Abstract

The Monte Carlo simulation method has been applied to generate the first data for bremsstrahlung buildup factor (BBUF) produced by the complete absorption of beta particles in different materials. The bremsstrahlung buildup factor was computed by the present method for different thicknesses of water, concrete, aluminum, tin and lead at the maximum bremsstrahlung energy of 2.2 MeV for $^{90}Sr/^{90}Y$ beta source. A computer program "BBF" was designed and improved to perform the calculations which solve the classical problem of a gamma ray reflection and transmission, the basic idea of this program is to create a series of life histories of the source particles using random sampling technique to sample the probability laws that describe the real particle's behaviour, and to trace out the particle's 'random walk' through the medium.

The effect of some considered parameters like the simulation parameters and physical parameters were studied. The bremsstrahlung buildup factor has been calculated for single layer shield. The simulation results indicate the following remarks:

- The bremsstrahlung buildup factor increases with the increase of thickness of the shield.
- The bremsstrahlung buildup factor for low atomic number material is lower than that for a high atomic number material at the same source energy.
- The relation between the bremsstrahlung buildup factor BBUF with the atomic number Z and thickness X of the shielding material is suggested to follow the semi-empirical formula:

$$BBUF = 1 + (a_1 Z + a_2) X^{(a_3 Z + a_4)}$$

where a_1, a_2, a_3 and a_4 are fitting parameters depends on the bremsstrahlung energy distribution. For $^{90}Sr/^{90}Y$ bremsstrahlung, these parameters are:

$a_1 = 0.0007$, $a_2 = 0.0022$, $a_3 = 0.0072$ and $a_4 = 0.4204$.

Acknowledgement

I would like to express my sincere thanks and deep gratitude to my supervisor **Prof. Dr. Mazin M. Elias** for suggesting the present project and for his guidance and advices throughout the research.

I am most grateful to **Dr. Laith A. Al-Ani** for his support and valuable discussions during the work.

Last but not least, I would like to record my deep affection and thanks to **my parents** for their moral support and patience throughout this work, and to **Mr. Ammar Al- Rawi** and **Mr. Muhammad Sahib** for their kind support and technical assistance.

Milad

Certification

I certify that this thesis entitled **"Simulation of Buildup Factor for Bremsstrahlung Produced by Complete Absorption of Beta Rays"** is prepared by **Milad Jathlan Ali Al-Ansari** under my supervision at the College of Science of Al-Nahrain University in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

Supervisor: Prof. Dr. Mazin M. Elias

Date: 27 / 3 / 2006

In view of the recommendations, we present this thesis for debate by the Examination Committee.

Dr. Ahmad K. Ahmad

Head of Physics Department

Date: 27/ 3 /2006

CHAPTER ONE

Theoretical Principles

1.1 Introduction

Natural radioactivity comes from those nuclei that are found in minerals in the earth's crust that (with a few exceptions) have atomic numbers Z greater than 82. These radioactive nuclides disintegrate spontaneously to produce fresh radioactive nuclides. They continue to do this until a stable nucleus is left. The disintegration of the atom takes the form of either the ejection from the atom of a positively charged alpha particle, or the ejection of a negatively/positively charged beta particle usually, but not necessarily accompanied by a gamma radiation [1].

One of the many ways in which different types of radiation are grouped together is in terms of ionizing and non ionizing radiation. Alpha and beta particles, neutrons, and gamma and x rays are ionizing radiations. That is, they interact with matter (all matter, including living tissues) by ionizing its atoms. An ion is an atom that has become charged by either gaining or losing an electron. The ionized atoms, if contained in biologically vital molecules such as DNA, result in damage that may be permanent or may be repaired by normal mechanisms [2].

Non ionizing radiation, refer to any type of radiation that does not carry enough energy to ionize the matter which pass through it. Visible light, near ultraviolet, infrared, and radio waves are all examples of non ionizing radiation. Gamma rays are more penetrating than either alpha or beta radiation, but less ionizing. It interacts with matter via three main processes, the photoelectric effect, Compton scattering, and the pair production.

In shielding calculations, the basic property of the attenuation of radiation is the exponential decrease of its intensity radiation as a homogeneous beam of radiation passes through the slab of the matter. This decrease is valid only when the beam of radiation is narrow. Therefore, the most widely used method of determination of the total effect on that radiation at the point of interest makes use of a parameter called "**Buildup Factor (B)**". In principle, the buildup factor concept is applicable to the gamma and neutron attenuation, but in practice, it is found to be much less successful when applied to the neutrons and consequently it's much less important concept in the context of neutron shielding calculation [3]. In the field of radiation shielding, the buildup factor can be assumed to refer to gamma radiation.

1.2 Beta particles

The beta particle is an identical with the electron. It has a rest mass of 9.1×10^{-28} g and a charge of 1.6×10^{-19} C. Thus, the principle distinction between an electron and a beta particle is the source or origin. An electron emitted from a nucleus is called a beta particle. The velocity of beta particle is dependent on its energy, which it is nearly equal to the velocity of light in vacuum. Classically, the energy of the beta particle of mass m and velocity ν is given by the expression

$$E_{ß} = m\nu^2 / 2$$

This expression is quite useful for small values of ν. It has been determined experimentally that beta particles have a continuous energy spectrum and that a part of the decay energy is carried away by neutrinos [4].

1.2.1 Beta decay

In nuclear physics, beta decay is a type of radioactive decay in which a beta particle (an electron or a positron) is emitted. In the case of electron emission, it is referred to as "beta minus" (ß$^-$), while in the case of a positron emission as "beta plus" (ß$^+$).

In ß$^-$ decay, the weak nuclear force converts a neutron into a proton while emitting an electron and an anti-neutrino:

$$n \rightarrow p + \beta^- + \overline{\upsilon} \tag{1.1}$$

In ß$^+$ decay, a proton is converted into a neutron, a positron and a neutrino:

$$p \rightarrow n + \beta^+ + \upsilon \tag{1.2}$$

Historically, the study of beta decay provided the first physical evidence of the neutrino. In 1911 Lise Meitner and Otto Hahn performed an experiment showed that the energies of electrons emitted by beta decay had a continuous rather than discrete spectrum. This was in apparent contribution to the law of conservation of energy, as it appeared that energy was lost in the beta decay process [5].

1.2.2 Interactions of beta particles

There are two main mechanisms of interaction that are important from the point of view of radiation protection:

i. The interactions with orbital electrons

The interaction between the electric field of a beta particle and the orbital electrons of the absorbing medium leads to inelastic collisions that generate electronic excitation and ionization. Because, the particles possessing like charges they repel each other. The Coulomb repulsion between a beta particle and one of the orbital electrons in the substance being traversed by a beta ray may be sufficient to expel the electron completely from its atom. The atom then becomes a positively charged ion. After this ionization process, the final energy of the electron E_f is less than the initial energy E_i by an amount equal to the sum of the binding energy of the ejected electron and its kinetic energy. That is [5],

$$E_f = E_i - (\phi + 1/2mv^2) \qquad (1.3)$$

where, ϕ is the binding energy of electron.

In collisions involving the expulsion of K, L or M electrons from an atom, characteristic x- rays are produced as electrons fall back into the ground state.

$$E_{photon} = h\nu = E_f - E_i \qquad (1.4)$$

ii. The interactions with nuclei (bremsstrahlung)

A collision of a beta particle with an atomic nucleus involves a Coulomb interaction in which the electron is sharply deflected in its path. When electrons are slowed down, decelerated, in the coulomb field of an atomic nucleus, a continuous electromagnetic radiation called bremsstrahlung is produced. Bremsstrahlung produced by the inelastic interaction of a beta particle with the nucleus. The total kinetic energy of the colliding systems is less by an amount equal to the energy radiated as a bremsstrahlung.

Bremsstrahlung is electromagnetic radiation similar to x-radiation. It is emitted by any charged particle as it decelerates in a series of collisions with atomic particles.

The phenomenon is described by

$$h\nu = E_i - E_f \tag{1.5}$$

where, hν is the energy of the photon of bremsstrahlung, E_i is the initial kinetic energy of the beta particle prior to the collision or deflection producing a final kinetic energy E_f of the electron

The percentage of bremsstrahlung production increases with the atomic number of the absorbing material. Hence, for shielding for protection against beta radiation, it is customary to use a material of low atomic number, such as plastics, and then gamma and bremsstrahlung radiation with heavy material (lead or other metal) [6].

The intensity of bremsstrahlung can be given as [7]

$$I \approx \frac{z^2 Z^2}{m^2} \tag{1.6}$$

where Z = the atomic number of the medium

z = the atomic number of the charged particle

m = mass of the charged particle

According to Eq. (1.6), bremsstrahlung intensity can be negligible for large ‘m’ particles (e.g., alpha particles).

1.3 Gamma Rays

Gamma rays are electromagnetic radiation and have no electric charge; they cannot be deflected by magnetic or electric fields, therefore they are similar in nature to light, but of a much higher energy. Gamma rays have greater penetration power and longer range in matter than the massive and charged alpha and beta particles of the same energy. Nevertheless, gamma rays are absorbed by matter, and the principle mechanism by which this type of radiation interacts with matter are discussed as the three main processes [8];

1.3.1 The photoelectric effect

In the photoelectric effect, the energy of photon is completely absorbed by an atom. Under such circumstance, the entire absorbed photon energy is transferred to an electron of the atom and the electron is released, consequently, the energy of the emitted electron is equal to the energy of the impinging photon less the binding energy of the electron. This is described by the photoelectric equation of the Einstein:

$$E_e = h\nu - \phi$$

where E_e is the energy of ejected electron, $h\nu$ is the energy of the incident photon and ϕ is the binding energy of the electron or the energy required to remove the electron from the atom. The ejected electron is identical to a beta particle and produces ionization (secondary ionization in this case) as it travels through matter.

