluated as follows

$$d\overline{\varepsilon} = \sqrt{\frac{2}{9}} \left[\left(d\varepsilon_x - d\varepsilon_\theta \right)^2 + \left(d\varepsilon_\theta - d\varepsilon_T \right)^2 + \left(d\varepsilon_T - d\varepsilon_x \right)^2 \right]$$
(2-38)

From the constancy of volume

$$d\varepsilon_x + d\varepsilon_\theta + d\varepsilon_T = 0 \tag{2-39}$$

Assume thickness to remain constant therefore $d\varepsilon_T = 0$ Hence

$$d\varepsilon_{\theta} = -d\varepsilon_x \tag{2-40}$$

Substituting equation (2-40) into equation (2-38) and simplifying

$$d\overline{\varepsilon} = \pm \frac{2}{\sqrt{3}} d\varepsilon_{\theta} = \mp \frac{2}{\sqrt{3}} d\varepsilon_{x}$$
(2-41)

Now

$$d\overline{\varepsilon} = \pm \frac{2}{\sqrt{3}} d\varepsilon_{\theta} = \pm \frac{2}{\sqrt{3}} \frac{dD}{D}$$

After integrating the above equation, the representative strain at a diameter D in the deformation zone becomes

$$\overline{\varepsilon} = \pm \frac{2}{\sqrt{3}} \ln \frac{D}{D_1}$$
(2-42)

Substitute for $\overline{\epsilon}$ in equation (2-34) to obtain

$$\overline{\sigma} = \overline{\sigma}_{o} + k_{o} \left(\frac{2}{\sqrt{3}} \ln \frac{D}{D_{1}}\right)^{n}$$
(2-43)

Now the work rate equation (2-32) for the element of the deformation zone can now written as

$$d\dot{w}_m = \frac{2}{\sqrt{3}} V \overline{\sigma} \frac{d\varepsilon_x}{dt}$$
(2-44)

For the moving element at a point in the deformation zone

$$d\varepsilon_x = \frac{d(dx)}{dx} \tag{2-45}$$

Substituting for the axial strain, volume of the element, and the representative stress in equation (2-44) gives

$$d\dot{w}_m = \frac{2}{\sqrt{3}}\pi DT \left[\overline{\sigma}_o + k_o \left(\frac{2}{\sqrt{3}}\ln\frac{D}{D_1}\right)^n\right] \frac{d(dx)}{dx\,dt}dx$$

Let

$$\frac{d(dx)}{dt} = dv$$

After simplification, the work rate equation for the element becomes

$$d\dot{w}_{m} = \frac{2}{\sqrt{3}}\pi DT \left[\overline{\sigma}_{o} + k_{o} \left(\frac{2}{3}\ln\frac{D}{D_{1}}\right)^{n}\right] dv \qquad (2-46)$$

From continuity of tube material [equation (2-30)]

$$v_1 D_1 = v D$$

Equation (2-46) can be written as

$$d\dot{w}_m = \frac{2}{\sqrt{3}}\pi v_1 D_1 T \left[\sigma_0 + k_0 \left(\frac{2}{3}\ln\frac{v_1}{v}\right)^n\right] \frac{dv}{v}$$

Re writing

$$\dot{w}_{m} = \int_{v_{1}}^{v_{2}} \frac{2}{\sqrt{3}} \pi v_{1} D_{1} T \left[\sigma_{o} + k_{o} \left(\frac{2}{\sqrt{3}} \ln \frac{v_{1}}{v} \right)^{n} \right] \frac{dv}{v}$$

By integrating and substituting the continuity equation of the tube material

$$\dot{w}_{m} = \frac{2}{\sqrt{3}}\pi v_{1} D_{1} T \left[\overline{\sigma}_{o} \ln \frac{D_{1}}{D_{2}} + \left(\frac{2}{\sqrt{3}}\right)^{n} \frac{k_{o}}{n+1} \left(\ln \frac{D_{1}}{D_{2}} \right)^{n+1} \right]$$
(2-47)

2.3.4 procedure of solution

The procedure of solution of the drawing process is explained in an algorithm diagram in appendix B.