A photon interacts with an entire atom, usually from the K, L shells of the atom. Momentum is conserved by the recoil of the residual atom.

When an electron from an inner atomic K or L shell is ejected, electrons from outer shells fall from their higher energy states to fill the resulting gap. These transitions in electron energy states require a release of energy by the atomic electrons which appear as soft (low- energy) x-rays [8].

Photoelectric effect is of greatest importance for the heavier atoms (larger Z) such as lead, especially at lower energies. The cross section (μ_{ph}) for each atom in the photoelectric absorption in the K shells represents 80 % from the sum reaction with other shells. It can be represented by the following relationship [9]

$$\mu_{ph} = \text{const.} \ \frac{Z^5}{(h\nu)^n} \tag{1.7}$$

where $n = 1$ if $h\nu \geq 0.5$ MeV

$n = 3$ if $h\nu < 0.5$ MeV

1.3.2 The Compton scattering

There is a second mechanism by which a photon (e.g., x-ray or gamma ray, etc) transfers its energy to an atomic orbital electron. In this interaction illustrated in Fig. (1.1), the photon, $E\gamma$, imparts only a fraction of its energy to the electron and in so doing is deflected with energy $E\gamma'$ at an angle θ, while the bombarded electron is ejected at an angle ϕ to the trajectory of the primary photon. This interaction is known as Compton scattering. However, the deflected photon continuous traveling through matter until it dissipates its entire kinetic energy by interacting with other electrons in a similar fashion or via other mechanisms of interaction with matter. The ejected electron, being identical in properties to a beta particle, loses its energy through the secondary ionization it causes according to mechanisms [8].

Photon losses some of its kinetic energy, then the conservation of energy gives

$$E_e = E\gamma - E\gamma' \qquad (1.8)$$

where,

E_e = energy of rejection electron

$E\gamma$ = energy of incident photon

$E\gamma'$ = energy of scattered photon

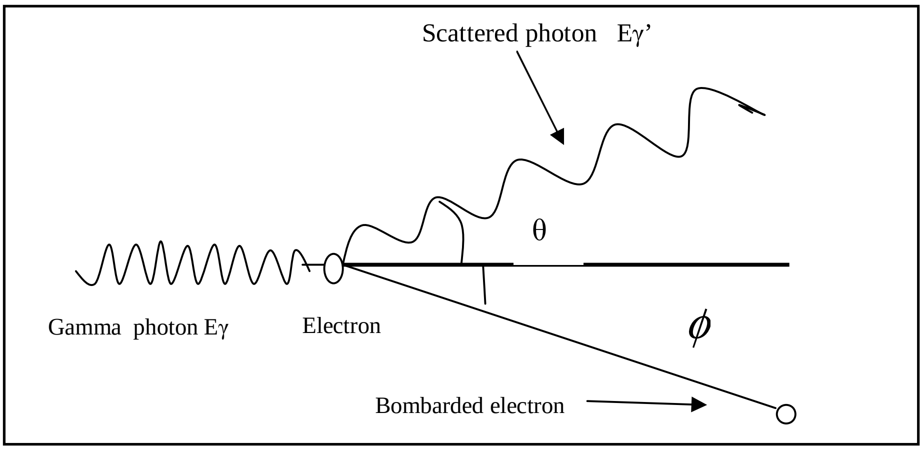

Fig. (1.1): The Compton scattering

Compton scattering is the predominant reaction for gamma photons with energies in the range (0.5-1 MeV) for elements of low and intermediate atomic number. The Compton cross section (μ_C) can be represented by the following relation [10]:

$$\mu_{C} = const \cdot \frac{Z}{E\gamma} \qquad (1.9)$$

1.3.3 Pair production

Pair production is a gamma-ray interaction in which the primary photon disappears as it transfers all its energy to the absorbing medium, in a single event that occurs generally in the vicinity of the nucleus of an atom of the absorber, the photon's energy is transformed into an electron-positron pair. The rest mass energy equivalent of each electron is 0.511 MeV. For this process to be energetically possible, the primary photons must have at least the minimum energy need to create the pair that is 1.022 MeV ($2m_oc^2$). Any excess over this threshold energy becomes kinetic energy shared by the electron and the positron. The electron and positron loss their kinetic energy via excitation and ionization, eventually, when the positron comes to rest, it interacts with a negatively charged electron, resulting dominantly in the formation of two oppositely directed 0.511 MeV annihilation photons that may escape from the absorber or may undergo Compton scattering, and finally photoelectric absorption within the absorber. Pair production, become an increasingly important mode of interaction with increasing energy of gamma above 1.022 MeV. The cross section of this process ($\mu_{P.P}$) can be represented by the following relation [11]:

$$\mu_{P.P} = \text{const.}\ Z^2 .\ E\gamma \tag{1.10}$$

1.4 Attenuation of Gamma Rays in a Medium

Attenuation is the removal of photons from a beam of x-or gamma rays as it pass through matter. Attenuation is caused by both absorption and scattering of the primary photons. At low photon energy the photoelectric effect dominates the attenuation process in soft tissue [12].

X-rays and gamma rays are attenuated exponentially; hence, they do not have a finite range. Depending on their energy and the type of absorber material, the ratio of intensity I of a beam of gamma rays that successfully passes through an absorber of thickness x to the incident intensity I_o of a collimated (narrow, parallel) beam of x-rays is given by the equation:

$$I / I_o = \exp(-\mu x) \tag{1.11}$$

where, μ is the linear attenuation coefficient.

The magnitude of the attenuation coefficient is a function of the atomic number of the absorbing medium and the energy of gamma rays; it's the sum of the attenuation coefficient for the three energy dependent gamma ray interaction processes just described. Thus we have:

$$\mu_{tot} = \mu_{photo} + \mu_{comp} + \mu_{pair} \tag{1.12}$$

Both the linear attenuation coefficient μ in reciprocal centimeters, and the mass attenuation coefficient μ/ρ (where ρ is the mass density of the absorber) in square centimeters per gram, are used in attenuation calculation [11].

It was found that, Eq. (1.11) represents the "exponential law" of the absorption observed when the attenuation measurement is applied on photons of a single energy under narrow beam conditions.

A narrow beam of photons can be defined in terms of passes of photon in two circular apertures in two or more massive shields or collimators. In narrow beam geometry, all photons that are incoherently scattered by a few degrees are prevented from reaching the detectors, such arrangement is called as “narrow beam” or “good geometrical” as shown in figure (1.2).