2.3.4.1 Evaluation of the tube reduction

The solution applied to the process of the deformation needs firstly an assumed deformation profile equation, which in this study is assumed of quadratic degree, as follows

$$D = D_1 - Z(x - x_o)^2$$
(2-48)

Where Z is a constant

Then the thickness of the polymer melt layer in the deformation zone is

$$h = d_1 / 2 - D / 2 \tag{2-49}$$

Now from the continuity of the tube material equation (2-30), the axial velocity of a point at the tube in the deformation zone becomes

$$v = v_1 D_1 / D \tag{2-50}$$

For the second part of the unit, the diameter of the deformed tube, the polymer melt thickness and the velocity of the drawn tube (drawing speed) are obtained according to the equations (2-48), (2-49), and (2-50) respectively.

$$D_{2} = D_{1} - Z(L_{1} - x_{o})^{2}$$
(2-51)

$$h_2^* = d_1 / 2 - D_2 / 2 \tag{2-52}$$

$$v_2 = v_1 D_1 / D_2 \tag{2-53}$$

Now substitution of v_2 , and h_2^* instead of v_1 and h_2 in equations (2-12), and (2-13) gives the shear stress on the deformed tube (τ_{c2}^*) and the flow rate of the polymer melt in the second part of the unit

$$\tau_{c2}^{*} = \left[\frac{-\eta v_{2}}{2kh_{2}^{*}} + \left(\frac{\eta^{2}v_{2}^{2}}{4k^{2}h_{2}^{*2}} + \frac{1}{27}\left(\frac{1}{k} + \frac{1}{4}p_{2}^{\prime 2}h_{2}^{*2}\right)^{3}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} + \left[\frac{-\eta v_{2}}{2kh_{2}^{*}} - \left(\frac{\eta^{2}v_{2}^{2}}{4k^{2}h_{2}^{*2}} + \frac{1}{27}\left(\frac{1}{k} + \frac{1}{4}p_{2}^{\prime 2}h_{2}^{*2}\right)^{3}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} + \frac{1}{2}p_{2}^{\prime}h_{2}^{*} \qquad (2-54)$$

$$Q_{2}^{*} = -\frac{p_{2}^{\prime}h_{2}^{*3}}{6\eta} + \frac{h_{2}^{*2}\tau_{c2}^{*}}{2\eta} + \frac{k}{\eta} \left(-\frac{1}{20}p_{2}^{\prime3}h_{2}^{*5} + \frac{1}{2}h_{2}^{*2}\tau_{c2}^{*3} + \frac{1}{2}h_{2}^{*3}\tau_{c2}^{*3} + \frac{1}{2}$$

Then the value of the pressure at the step (p_m^*) can be iterated (note that $p'_2 = p_m^* / L_1$) until $Q_2^* = Q_1$.

To calculate the required work rate due to shearing the polymer melt (for the assumed profile of deformation), for zone I the shear stress (τ_{c1}) is the same as in the analysis of no tube deformation, and hence it can be calculated from equation (2-25), whereas, the work rate in zone III can be calculated from equation (2-26). The work rate for shearing the polymer melt in zone II can be found by integrating equation (2-31) numerically, but before done so, the pressure distribution ,the pressure gradient, and the tangent of the profile should be known.

From results obtained by Al-Rawi [15] the pressure distribution can assumed linear along the deformation zone (zone II), thus it becomes

$$p = (p_m^* - p_o)(x - x_o) / (L_1 - x_o)$$
(2-56)

And the pressure gradient in the deformation zone is

$$p' = (p_m^* - p_o) / (L_1 - x_o)$$
(2-57)

Now the shear stress at the surface of an element of the tube in the deformation zone is

$$\tau_{C} = \left[\frac{-\eta v}{2kh} + \left(\frac{\eta^{2} v^{2}}{4k^{2} h^{2}} + \frac{1}{27} \left(\frac{1}{k} + \frac{1}{4} \left((p_{m}^{*} - p_{o})/(L_{1} - x_{o})\right)^{2} h^{2}\right)^{3}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$$\left[\frac{-\eta v}{2kh} - \left(\frac{\eta^{2} v^{2}}{4k^{2} h^{2}} + \frac{1}{27} \left(\frac{1}{k} + \frac{1}{4} \left((p_{m}^{*} - p_{o})/(L_{1} - x_{o})\right)^{2} h^{2}\right)^{3}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} (2-58)$$

$$\pm \frac{1}{2} (p_{m}^{*} - p_{o})/(L_{1} - x_{o})h$$

The above sign is (+) ve when $p_m^* < p_o$ and (-) when $p_m^* > p_o$ The tangent of the profile of the tube is

$$\tan \alpha = \frac{dD}{2dx} \tag{2-59}$$

From equation (2-48)

$$\frac{dD}{dx} = -Z(x - x_o)$$

Then

$$\tan \alpha = -Z(x - x_o) \tag{2-60}$$

The total work rate spent on the system for the assumed profile of deformation is the summations of the work rate for shearing the polymer melt, in the three zones and the work rate of deformation of the tube material

$$\dot{w} = \dot{w}_{p1} + \dot{w}_{p2} + \dot{w}_{p3} + \dot{w}_m \tag{2-61}$$

According to the principle of minimum work rate there is one assumed profile of deformation that gives minimum work rate and it is the nearest to the true deformation profile.