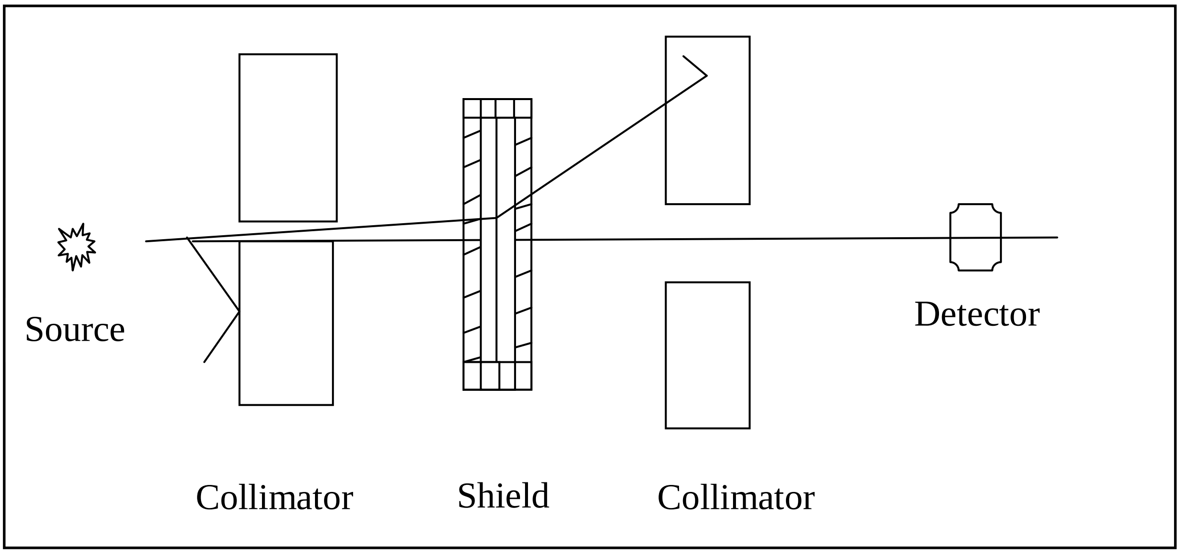

Fig. (1.2): The good geometry arrangement

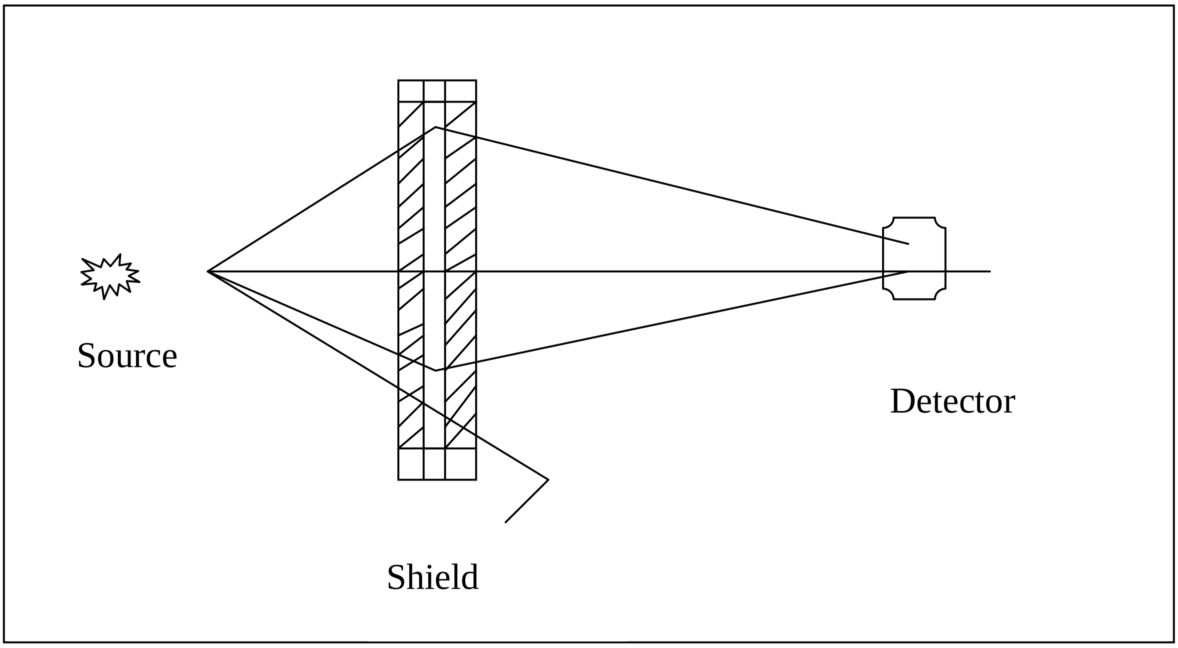

Fig. (1.3): The poor geometry arrangement

When a large part of scattered and subsidiary radiation is arrived to the detector, then the arrangement is called “broad beam” or “poor geometry” as shown in Fig. (1.3), so Eq. (1.11) will become

$$I / I_o = B\, e^{-\mu x} \tag{1.13}$$

where B, is defined as the buildup factor which is a dimensionless quantity.

1.5 Buildup Factor

Since in the case of the interaction of uncharged particles with matter, scattering processes are always significant, the contribution from the scattered photons or neutrons has to be taken into account. Then the concept of buildup factor B is very useful in describing broad-beam attenuation quantitatively. In experiments, one must be aware of geometry arrangements and other factors which may affect the measured number of particles or the values of other radiation quantities under consideration (e.g, exposure, absorbed dose, etc).

In photon beam attenuation, for example, the conditions may correspond to a narrow-beam geometry where exponential attenuation takes place or to abroad-beam geometry where the buildup factor must be considered [13].

In general, the buildup factor can be defined as follows:

B= [Primary +Scattered +Secondary] radiation / [Primary radiation]

For narrow-beam geometry it follows that B=1 exactly, and for broad-beam geometry B>1. The value of buildup factor is a function of radiation type and energy, attenuating medium and depth, geometry, and the measured quantity [14].

Buildup factor may be expressed in terms of three main quantities [15].

1. The Number Buildup Factor B_N, defined as:

$$B_N = \frac{\int N_t dE}{\int N_{uns}.dE} \tag{1.13}$$

where, N_t = total number of photons reached to detector

$N_{uns.}$ = number of unscattered photons reached to the detector

2. The Energy Buildup Factor B_E, defined as:

$$B_E = \frac{\int E_t dE}{\int E_{uns}.dE} \tag{1.14}$$

where, E_t = total energy of photons reached to detector

E_{uns}. = energy of unscattered photon reached to detector

3. The Dose Buildup Factor B_D, defined as:

$$B_D = \frac{\int \mu_{air} E_t dE}{\int \mu_{air} E_{uns}.dE} \tag{1.15}$$

where, μ_{air} = the energy absorption coefficient of air

1.6 Empirical Formula for B (μx)

In calculations involving the buildup factor (B), it is convenient to have a mathematical expression for B, and a variety of such formula have been proposed. Some of the more commonly used of these formulas are briefly reviewed:

1.6.1 For a single layer shields

a) Linear formula [16]

$$B = 1 + k.(\mu x) \tag{1.16}$$

where,

k = Constant depends on source energy and atomic number of the shield

μ = Linear attenuation coefficient for the shield material

x = Shield thickness in cm

μx = Shield thickness in mean free path (mfp).

b) Taylor Formula

For a point isotropic source in infinite medium, the buildup factor is defined as [17, 9]:

$$B = A \exp(-\alpha_1 \mu x) + (1-A) \exp(-\alpha_2 \mu x) \tag{1.17}$$

The coefficients values of A, α_1 and α_2 in Eq. (1.17) were published for many materials; the importance of this equation comes from its simplicity, therefore it is considered in a wide range of applications.

c) Berger Formula

For low Z media, the Berger formula provides a very good fit for high source photon energies at ~ 1 MeV and higher and for low energies below ~ 0.06 MeV. The buildup factor according to this formula is given by [18]:

$$B = 1 + a\,(\mu x)\,\exp\,[b\,(\mu x)] \tag{1.18}$$

where a and b are the parameters, function of source photon energy and attenuating medium. For a medium Z material such as iron, the Berger formula provides an excellent fit through the whole range of photon energies studied. Furthermore, the formulation is such that the contribution from the uncollided radiation is provided by the first term while the second term determines the scattered contribution.

d) Capo Formula

In this formula, the buildup factor is given by [16]:

$$B(E_O, \mu x) = \sum_{i=0}^{3} B_i\,(\mu x)^i \tag{1.19}$$

where;

$$B_i = \sum_{j=0}^{4} C_{ij}\left(\frac{1}{E_O}\right)^j \tag{1.20}$$

The coefficients values of Eq. (1.19) were published [19] for many materials. The equation stated the Bi can be considered as a function for the primary photons energy E_0 and the rate of the atomic number of the medium C_{ij}.

1.6.2 For multi-layered shields

The variation of the buildup factor with penetration distance in multi-layered shields differs from that in homogeneous media, mainly because of the change in the angular and energy distribution of the radiation in the vicinity of regional boundaries. This means that in multi-layered systems the buildup effect on the incident radiation depends on the previously penetrated layers as well as on the layer under consideration and that the order in which the layers occur may be significant [16].

a) Goldstein Method of an Effective Atomic Number

This method proposes the homogenization of the shield layers by specifying a single effective atomic number, $\overline{Z}$ for the shield. The buildup factor for the composite shield depends on the actual number of mfp's penetrated by the radiation and on $\overline{Z}$ [16].

b) Blizard Method

The proposal is to take the atomic number, Z, of the last material penetrated by the radiation, and the total attenuation thickness of the shield, ($\sum_i \mu_i x_i$), as the arguments in entering the basic buildup factor tables.

The principle merit of the method is that it is easy to apply; obviously; it should only be used with caution [16].

c) Border Formula

A more refined approach to the multi-layer problem is that of Border [20]. Here an attempt is made to allow for the passage of the photons through the previous layer, by assuming the buildup contribution of each layer is additive and that it can be found as a result of a simple differencing procedure. Thus, for a double layer shield and a point isotropic source, the buildup factor is given by [15]:

$$B = \sum_{n=1}^{N} B_n\left(\sum_{i=1}^{n} X_i\right) - \sum_{n=2}^{N} B_n\left(\sum_{i=1}^{n-1} X_i\right) \tag{1.21}$$

where

X_i = layer thickness in mfp

B_n = dose buildup factor for N layers with thickness X_i

1.7 Literature survey

The literature since 1950 on gamma-ray buildup factor using different radiation sources and different media is surveyed here. Many of these studies were treated and processed by Monte Carlo calculation. Because the buildup factor depending on many factors, so the nature of studies was varied. Some of the published studies were concentrated on the method of calculation, and some others concerned with the type of radioactive source used, whereas the others concerned with the type of secondary radiation that resulting from gamma-ray interaction with matter. Here we have a historical review on the most important performed studies.

In 1950 White [21], started a first study for the buildup factor for water. She used the Co-60 source with two types of detectors, G.M. tube and ionization chamber where the results appeared that the buildup factor increases exponentially with increasing the thickness of the layer of water that used in measurement. And the buildup factor value for 46 cm thickness of water was 4.51.

Dixon [22], presented an experimental study about the buildup factor calculation for concrete and lead with Co-60 source with an effective strength of 1000 Ci using the ionization chamber. From this study the value of buildup factor for lead is seen to be much lower than for concrete which give the result that the lead is a good absorber for gamma radiation and other secondary radiation which result from the interaction with it.

Monte Carlo method was used to calculate the longitudinal development of the showers in study presented by Zerby and Moran [23] in 1963. The described method of the performing the complete Monte Carlo calculation, and a description of how the individual selection techniques where used was also included.

Hubbell [24], at the same year investigated and formulated a buildup factor form using a power series technique. He evaluated the response of an isotropic detector to primary radiation from a finite plane source as the sum over an infinite series. This study was investigated as an application to rectangular and off- axis disk source problems.