To find this profile the value of (Z) in equation (2-48) is iterated until equation (2-61) gives minimum work rate.

The diameter of the tube after deformation (D_2) can be evaluated using equation (2-51) for this value of Z.

2.3.4.2 Evaluation of the drawing stress and the coating thickness

The drawing stress is the axial stress in the drawn tube at the exit of the unit, it is

$$\sigma_{xe} = F_e / A_e$$

Where

 F_{e} is the drag force

 A_{a} is the cross sectional area of the drawn tube at the exit of the unit

The drawing force is the total drag applied on the tube for the three zones of the unit

For zone I, the drag force is obtained by integration of equation (2-22)

$$F_1 = \int_0^{x_0} \pi D_1 \tau_{c1} dx$$

Since (τ_{c1}) is independent variable of x then after integration, the drag force for zone I is

$$F_1 = \pi D_1 \tau_{c1} x_0 \tag{2-62}$$

In the same way the drag force (F_2) for zone II can be found from equation (2-28)

$$F_{2} = \int_{x_{0}}^{L_{1}} \pi D(\tau_{c} + 2p \tan \alpha) dx \qquad (2-63)$$

Substituting for $D, \tau_c, p, and \tan \alpha$ from equations (2-48), (2-58), (2-56), and (2-60) respectively in equation (2-63) then integrate numerically to obtain F_2 . As in zone I the drag force in zone III is

$$F_{3} = \int_{0}^{L_{2}} \pi D_{2} \tau_{c2}^{*} dx$$

 τ_{c2}^{*} in zone III, is independent of x

$$F_3 = \pi D_2 \tau_{c2}^* L_2 \tag{2-64}$$

Now the total drag force is

$$F_e = F_1 + F_2 + F_3 \tag{2-65}$$

Hence the drawing stress becomes

$$\sigma_{xe} = \frac{F_1 + F_2 + F_3}{A_e}$$

Where

$$A_e = \pi D_2 T$$

Hence

$$\sigma_{xe} = \frac{F_1 + F_2 + F_3}{\pi D_2 T}$$
(2-66)

Other important variable in the die less is the coating thickness, which is the thickness of the polymer melt after solidification on the surface of the drawing tube, it can be evaluated from equation (2-52).



Figure 2.1 the DRU unit and the tube with the assumption of no tube deformation



Figure 2.2 showing the three zones of the tube in the DRU when tube deformation occurs



Figure 2.3 show the stresses acting on a small element of the tube in zone I



a



b

Figure 2.4 the axial equilibrium of forces for an element of the tube **a**-in zone I **b**-in zone II



Figure 2.5 isometric element of the tube in zone II



Figure 3.1 the variation of the pressure at the step versus the drawing speed for the assumption of no tube deformation



Figure 3.2 the variation of the distance from the entry to the point where deformation commences versus the drawing speed



Figure 3.3 the variation of the percentage reduction in diameter versus the drawing speed



Figure 3.4 the variation of the percentage reduction in diameter versus the drawing speed for different values of length ratio (L_1/L_2)



Figure 3.5 the variation of percentage reduction in diameter versus the drawing speed for different values of gap ratio (h_1/h_2)



Figure 3.6 the variation of the percentage reduction in diameter versus the drawing speed for different values of shear stress constant (k)



Figure 3.7 the variation of the coating thickness versus the drawing speed for different values of the length ratio (L_1/L_2)



Figure 3.8 the variation of coating thickness versus the drawing speed for different values of the gap ratio (h_1/h_2)



Figure 3.9 the variation of coating thickness versus the drawing speed for different values of shear stress constant



Figure 3.10 the variation of the drawing stress versus the drawing speed



Figure 3.11 the variation of drawing stress versus the drawing speed for different values of length ratio (L_1/L_2)



Figure 3.12 the variation of the drawing stress versus the drawing speed for different values of gap ratio (h_1/h_2)



Figure 3.13 the variation of the drawing stress versus the drawing speed for different values of shear stress constant (k)

CHAPTER FOUR CONCLUTIONS AND RECOMMENDATIONS FOR FUTURE WORK

4.1 Conclusions

A theoretical analysis of die-less tube sinking of copper tubes based on non-Newtonian behavior of the polymer melt using the work rate method is presented in this study. The main conclusions can be summarized as follows:

- 1. The nearest assumed profile to the true profile is of a quadratic degree, and the value of Z in the equation representing the profile is nearly the same for different value of the drawing speed.
- 2. The reduction ratio in the tube diameter depends on where yield of the tube commences (i.e. on x_0) which in turn is affected by the drawing speed.
- 3. For a given DRU and shear stress constant of polymer, increasing the drawing speed raises slightly the reduction ratio in the tube diameter, the coating thickness, and drawing stress.
- 4. The dimensions of the DRU have a large effect on the various process parameters. Increasing the gap ratio, the length ratio, or both of them increases the reduction in tube diameter, coating thickness, and the drawing stress.
- 5. Increasing the shear stress constant of the polymer causes the reduction ratio to decrease as well as the coating thickness and the drawing stress.

4.2 Recommendations for future work

- 1. An investigation into the tube reduction with tapered die-less unit using the work rate method can be made.
- 2. Analyzing the die-less wire drawing process using the work rate method.
- 3. Applying the work rate method for die-less drawing using a strain-rate sensitive material as the pressure medium.
- 4. Analysis of die-less (tube or wire) drawing of strain-rate sensitive material using the work rate method can be presented.
- 5. Analyzing a system containing a number of die-less reduction units for tube or wire drawing.
- 6. Studying the effects of back tension in the die-less drawing process.
- 7. Study the die-less drawing process using polymer melt and taking into account the effect the pressure and shear rate on the viscosity of the polymer melt at every point in the unit.
- 8. Evaluate a solution for the die-less drawing process using finite element method

CHAPTER THREE RESULTS AND DISCUSSION

The effects of the different process parameters such as the drawing speed, the DRU dimensions, and type of the polymer on the reduction ratio of the tube diameter, coating thickness and the drawing stress are discussed in this chapter.

The effect of the assumed profile on the work rate and on other process parameters will also be discussed. The standard values of the process parameters is found in appendix B.

3.1 Drawing with no tube deformation

As mentioned in the theoretical analysis the importance of the analysis of no tube deformation is to define and fined the different process parameters of zone I for a particular value of the drawing speed, DRU dimensions, and the type of the polymer, it was found that by increasing the value of the iterated pressure at the step (P_m), increase the pressure gradient in the first part of the unit. Hence according to equation (2-7) the flow rate in the first part of the unit (Q_1) decreases. For the second part of the unit, increasing (P_m) causes an increase in the pressure gradient and hence according to equation (2-13) the flow rate increases too. Further increasing in P_m makes $Q_1=Q_2$ at a specific value of P_m . evaluation of P_m leads to evaluating the pressure distribution for the whole unit

since the pressure gradient is constant $(p'_1 = \frac{p_m}{L_1}, p'_2 = \frac{p_m}{L_1})$. For known pressure

distribution in the unit, (x_0) the location where the deformation commences can also be found.

Increasing the drawing speed of the tube raises the velocity gradient in the polymer melt, which in turns increases the shear stress in the polymer, causing hydrodynamic pressure to increase also. The value of x_0 is affected by changing the hydrodynamic pressure and the shear stress on the surface of the tube, [see equation (2-19)]. The effect of changing the drawing speed on P_m and x_0 is shown in figures (3.1) and (3.2) respectively.

On the other hand, for a fixed value of the drawing speed, increasing the gap ratio or / and the length ratio leads to an increase in the pressure in the unit and hence p_m to satisfy the continuity equation of the polymer melt (Q₁=Q₂).

Increasing the shear stress constant causes the viscosity of the polymer melt to decrease and hence the shear stress in the polymer decreases. This in turn causes the hydrodynamic pressure to decrease which with the decrease in the shear stress, the value of x_0 increases according to equation (2-19).

3-2 Effect of the assumed profile

To obtain the true profile of deformation is very difficult because the profile is not a straight line (from early studies). The profile should be a continuous function with a point of inflection somewhere in the middle of profile, the position of which is difficult to estimate.

It is clear that the profile of deformation affects the amount of the work rate which is required to undergo deformation (note that the work rate spent in zone I is not affected by the type of the profile). It is also known that the deformation requires minimum work rate and accordingly, the nearest assumed profile to the true profile gives minimum work rate.

The profile determines the gap thickness between the tube and the unit for zones II, and III. Also the profile determines the axial velocity. The gap thickness and the axial velocity affect the shear stress on the surface of the tube. For new gap thickness in the second part of the unit (h_2^*) the continuity principle for the polymer melt must be satisfied. As in the process of no tube deformation the pressure at the step must be iterated until $Q_1=Q_2^*$.

It is found that the pressure at the step decreases as the gap thickness (h_2^*) increases. Since the pressure distribution is assumed changes linearly along the unit in zone II, the pressure distribution can therefore evaluated.