In 1970 Morris and Chilton [25], calculated the buildup factor for the water using two different methods, Monte Carlo and moment, with two point sources of energies, respectively, 0.5 and 1MeV. The comparison between these results with the results that obtained from the study of Goldstein and Wilkins in (1954), which was by moment method, and the standard deviation between results was 5%. From this study, the value of buildup factor for water at 2 mfp thickness at 1 MeV energy was calculated using Monte Carlo and moment methods.

In 1975, Morris et al. [26] presented a measurement for buildup factor for energy deposition in water and aluminum and for exposure in concrete for monoenergetic, point isotropic gamma ray sources. The buildup factor were evaluated using the method of moments and are tabulated for energies from 0.03 to 10 MeV, and distance out to 50 mfp from the source. Empirical representations of these buildup factors are developed using Berger's formula. Fitting parameters are tabulated for each material as a function of source energy.

Eisenhauer and Simmons [27], presented in 1975, a theoretical study about the energetic buildup factor calculation, absorbed energy, and dose for concrete in accordance of that the source was a point and symmetric with energy range from 0.015 to 15 MeV, using the moment method. They concluded that the buildup factor with existing the annihilation radiation is higher than that without it.

In 1977, Shure and Wallace [28], measured Taylor's factors for buildup factor of concrete that measured previously by Eisenhower and Simons. Also, that standard deviation between the values measured by Taylor method and also measured by Eisenhower was greater than 10%.

In 1978, Metghalchi [29] determined the coefficients for Berger's two-parameter formula for the Eisenhower-Simmons gamma-ray buildup factor in ordinary concrete. His work showed that the coefficients available for Taylor formula has improved considerably since Chilton [30] concluded that the accuracy that can be achieved using fitted values of the two parameters is better in general than previously suggested formulas of three parameters or less.

Chilton [30] calculated the Berger coefficients of point source and infinite medium buildup factor for gamma rays through ordinary concrete, which have been provided by Eisenhauer and Simmons. Their results are quite comprehensive both with respect to source photon energy (0.015 to

15MeV), and penetration depth (0 to 40 mfp), also these buildup factor data have been fitted to the Taylor formula by Shure and Wallace.

In 1980, Chilton et al. [18] calculated the buildup factor for photons of point isotropic sources in infinite homogeneous samples of air, water and iron by a moment's method code. Then the results gives the parameters in the Berger empirical formula for buildup factors have been evaluated. The Berger formula was shown to fit the calculational results for nuclei of low atomic number at energies above 1MeV and below 0.06 MeV. In mid energy range, differences of as much as 40% are observed. The concrete buildup factor data have been fitted to both the Taylor and the Berger empirical formulae.

Foderaro and Hall [31] suggested in 1981 a new formulation to fitting the results of buildup factor because of the high relative standard deviation for the Taylor and Berger formula at increasing the thickness. This new formula was called; "The three-exponential representation" which was better fit the raw buildup factor data than either the Berger or Taylor representation when this method was applied on the calculation of buildup factor of water.

Hirayama [32] presented a comparison between gamma ray buildup factor for low Z-material and for low energies using the discrete ordinates and point Monte Carlo method. They calculated the exposure and absorbed dose buildup factor for a point source in an infinite Beryllium in the low energy range of 0.03 to 0.3 MeV, for penetration depth up to 40 mfp. The study showed a reasonable agreement for two types of sources; normally incident and point isotropic sources from the compressions of both results two values obtained by point Monte Carlo calculations using the "Electron gamma shower version (4) codes". They conclude that for both methods, the results up to 10 mfp were in good agreement for normally incident and point sources with those of the EGS4 as a test standard by the point Monte Carlo method.

Bishop [33] presented a measurement for buildup factor of energies 1.43, 2.57 and 6.13 MeV, where the resulted spectrum from analyzing the slabs of concrete, lead, and iron was investigating using NaI (TI) detector.

In 1988 Herbold et al. [34], the dose build-up factors in the range from 15 to 100 keV of point isotropic gamma sources in water have been calculated based on the formalism given by Berger and using the Monte Carlo code EGS4. All effects of penetration, especially electron transport and Rayleigh scattering have been included. It was found that the new build-up factors show considerable deviations relative to Berger's. The behavior of build-up factors in finite, spherically shaped media was also investigated. Results were applied to the dose distribution of I-125 seeds.

The buildup factor for concrete was measured by Fourine and Chilton [35] from monoenergetic point source and penetration depth (10 mfp) with different angles of incidence on the slab at the energy range (0.661 MeV up to 6.13 MeV).

In 1989 Al-Ani [36] presented the measurement of buildup factor for Fe, Cu, Al and concrete using Co-60 and Cs-137 gamma sources. The effects of geometrical factor on the buildup factor in addition to the effect of detector type were considered.

Hattif in 1994 [37], measured the buildup factor for aluminum, iron, copper, brass, and lead using Cs-137 and Co-60 gamma sources with NaI(TI) detector.

In 1994 Bakos [38] calculated the buildup factor for photon penetration through multilayer shielding slab with energies 1.6, 6.13, and 7.10 MeV. Theoretical and experimental calculation of dose was determined for penetrations through various thicknesses of aluminum Al, Steel Fe, and lead Pb. He concludes from this investigation that in the case where lead forms the outer layer the buildup factor is reduced. Where as, when steel forms the outer layer the dose buildup factor approached the buildup factor of an all steel shield of the same total mfp penetration.

One year later, Bakos [39] presented a study for the angular properties of the scattered photons for combined 1.43 and 2.75 MeV source photons penetrating single and double-layer shields. He determined experimentally angular flux spectra for these energies. From his work, he noticed that for any given polar angle, the scattered photon properties decrease exponentially with increasing shield thickness.

Al-Ammar [40], measured the buildup factor for iron by Co-60 source for the single and double layer with NaI (TI) detector. His results were in agreement with the theoretical results.

In 2000 Sidhu et al. [41] presented a theoretical method to determine the gamma radiation buildup factor in various biological materials. In this study, the gamma energy range was 0.015 to 15 MeV, with penetration depth up to 40 mfp. The dependence of the exposure buildup factor on the incident photon energy and the effective atomic number (Z_{eff}) was also assessed.

Al-Baite in 2001 [42] measured the buildup factor for single and multi-layer shield for Al, Fe, Copper, Brass, and Pb materials. He calculated the buildup factor using the moment method with the information of the linear formula. He used the Kalos method for calculate the buildup factor for double layers.

In 2002 Al-Samaraey [43] presented the study of gamma ray buildup factor for the conical beam using Monte Carlo method for Al, Fe, and Pb shields .The number buildup factors from Cs-137 and Co-60 was calculated.

In the same year, Shimizu [44] presented a method of invariant embedding to calculate the gamma ray buildup factors for point isotropic source in infinite homogeneous media up to depth of 100 mfp without bremsstrahlung. The exposure buildup factors for water, iron and lead for typical source energies of 10, 1 and 0.1 MeV were provided. The calculated buildup factors are found to agree well with other data including the moment's method calculations and the Monte Carlo calculations by EGS4.

After one year, Shimizu and Hirayama [45] reported an improved set of gamma ray buildup factors for point isotropic sources in the infinite homogeneous media based on the method of invariant embedding (IE method). In this study the exposure buildup factors including bremsstrahlung were computed for lead, iron and water at the source energy of 10 MeV up to depth of 100 mfp. The accuracy of the present method was checked by comparison with the calculations by use of EGS4. Excellent agreement was obtained between the calculations by both methods about the exposure buildup factors per energy for lead up to depth of 10 mfp and the ratio of the exposure buildup factor with bremsstrahlung to that without bremsstrahlung for lead, iron and water up to depths of 40 mfp.

In 2004, Al-Rawi [46], calculated the number buildup factor for gamma ray in two materials, water and lead, for single and double-layer shields with Cs-137and Co-60 gamma sources using the Monte Carlo Method. A computer program "MONTRAY" was designed and implemented to perform these calculations. His method is adapted for photon transport, and simulates the transmission and reflection of incident gamma rays on an infinite slab of arbitrary material.

In the same year, Shimizu et al. [47] presented an improved data set of gamma ray buildup factors for 26 materials and for point isotropic sources in infinite homogenous media. The method of invariant embedding (IE) was employed. In this study, the following was considered; (1) extension of the buildup factors up to depth of 100 mfp, (2) improved treatment of bremsstrahlung, (3) addition of the effective does buildup factors, (4) consistent use of the cross section to all materials and (5) a quantitative evaluation about the accuracy in transport calculation.

1.8 The Aim of the present work

Since 1950s, the calculations of gamma ray buildup factors considered different gamma sources and different media. Most of these calculations used monoenergetic sources in finite or infinite media with single or multi-layer shields, and different penetration depths. No single study of the buildup factor for a continuous energy source was performed until 2004 at which Elias et al. [15, 48] presented the first experimental study of gamma ray buildup factor for bremsstrahlung produced by complete absorption of beta rays emitted from ($^{90}Sr/^{90}Y$) source for different materials in single and multi-layer shields with different penetration depths. They found that the buildup factors of a continuous energy source are different from that of the monoenergetic sources.

The aim of this project is to simulate and calculate the gamma ray buildup factor for bremsstrahlung produced by absorption of beta particles in different materials by designing a new computer program based on the

Monte Carlo method. This method depends on the photon transport, and simulates the transmission and reflection of photons on an infinite slab of a homogenous material.

This work also aims to find a general formula to calculate the bremsstrahlung buildup factor as a function of the material atomic number and its thickness.

Table (3.1) The mass attenuation coefficient (μ/ρ) and Compton cross section for different materials and air, in cm2/g corresponding to energy interval (in MeV) mesh data. These data were extracted from Hubble [54].

Energy mesh	H_2O		Concrete		Al		Sn		Pb		Air
	$\mu_{Compton}$	μ_{Total}	$\mu_{Compton}$	μ_{Total}	$\mu_{Compton}$	μ_{Total}	$\mu_{Compton}$	μ_{Total}	$\mu_{Compton}$	μ_{Total}	μ_{Total}
15	0.0127	0.0194	0.0116	0.0210	0.0110	0.0219	0.0096	0.0431	0.00902	0.0566	0.018
10	0.0171	0.0222	0.0157	0.0228	0.0148	0.0232	0.013	0.0389	0.0122	0.0497	0.021
8	0.0201	0.0243	0.0184	0.0243	0.0174	0.0244	0.0152	0.0372	0.0143	0.0467	0.022
6	0.0245	0.0277	0.0225	0.0270	0.0213	0.0265	0.0186	0.0358	0.0175	0.0438	0.025
5	0.0278	0.0303	0.0255	0.0291	0.0241	0.0284	0.0211	0.0354	0.0198	0.0426	0.028
4	0.0322	0.0340	0.0295	0.0322	0.0279	0.0311	0.0244	0.0355	0.0229	0.0418	0.031
3	0.0385	0.0397	0.0354	0.0370	0.0335	0.0354	0.0292	0.0367	0.0274	0.0420	0.036
2	0.0490	0.0494	0.0450	0.0455	0.0425	0.0432	0.0371	0.0408	0.0348	0.0453	0.045
1.5	0.0574	0.0575	0.0527	0.0528	0.0498	0.05	0.0435	0.0458	0.0407	0.0509	0.052
1	0.0707	0.0707	0.0648	0.0648	0.0613	0.0613	0.0534	0.0567	0.0499	0.0680	0.064
0.8	0.0786	0.0786	0.0721	0.0721	0.0681	0.0862	0.0593	0.0647	0.0554	0.0841	0.071
0.6	0.0894	0.0894	0.0820	0.0820	0.0775	0.0776	0.0672	0.0777	0.0626	0.117	0.081
0.5	0.0966	0.0966	0.0886	0.0887	0.0837	0.0839	0.0724	0.0888	0.0673	0.150	0.087
0.4	0.106	0.106	0.0969	0.0971	0.0916	0.0919	0.0789	0.108	0.0731	0.215	0.096
0.3	0.118	0.118	0.108	0.109	0.102	0.103	0.0872	0.151	0.0804	0.373	0.107
0.2	0.135	0.136	0.124	0.126	0.117	0.119	0.0982	0.298	0.0897	0.936	0.123
0.15	0.147	0.148	0.134	0.139	0.127	0.132	0.1105	0.561	0.0948	1.91	0.136
0.1	0.163	0.165	0.148	0.164	0.139	0.157	0.112	1.58	0.0989	5.34	0.154
0.08	0.170	0.175	0.154	0.186	0.144	0.182	0.113	2.88	0.0992	2.11	0.166
0.06	0.177	0.192	0.159	0.241	0.148	0.244	0.113	6.33	0.0973	4.53	0.186
0.05	0.180	0.208	0.161	0.306	0.150	0.321	0.111	10.4	0.0948	7.39	0.205
0.04	0.183	0.240	0.162	0.455	0.149	0.5	0.108	19	0.0902	13.4	0.246
0.03	0.183	0.329	0.160	0.880	0.146	1.02	0.101	40.5	0.0823	28.9	0.340
0.02	0.177	0.721	0.152	2.66	0.137	3.24	0.0881	20.2	0.0690	84	0.733
0.015	0.170	1.54	0.143	6.14	0.127	7.64	0.0773	44.9	0.0592	108	1.522
0.01	0.155	5.10	0.125	20.1	0.106	25.7	0.0607	136	0.0454	126	4.910
0.008	0.144	10.1	0.114	38.4	0.0929	49.6	0.0516	247	0.0381	223	9.005
0.006	0.126	24.2	0.0968	83.3	0.0770	114	0.0412	525	0.0297	460	22.154

Table (3.3) Calculated BBUF for different thicknesses of water, concrete, aluminum, tin and lead shielding materials

Thickness (cm)	Water		Concrete		Aluminum		Tin		Lead	
	Buildup factor	STD in buildup factor	Buildup factor	STD in buildup factor	Buildup factor	STD in buildup factor	Buildup factor	STD in buildup factor	Buildup factor	STD in buildup factor
0.5	1.0044	0.0005	1.0071	0.0007	1.0073	0.0007	1.0211	0.0030	1.0284	0.0052
1	1.0056	0.0006	1.0114	0.0008	1.0120	0.0009	1.0374	0.0040	1.0562	0.0085
1.5	1.0061	0.0007	1.0129	0.0010	1.0145	0.0011	1.0609	0.0051	1.0840	0.0127
2	1.0077	0.0008	1.0167	0.0012	1.0179	0.0013	1.0618	0.0063	1.1149	0.0181
2.5	1.0084	0.0009	1.0177	0.0014	1.0187	0.0015	1.0758	0.0076	1.1397	0.0252
3	1.0098	0.0010	1.0196	0.0016	1.0200	0.0017	1.0888	0.0092	1.1447	0.0328
3.5	1.0098	0.0010	1.0198	0.0018	1.0201	0.0019	1.1071	0.0110	1.1756	0.0420
4	1.0105	0.0011	1.0199	0.0020	1.0206	0.0022	1.1164	0.0131	1.2039	0.0553
4.5	1.0118	0.0012	1.0210	0.0022	1.0227	0.0025	1.1220	0.0154	1.2394	0.0706
5	1.0135	0.0013	1.0265	0.0025	1.0245	0.0028	1.1302	0.0181	1.3842	0.0926

CHAPTER THREE

Results, Discussion and Conclusions

3.1 Introduction

This chapter contains the first ever been published simulated data of bremsstrahlung buildup factors. These data were extracted from simulation of the system based on the Monte Carlo method produced by complete absorption of beta particles for single layer shield of different materials and thicknesses using the good and poor geometrical arrangements, and contains the parameters affecting the continuous gamma ray buildup factor.

Before presenting the results, we shall demonstrate the input data of the 'BBF' program.

3.2 Input Data

These data can be classified into two types:

3.2.1 The Simulation Parameters

The following simulation parameters were considered:

1. The iteration number, i.e., the maximum number of tracks used in Monte Carlo simulation which is the particle histories to be followed. The value 10^6 was considered in the present work. These particles distributed as the fractional weight of the bremsstrahlung spectrum of $^{90}Sr/^{90}Y$ beta source.
2. Total number of (coarse mesh) energies for which the corresponding basic cross section data has to be assigned into program. In this work, the coarse mesh was 28.
3. Interval number of energy used to cover the photon energy spectrum. (It is taken 38).

3.2.2 The Physical Parameters

The following physical parameters were considered:

1. The atomic number of shielding materials. Different materials were chosen in this work, namely: water, concrete, aluminum, tin and lead. These materials are usually used for shielding.
2. The shield thicknesses, (0.5-5 cm) considered in the present work.
3. Density of the shielding materials (1, 2.3, 2.699, 7.31 and 11.34 g/cm^3 for water, concrete, Al, Sn and Pb respectively).
4. Cutoff energy, when photon energy is reduced less than cutoff energy its life history is terminated. In This work, the cutoff energy is taken 0.019 for Al, H_2O, concrete and 0.022 for Sn and 0.025 MeV for Pb.
5. The energy interval mesh; the applied energy was partitioned into 28 interval which cover the range 0.006 to 15 MeV.
6. The corresponding Compton cross section data at each energy interval mesh in cm^2/g were extracted from those given in Ref. [54].
7. The corresponding total mass attenuation coefficient data in cm^2/g for all materials used in this work. The data are from Ref. [54].
8. The corresponding total mass attenuation coefficient data for air in cm^2/g. The data are from Ref. [54]. (Table 3.1).

3.3 Simulated Results of Continuous Energy Buildup Factors (BBUF)

3.3.1 The effect of simulated parameters

These parameters are concerned within the accuracy of the program results obtained by applying Monte Carlo method. It can be summarized as follow:

Table (3.1) The mass attenuation coefficient (μ/ρ) and Compton cross section for different materials and air, in cm^2/g corresponding to energy interval (in MeV) mesh data. These data were extracted from Hubbel [54].

I. The effect of iteration number

The iteration number is the number of particles to be followed, i.e., the model is recalculated to produce the simulation result. Table (3.2) presents the values of the standard deviation of the simulated buildup factor versus the iteration number.