As mentioned above the pressure at the step depends on the assumed profile, this means that the pressure gradient depends on the profile too, hence according to equations (2-58), and (2-54), the shear stress at the surface of the tube in zones II, and III are effected.

The assumed profile affects the shear stress on the surface of the tube in zone II and III, the tangent of the angle of deformation, the axial velocity, and the pressure distribution in the unit for tube deformation which in turns affect the work rate spent on the system (due to shearing the polymer melt) according to equations (2-26) and (2-31).

The work rate spent on deformation of the tube increases when the reduction in the tube diameter increases [equation (2-47)] and does not depend on the type of the profile.

In this study the assumed profile is assumed similar to the profiles obtained in early studies of quadratic degree with a constant (Z). Now it is found that in the solution, decreasing the value Z decreases the work rate until

minimum value is reached and then increases again. Other assumed profiles (3rd or 4th degree polynomials) where found to give higher work rates which keep on increasing irrespective of the constants used in the equations defining the profiles.

3.3 Reduction in tube diameter and coating thickness

The amount of reduction in tube diameter during the drawing process is the most important parameter, which is a measure of the process success. In the conventional tube sinking, the diameter of the drawn tube takes the bore diameter of the die, which is already known. But the diameter reduction in dieless tube sinking is not so, since the diameter of the drawn tube depends on the hydrodynamic pressure and the shear stress on the surface of the tube. These two parameters can be simply controlled by the value of the drawing speed. Increasing the drawing speed increases both of the hydrodynamic and the shear stress on the surface of the tube.

The relation of between the drawing speed and the percentage reduction in diameter is shown Fig. (3.3) where the percentage reduction in diameter is increases slightly as the drawing speed increases due to the change in hydrodynamic pressure and the shear stress. The value of Z which gives minimum work is nearly the same for different values of the drawing speed, thus the little change in reduction ratio versus the drawing speed is not affected by the value of Z, but it is affected by changing the value of x_o [see Fig. (3.2)].

The trend of variation of the reduction in diameter is similar to the trend obtained by M. I. Paranhwar et al. [14] and the result predicted by Al-Rwai [15] when using non-strain rate sensitive material.

Another parameter, which affects the tube reduction, is the dimension of the DRU. It was seen when considering the process with no tube deformation, the viscosity, the distance at which yield commences, and the shear stress on the surface of the tube for zone I where affected by the hydrodynamic pressure which raise as the dimensions of the DRU change, increasing the length ratio (L_1/L_2) or/and the gap ratio (h_1/h_2) raises the hydrodynamic pressure. Also when considering tube deformation, increasing the length ratio (L_1/L_2) or/and the gap ratio (h_1/h_2) raises the hydrodynamic pressure in the unit and increases the reduction in tube diameter. This effect is shown in Figs. (3.4) and (3.5), this trend is the same as these obtained by M. I. Paranhwar et al. [14] and by Al-Rwai [15].

Another effective parameter affecting the tube deformation is the shear stress constant, which depends on the characteristics of the polymer, which is shown in Fig. (3.6). It can be seen that as the shear stress constant increases the reduction ratio decreases. Increasing the shear stress constant causes the viscosity of the polymer melt to decrease; hence the shear stress decreases too. The decrease in shear stress at the polymer melt causes the hydrodynamic pressure in the unit to decrease. Thus decreasing the reduction in diameter.

It is clear that the reduction in diameter of the tube and coating thickness are complementary to each other in making-up the overall of the coated tube. Figs (3.7), (3.8), and (3.9) show the effect of the different parameters in evaluating the coating thickness.
3.4 Drawing stress

Drawing stress is an important parameter, which is considered in the drawing of tubes, which has to be less than the yield stress at the tube. The drawing stress is the summation of the total drag force on the surface of the tube divided by the cross sectional area of the deformed tube. Increasing the drawing speed raises the hydrodynamic pressure and the shear stress on the surface of the tube, thus causing a slight increase in the drawing stress as shown in Fig. (3.10). The trend of this curve reasonable the trend of the results obtained by M. I. Paranhwar et. al [14].

The effect of the DRU dimensions is as mentioned previously, i.e. when increasing the gap ratio or/and the length ratio, the hydrodynamic pressure increases, thus causing the drawing force to increase which in turn cause the drawing stress to increase, as shown in Figs. (3.11), and (3.12). The figures obtained are similar to these obtained by M. I. Paranhwar et. al [14].

As in the previous section the effect of increasing the shear stress constant leads to decreasing the hydrodynamic pressure, hence the drawing stress decreases, as shown in Fig. (3.13).