Table (3.2) The standard deviation of buildup factor versus the iteration number for 0.5 cm shield thickness of water

Iteration number	Standard deviation
5×10^4	0.0060
1×10^5	0.0035
3×10^5	0.0020
5×10^5	0.0011
7×10^5	0.0007
9×10^5	0.0006
1×10^6	0.0005

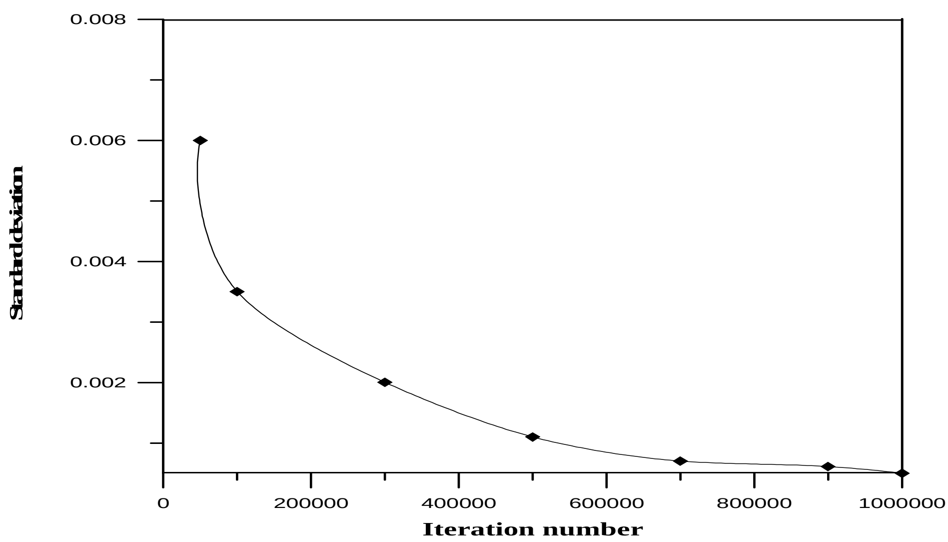

Fig. (3.1): The effect of iteration number on the standard deviation of simulated buildup factor.

It is obvious from Table (3.2) and Fig. (3.1) that the standard deviation is significantly decreases with the increase in the iteration number. In this simulation we need to obtain the simulated bremsstrahlung buildup factor with more convergence with its expected value then the largest number of iteration (10^6) used with longer run time. Then the value of standard deviation indicates for the uncertainty may exist in the expected value.

II. Number of energy intervals

This parameter represents the number of intervals used to describe the considered range of photon energy spectrum. Using different number of intervals for different penetrating slab thickness materials, one can conclude that there is no variation in standard deviation of the simulated buildup factor when the interval number increased or decreased. As a result the interval number 38 was chosen as affixed parameter in all present work calculations to obtain the photon energy spectrum distributed at 0.05 MeV for maximum bremsstrahlung energy 2.2 MeV.

III. Distribution of photons number with their energy

The total number of photons distributed as the fractional weights of bremsstrahlung spectrum for $^{90}Sr/^{90}Y$ beta source with their energies, shown in Fig. (3.2), this distribution describes the continuous energy spectrum of photons produced by complete absorption of beta rays by lead shield with maximum bremsstrahlung energy 2.2 MeV beta source distributed at 0.05 MeV for each energy interval. This distribution is regarded as the continuous energy source of gamma rays in “BBF” program.

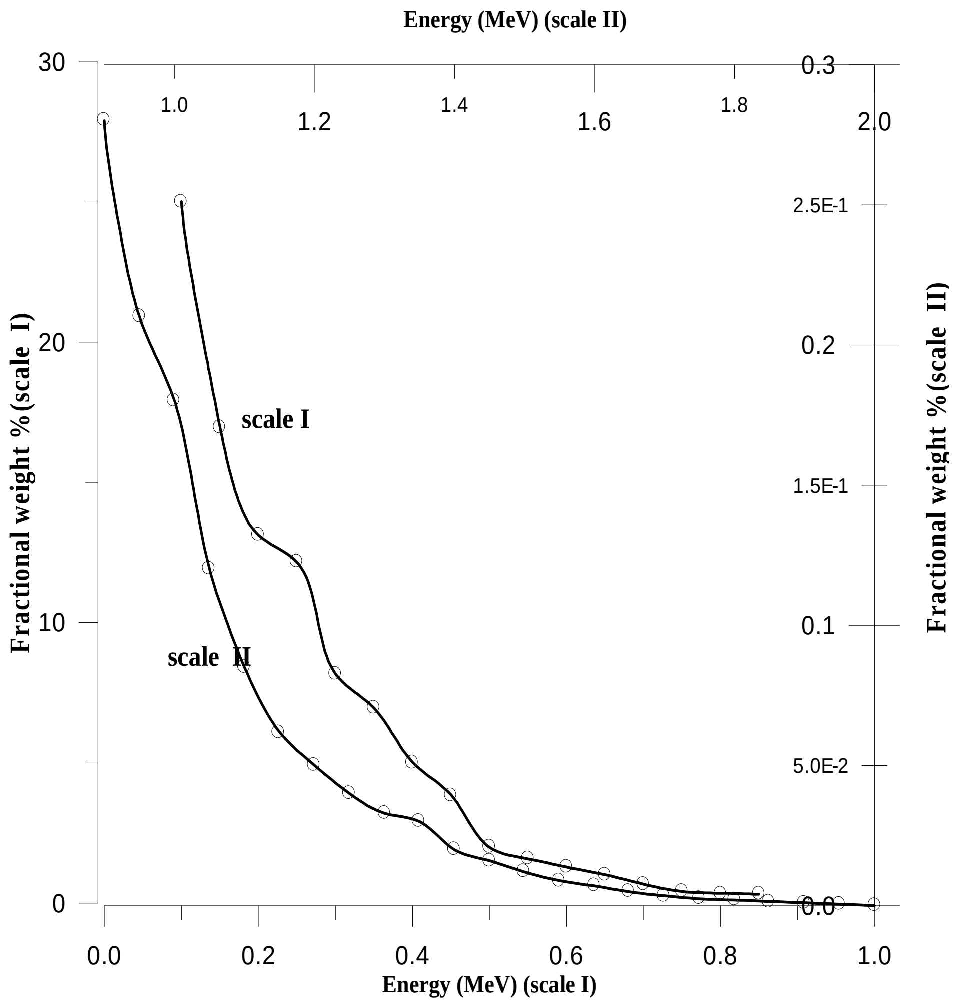

Fig. (3.2): The fractional weight of bremsstrahlung spectrum of ^{90}Sr/^{90}Y source as a function of photon energy

3.3.2 The Effect of Material Physical Parameters

The following parameters affect the value of bremsstrahlung buildup factor BBUF.

I. The effect of thickness

Table (3.3) and Fig. (3.3) show the relation between the simulated buildup factor for continuous “bremsstrahlung” source BBUF and thickness and type of the shielding materials for the beta source ^{90}Sr/^{90}Y of maximum bremsstrahlung energy 2.2 MeV.

Table 3.3

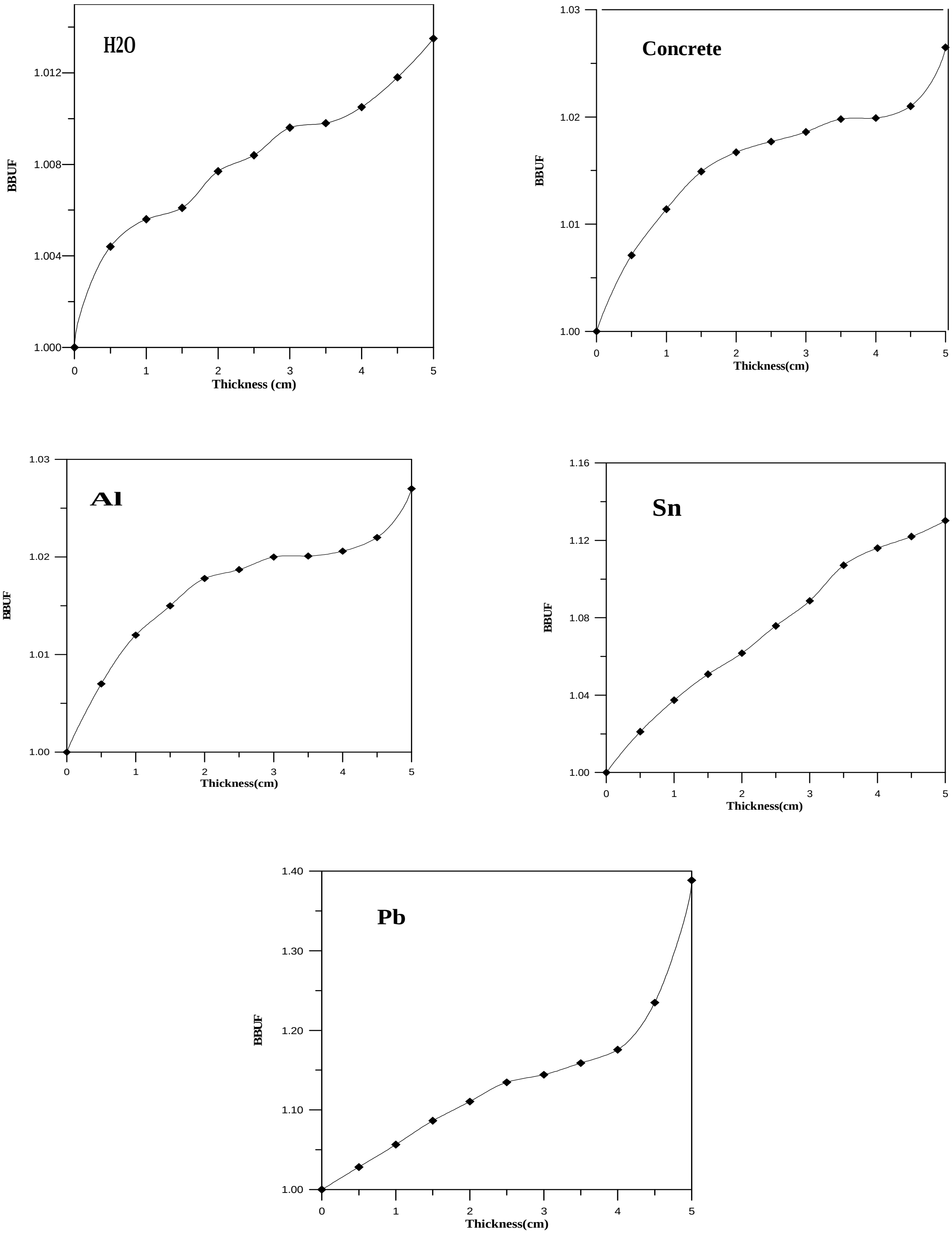

Fig. (3.3): BBUF for different materials as a function of shield thickness (in cm).