NOMENCLATURE

SYMBOLE	DEFFENTION	UNITS
a, b	Constants depending on polymer characteristics	
Aa	Cross sectional area of the deformed tube at	m^2
	the exit of the unit	
D	The diameter of the tube in the deformation	m
	zone	
D ₁	The diameter of the tube before deformation	m
d ₁	Bore diameter of the first part of the unit	m
D ₂	The diameter of the tube after deformation	m
d ₂	Bore diameter of the second part of the unit	m
DRU	Die-less reduction unit	
F ₁	Drag force in zone I	Ν
F ₂	Drag force in zone II	Ν
F ₃	Drag force in zone III	Ν
F _e	Drawing force	Ν
h	Gap thickness in the deformation zone	m
h ₁	Gap thickness between the un deformed tube	m
	and the first part of the unit	
h ₂	Gap thickness between the un deformed tube	m
	and the second part of the unit	
h_2^*	Gap thickness between the deformed tube and	m
	the second part of the unit	
k	Non Newtonian factor (shear stress constant)	m^4/N^2
ko	Constant relates the tube material properties	N/m^2
L ₁	Length of the first part of the unit	m
L ₂	Length of the second part of the unit	m
n	Constant relates the tube material properties	
р	Pressure of the polymer melt	N/m^2
p_m	Pressure at the step with no tube deformation	N/m ²
p_m^*	Pressure at the step with tube deformation	N/m ²
p_o	Pressure of the polymer at $x=x_0$	N/m^2
Q_1	Flow rate of the polymer melt in the first part	m ² /sec
	of the unit	

SYMBOLE	DEFFENTION	UNITS
Q ₂	Flow rate of the polymer melt in the second part of the unit with no tube deformation	m ² /sec
Q2*	Flow rate of the polymer melt in the second part of the unit with tube deformation	m ² /sec
Т	Thickness of the tube	m
u	Velocity of the polymer melt	m/sec
u ₁	Velocity of the polymer melt in the first part of the unit	m/sec
V	Velocity of the tube in the deformation zone	m/sec
V	Volume of an element of the tube in the deformation zone	m ³
v ₁	Velocity of the tube before deformation	m/sec
V2	Velocity of the tube after deformation	m/sec
X _o	Distance from the entry to the point were deformation commences	m
Y _o	Initial yield stress of the tube material	N/m^2
Z	Constant in the equation of the assumed profile	
Q1	Flow rate of the polymer melt in the first part of the unit	m ² /sec
Q ₂	Flow rate of the polymer melt in the second part of the unit with no tube deformation	m ² /sec
Q ₂ *	Flow rate of the polymer melt in the second part of the unit with tube deformation	m ² /sec
Т	Thickness of the tube	m
u	Velocity of the polymer melt	m/sec
τ	Shear stress of the polymer melt	N/m ²
η	Viscosity of the polymer melt	$N.sec/m^2$
τ_1	Shear stress of the polymer melt in the first part of the unit	N/m ²
τ_{c1}	Shear stress on the surface of the tube in the first part of the unit	N/m^2
<i>p</i> '	Pressure gradient in the first part of the unit for the un deformed tube	N/m ³
τ2	Shear stress of the polymer melt in the second part of the unit	N/m ²
τ _{c2}	Shear stress on the surface of the un- deformed tube in the second part of the unit	N/m ²

SYMBOLE	DEFFENTION	UNITS
τ_c	Shear stress on the surface of the tube in the deformation zone	N/m ²
η₀	Initial viscosity of the polymer melt	N.sec/m ²
γ̈́	Apparent shear rate in the unit	1/sec
σ	Axial stress in the drawn tube	N/m ²
σ_{θ}	Hoop stress on the drawn tube	N/m ²
$\sigma_{_{ heta o}}$	Hoop stress on the drawn tube at $x=x_0$	N/m ²
\dot{w}_{p1}	Work rate for shearing the polymer in zone I	Nm/sec
\dot{w}_{p2}	Work rate for shearing the polymer in zone II	Nm/sec
\dot{w}_{p3}	Work rate for shearing the polymer in zone III	Nm/sec
$ au_{c2}^*$	Shear stress on the surface of the deformed tube in the second part of the unit	N/m^2
α	Angle of tube deformation	
\dot{W}_m	Work rate of deformation in the tube material	Nm/sec
σ	Representative stress	N/m ²
3	Representative strain	
$\overline{\sigma}_{o}$	Representative stress at $x=x_o$	N/m^2
σ _{το}	Thickness stress at x=x _o	N/m ²
σ _{xe}	Drawing stress	N/m ²
Ŵ	Total work rate spent on the system	Nm/sec
σ _{xo}	Axial stress in the drawn tube at $x=x_0$	N/m ²
p_2	Pressure gradient in the second part of the unit	N/m ³

REFERENCES

[1] John A. Schey "Introduction to manufacturing process" second edition, Mc Grawhill Book company, 1987.

[2] M.S.J.Hashmi, " Anote on the prospects of plasto-hydrodynamic die-less tube sinking", J.of Mech.Working Tech., Vol.11, PP. 237-242, 1985.

[3] W.L.Wilkinson, "*Non-Newtonian flow and heat transfer*", Proc. Inst. Mech. Engrs., Vol.186, 1972.

[4] Abdul-Wahab Mohammed Abdullah, "*Factors affecting the lubrication and the mechanics of drawing rod at high speeds*", Ph.D Thesis, University of Bradford, 1982.

[5] D.G. Christopherson and H. Naylor, "*Promotion of fluid lubrication in wire drawing*", Proc. Inst. Mech. Engrs. Vol. 169, pp 643 – 653, 1955.

[6] Susan M.Bloor, D.Dowson and B. Parsons," *An elasto-plasto-hydrodynamic lubrication analysis of the plane strain drawing process*", J.of Mech. Engng. Sci., Vol. 12, PP.178-190, 1970.

[7] P.J.Thompson and G.R.Symmons, "*A plasto – hydrodynamic analysis of the lubrication and coating of wire using a polymer melt during drawing*", Proc. Inst. Mech. Engrs. Vol. 191, PP 113 –123. 1977.

[8] R. Crampton, G. R. Symmons and M.S.J. Hashmi, "*A non – Newtonian plasto – hydrodynamic analysis of the lubrication and coating of wire using a polymer melt during drawing*", Int. Symposium on metal working lubrication, San Francisco, U.S.A, PP 107 – 115 ,August 1980.

[9] M.S.J. Hashmi, R.Crampton and G.R. Symmons, "*Effect of strain hardening and strain rate sensitivity of the wire using a polymer melt during drawing under non – Newtonian plasto – hydrodynamic lubrication conditions*", Inst. J. Mech. Tool Des. Res., Vol. 21, PP 71 – 86, 1981.

[10] M.S.J. Hashmi, G.R. Symmons and H. Parvinmehr," A Noval technique for wire drawing ", J. of Mech. Eng. Sci, vol. 24, pp 1 - 4, 1982.

[11] M.S.J. Hashmi and G.R. Symmons, "A numerical Solution for the plasto – hydrodynamic drawing for rigid non – linearly strain hardening continuum through a conical orifice", Proc. 2nd. Int. Conf. On numerical methods for non – linear problem, Spain. PP 1048 –1059, 1984.

[12] M.S.J.Hashmi, "Anote on the prospects of plasto-hydrodynamic die-less tube sinking", J.of Mech.Working Tech., Vol.11, PP. 237-242, 1985.

[13] H. Parvinmehr, G.R. Symmons and M.S.J. Hashmi, "A non – Newtonian plasto – hydrodynamic analysis of dieless wire drawing process using a stepped bore unit", Int. J. Mech. Sci. Vol. 29. PP 239 – 257, 1987.

[14] M.I.Panhwar, R.Crampton, M.S.J.Hashmi, "Analysis of the die-less tube sinking process based on non-Newtonian characteristic of fluid medium", J.of Material Processing Technology, Vol.21, PP.155-175, 1990.

[15] Bashar Hassan AL – Rawi, "Dieless tube and wire drawing of strain rate sensitive materials", M.Sc. Thesis, University of Saddam, 2001.

[16] A. S. M. Al-Samarrai, "*die-less wire drawing of mild steel*" M.Sc. Thesis, University of Saddam, 2002.

[17] W. Johnson, P. B. Mellor " engineering plasticity ", New York Van Nostrand Reinhold company, London, 1973.

Appendix A

This appendix shows the details used to obtain the final form of equations (2.5) and (2.12) which gave the shear stress on surface of the tube in the first and the second part of the unit [14].

a) Shear stress on the tube in the first section of the unit:

$$\tau_{c1}^{3} + \tau_{c1}^{2} \left(\frac{3}{2} p_{1}' h_{1}\right) + \tau_{c1} \left(\frac{1}{k} + {p_{1}'}^{2} h_{1}^{2}\right) + \left(\frac{\eta v}{k h_{1}} + \frac{p_{1}' h_{1}}{2k} + \frac{1}{4} {p_{1}'}^{3} h_{1}^{3}\right) = 0$$

Let

$$J_{1} = \frac{3}{2} p_{1}' h_{1} , M_{1} = \frac{1}{k} + p_{1}'^{2} h_{1}^{2}$$
$$N_{1} = \frac{\eta v}{k h_{1}} + \frac{p_{1}' h_{1}}{2k} + \frac{1}{4} p_{1}'^{3} h_{1}^{3}$$