The results show that the bremsstrahlung buildup factor increases with the increase of thickness for all types of the investigated shields. The increase of buildup factor occurred because the increasing of the scattering cross section with small angles with increasing of thicknesses.

II. The effect of the atomic number

As shown in Fig. (3.4) and Table (3.3), the bremsstrahlung buildup factor increases by increasing the atomic number of shielding material. The buildup factor of water is less than that for lead when the shielding material measured in given cm units. A behavior is due to result caused by appreciable the mean free path of Pb is much shorter less than that in water.

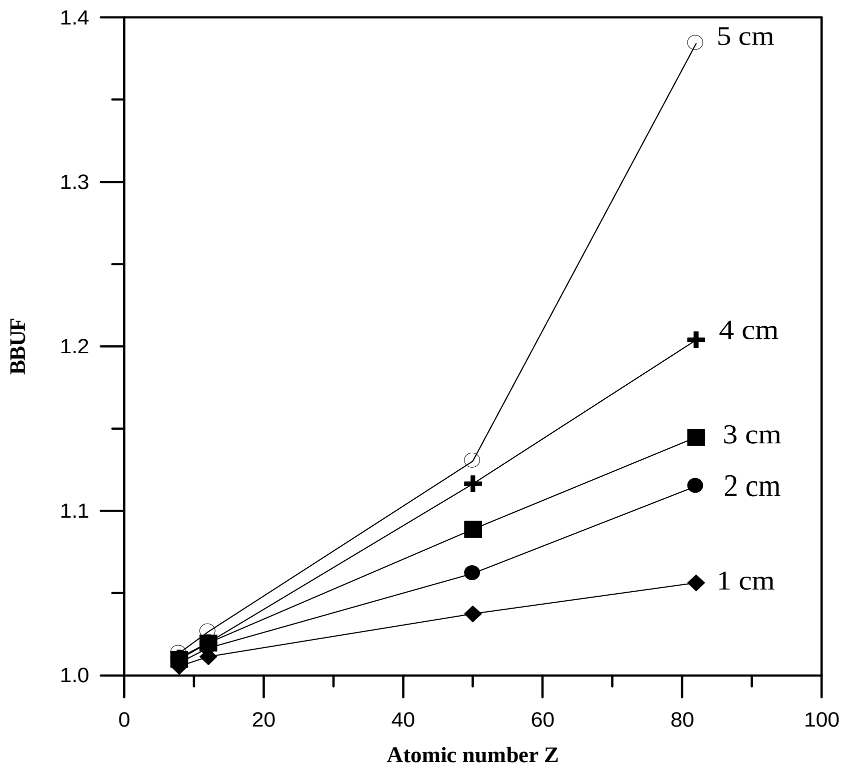

Fig. (3.4): The effect of atomic number Z on BBUF for different thicknesses

3.4 The concluded Semi Empirical Formula

As shown in Fig. (3.3), the relation between bremsstrahlung buildup factor and shield thickness can be fitted with the potential equation and given as

$$BBUF = 1 + a\,X^{b} \qquad (3.1)$$

where, a and b are the fitting parameters depends on the shielding material atomic number Z and maximum bremsstrahlung energy, X, the shield thickness in cm. For $^{90}Sr/^{90}Y$ beta source the values of a and b were calculated and given versus the atomic number Z for water, concrete, Al, Sn and Pb shown in Table (3.4) and Fig. (3.5).

Table (3.4) The relation between a, b parameter and atomic number Z for $^{90}Sr/^{90}Y$ beta source for water, concrete, Al, Sn and Pb shielding materials.

Shield material	Atomic Number Z	a parameter	b parameter
Water	7.89	0.0056	0.4751
Concrete	12.07	0.0108	0.5083
Aluminum	13	0.0111	0.5128
Tin	50	0.0380	0.7897
Lead	82	0.0553	1.0092

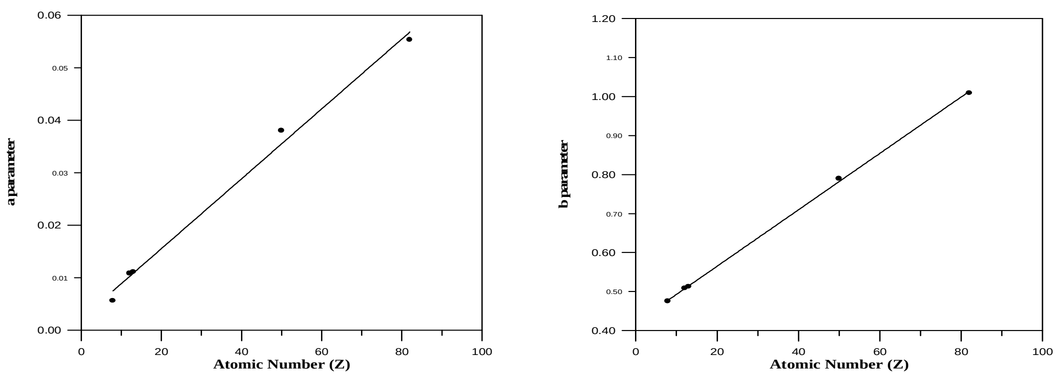

Fig. (3.5): The relation between the fitting parameters a and b versus the atomic number Z of shield material.

The results show that the value of these parameters increase with the increase of material atomic number Z. Fitting these relations with the linear equation Fig. (3.5) and substituting in Eq. (3.1), then the bremsstrahlung buildup factor "BBUF" can be expressed as function of both material thickness X and atomic number Z as follow

$$BBUF = 1 + (a_1 Z + a_2) X^{(a_3 Z + a_4)} \qquad (3.2)$$

where a_1, a_2, a_3, and a_4 are now, new constants depend on the bremsstrahlung energy distribution. For $^{90}Sr/^{90}Y$ beta source these parameters are;

$a_1 = 0.0007$ $a_3 = 0.0072$

$a_2 = 0.0022$ $a_4 = 0.4204$

3.5 Conclusions

The first data set for bremsstrahlung buildup factor BBUF are generated for five materials by the Monte Carlo simulation method. The data cover the maximum bremsstrahlung energy of 2.2 MeV emitted due to the complete absorption of $^{90}Sr/^{90}Y$ beta particles and depth 0.5-5cm shield thickness. The following remarks were concluded:

1. The bremsstrahlung buildup factor increases with the increase of shield thickness.
2. The bremsstrahlung buildup factor increase with the increases of the atomic number Z. Since, the thickness of materials measured in cm

units then the mean free path increase with the increase the atomic number.

3. The bremsstrahlung buildup factor is in general less than that for monoenergetic source, due to the fact that in continuous energy distribution photons are distributed from zero to maximum bremsstrahlung energy with high probability only at the lower region of energy spectrum, while in monoenergetic source all emitted photons carry the same energy.
4. The relation between the bremsstrahlung buildup factor and both the atomic number Z and thickness X of the shielding material can be expressed in the following semi-empirical formula

$$BBUF = 1 + (a_1 Z + a_2) X^{(a_3 Z + a_4)}$$

where a_1, a_2, a_3 and a_4 are constants depend on the bremsstrahlung energy distribution. For $^{90}Sr/^{90}Y$ bremsstrahlung, these parameters are $a_1 = 0.0007$, $a_2 = 0.0022$, $a_3 = 0.0072$ and $a_4 = 0.4204$

3.6 Recommendations

1. Considering the effect of the pair production and annihilation radiation in the bremsstrahlung buildup factor.
2. Developing a program to calculate the buildup factor for double and multi-layer shield using the Monte Carlo method.
3. Calculating the bremsstrahlung buildup factor for other bremsstrahlung sources like Tl-204, Y-91 and Ca-45. A more general semi-empirical formula can then be obtained by introducing the effect of bremsstrahlung energy distribution.

CHAPTER TWO
Simulation Details

2.1 Introduction

Simulation has long been an important tool of designers, whether they are simulating a supersonic jet flight, a telephone communication system, a business game, or a maintenance operation. Although simulation is often viewed as a “method of last resort” to be employed when everything else has failed, recent advances in simulation methodologies, availability of software, and technical developments have made simulation one of the most widely used and accepted tools in system analysis and operations research [49].