Then equation (2.6) becomes

$$\tau_{c1}^3 + J_1 \tau_{c1}^2 + M_1 \tau_{c1} + N_1 = 0 \tag{A-1}$$

Also, let

$$\tau_{c1} = \phi_1 - J_1 / 3 \tag{A-2}$$

Substituting for τ_{c1} from equation (A-2) into equation (A-1) gives:

$$\left(\phi_1 - \frac{J_1}{3}\right)^3 + J_1(\phi_1 - \frac{J_1}{3})^2 + M_1(\phi_1 - \frac{J_1}{3}) + N_1 = 0$$

Which after substitution for J₁, M₁ and N₁ from above and simplification becomes:

$$\phi_1^3 + A_1 \phi_1 + B_1 = 0 \tag{A-3}$$

Where;

$$A_1 = \frac{1}{k} + \frac{1}{4} p_1'^2 h_1^2$$

and

$$B_1 = \frac{\eta v}{kh_1}$$

Equation (A-3) can be solved by applying the solution of cubic equation

$$\phi_1^3 + 3p\phi_1 + 2q = 0$$

Which can be written as

$$\phi_1 = [-q + (q^2 + p^3)^{1/2}]^{1/3} + [-q - (q^2 + p^3)^{1/2}]^{1/3}$$

Substituting the values of ϕ_1 , A_1 , B_1 and J_1 in the above equation gives:

$$\tau_{c1} = \left[\frac{-\eta v}{2kh_{1}} + \left(\frac{\eta^{2} v^{2}}{4k^{2}h_{1}^{2}} + \frac{1}{27}\left(\frac{1}{k} + \frac{1}{4}p_{1}^{\prime 2}h_{1}^{2}\right)^{3}\right)^{1/2}\right]^{1/3} + \left[\frac{-\eta v}{2kh_{1}} - \left(\frac{\eta^{2} v^{2}}{4k^{2}h_{1}^{2}} + \frac{1}{27}\left(\frac{1}{k} + \frac{1}{4}p_{1}^{\prime 2}h_{1}^{2}\right)^{3}\right)^{1/2}\right]^{1/3} - \frac{1}{2}p_{1}^{\prime}h_{1}$$
(A-4)

Equation (A-4) gives the shear stress on the surface of the tube for no tube deformation for known values of p'_1 .

b) Shear stress on the tube in the second part of the unit

By following the same way as in the first part the following equation is obtained:

$$\tau_{c2} = \left[\frac{-\eta v}{2kh_2} + \left(\frac{\eta^2 v^2}{4k^2 h_2^2} + \frac{1}{27}\left(\frac{1}{k} + \frac{1}{4}{p'_2}^2 h_2^2\right)^3\right)^{1/2}\right]^{1/3} + \left[\frac{-\eta v}{2kh_2} - \left(\frac{\eta^2 v^2}{4k^2 h_2^2} + \frac{1}{27}\left(\frac{1}{k} + \frac{1}{4}{p'_2}^2 h_2^2\right)^3\right)^{1/2}\right]^{1/3} + \frac{1}{2}{p'_{21}}h_2$$
(A-5)

Equation (A-5) gives the shear stress on the tube in the second part of the unit for a known value of p'_2 .

Appendix B

This appendix contain the algorithm diagrams which shows the steps leads to solve the governing equations in this study, and also contains the standard values of the polymer melt properties, the dimensions of the DRU, and the tube material properties.

(a) Algorithm diagram for the analysis with the assumption of no tube deformation



(b) Algorithm diagram for the analysis of the tube deformation



(c) The standard values of the parameters were assumed to be as follows:

(i)Data for the polymer (Polyethylene WVG23) $\eta_o = 100 \text{ N.s/m}^2$; k=5.6*10⁻¹¹ m⁴/N²; a=12*10⁴N/m²; b=4*10⁻¹¹ m²/N

(ii)Data for tube material $D_1=13.52*10^{-3} \text{ m}$; T=2.5*10⁻³ m; Y_o=180*10⁶ N/m²; k_o=400*10⁶ N/m²

(iii)Dimensions of the DRU $h_1=0.5*10^{-3} \text{ m}$; $h_2=0.1*10^{-3} \text{ m}$; $L_1=360*10^{-3} \text{ m}$; $L_2=15*10^{-3} \text{ m}$

الخلاصة

(DRU)) (

•

(plasto-hydrodynamic)

(Leading end)

.

•

.

•

[(non-Newtonian)

•

.

]

(h₁/h₂)

•

 (L_1/L_2)

•



صفر نیسان

1425 2004

.....

.....

.

.....

.....