Simulation is a numerical technique for conducting experiments on a digital computer which involves certain types of mathematical and logical models that describe the behavior of a system [50]. Computer simulation also enables us to replicate an experiment. Replication means rerunning an experiment with selected changes in parameters or operating conditions being made by the investigator. In addition, computer simulation often allows us to induce correlation between these random number sequences to improve the statistical analysis of the output of a simulation.

Because sampling from a particular distribution involves the use of random numbers, stochastic simulation is sometimes called Monte Carlo simulation. As it was mentioned in the first chapter, the calculation of gamma attenuation when it passes through a shield of material can be performed using simulation model based on the Monte Carlo method, which is discussed in this chapter.

2.2 Monte Carlo simulation techniques

The term “Monte Carlo” was introduced by von Neumann and Ulam during World War II, as a code word for the secret work at Los Alamos [49]. The general accepted birth date of the Monte Carlo method is 1949, when an article entitled “The Carlo Method” appeared. In the former Soviet Union, the first articles on the Monte Carlo method were published in 1955 and 1956[51].

The Monte Carlo method is a mathematical tool utilizing random numbers. This makes possible the simulation of any process influenced by random factor. This method can obtain accurate results about the absorption, backscattering and transmission of the photons, electrons penetrating both supported and unsupported thin films, and can also provide implantation profiles of the absorbed electrons or photons, the energy and the angular distributions of backscattered and transmitted photons, and the spectra of secondary electrons, so they can be used in the Monte Carlo code.

The Monte Carlo method is now the most powerful and commonly used technique for analyzing complex problems. The primary components of a Monte Carlo simulation method include first the probability distribution functions (pdf’s), which the any physical or mathematical system must be described by a set of pdf’s. Second, random number generator, which a source of random number uniformly distributed on the unit interval, must be available. As the Monte Carlo method is statistical then the accuracy of its results depends on the number of simulated trajectories. The recent evolution in computer calculation capability means we are now able to obtain statistically significant results in very short time of calculation [52].

2.3 Simulation of buildup factor for continuous energy photons

2.3.1 The particle trajectory

The life history of a particle is built up from knowledge of its trajectory through the particular system of interest. Consider the path of a particle as it travels through some homogeneous medium. Since the typical particle scatters frequently, the path will zigzag rather in the manner indicated in Fig. (2.1). Here the particle originates at a known direction and energy. It has a free-flight until it has a collision with an atom of the medium. This collision could result in the absorption of the particle and the immediate termination of its history, but shall assume a scattering interaction occurs and the particle continues with a new direction and a change of energy (or wavelength). This change of energy and direction is a statistical process, that is, there is not a unique energy and direction after a scattering, rather there is a probability distribution for each of these variables discussed later. After the first scattering the same particle makes another free-flight and experiences another collision, and so on. In order to track the particle during its journey we require knowing: its spatial coordinates (x, y, z); the spherical coordinates (θ, ϕ) of its direction; and its energy. These variables are sufficient to define the state, α, of the particle, where

$$\alpha=\alpha\ (x, y, z, E, \theta, \phi) \tag{2.1}$$

The spherical coordinate system for defining the particle's direction is illustrated in Fig. (2.2). A particle's trajectory from collision to collision can be constructed as a succession of states α_o, α_1… α_n, where the i^{th} state is

$$\alpha_{\mathbf{i}} \equiv \alpha_{\mathbf{i}} (x_{\mathbf{i}}, y_{\mathbf{i}}, z_{\mathbf{i}}, E_{\mathbf{i}}, \theta_i, \phi_i) \tag{2.2}$$

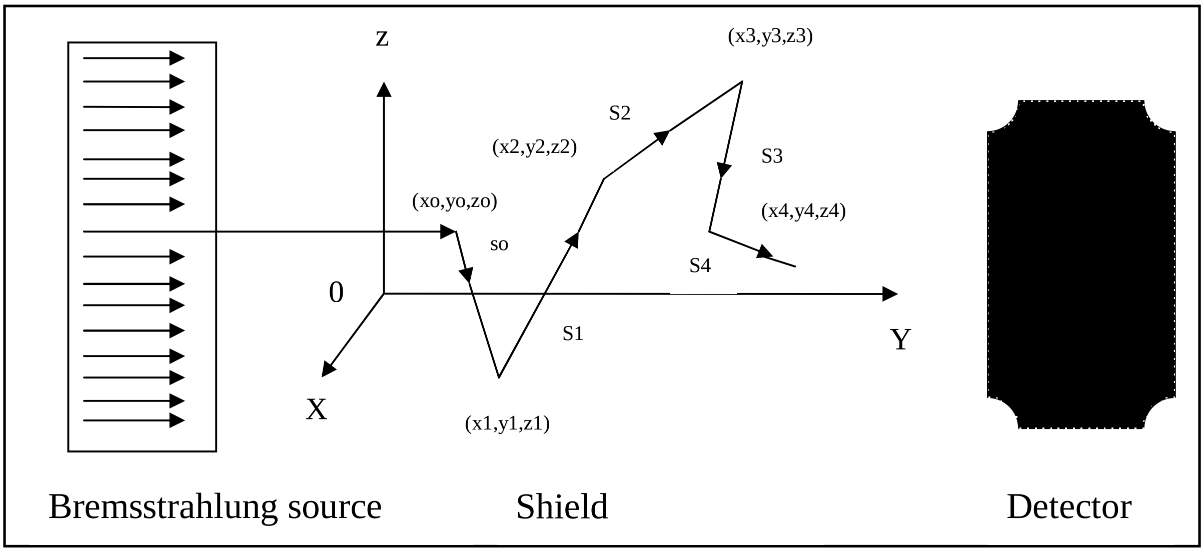

Fig. (2.1): A typical particle's 'random walk' through the medium (shield)

That is, in the i^{th} state, a particle has the spatial coordinates of the i^{th} collision point and the energy and direction of the particle after the i^{th} collision. Thus one could commence with initial, or source, conditions which define $\alpha_{\mathbf{o}}$, choose by random sampling from the relevant probability distributions which has been discussed, the new values of the variables which determine $\alpha_{\mathbf{1}}$, and soon. In this way the individual life history can be constructed. In most Monte Carlo calculations it is not necessary to store simultaneously every detail of the life history of every particle studied: usually all that is required is the latest state of the particle being currently followed.

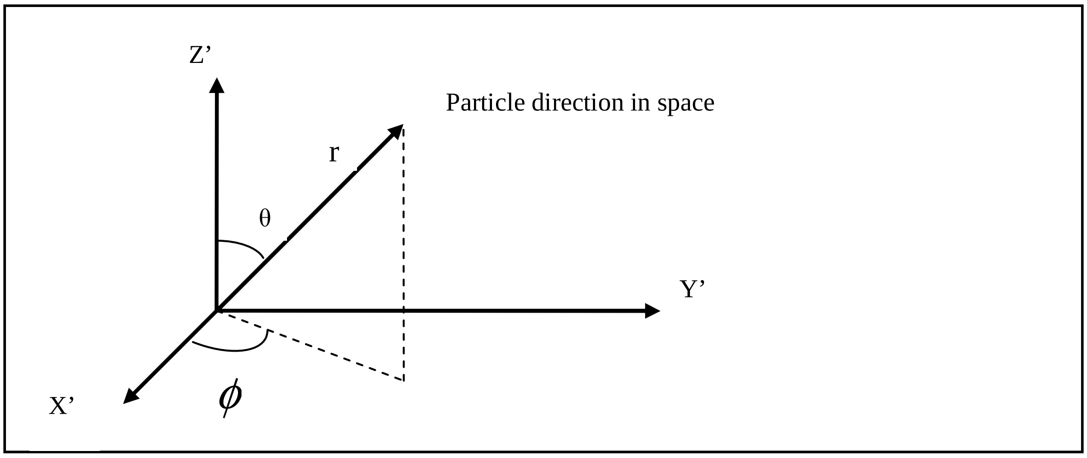

Fig. (2.2): Particles direction in spherical coordinates (θ, ϕ)

Obviously what we require are specific mathematical procedures for selecting the position of the next collision point, and the new energy and direction of the particle if it survives that collision. Let us consider a particle which has just undergone its i^{th} collision (a scattering), and begin by finding the spatial coordinates of its next collision point [16].

If we denote by S the path length of the particle to the next collision point, the probability of a particle traveling a distance S without having an interaction is $e^{-\Sigma s}$. The probability that a particle will have an interaction in the interval ds is Σds. Therefore, the probability that a particle will have an interaction between S and S+ds is

$$\int_0^\infty \Sigma e^{-\Sigma s} ds = 1 \qquad (2.3)$$

where, Σ the particle's total cross section (denoted by μ in the case of a photon). Hence we must establish a procedure for picking at random a value of S from the probability function implied by Eq. (2.3). For the moment we shall assume that such a procedure is available and that we have selected a particular value of S to assign to S_i. Once the value of S_i is determined, the coordinates of the next collision point are readily found from

$$X_{i+1} = X_i + S_i\,(\sin\theta_i \cos\phi_i),$$

$$Y_{i+1} = Y_i + S_i\,(\sin\theta_i \sin\phi_i), \qquad (2.4)$$

$$Z_{i+1} = Z_i + S_i\,(\cos\theta_i).$$

Next, the type of interaction must be decided, again by sampling from the appropriate probability function. Let as for the sake of illustration assume that it is again a scattering event. The particle's energy and direction after scattering are obtained by sampling from the appropriate scattering function. Then the probability function is given by the Klein-Nishina theory K-N in the case of a photon when its scattering process is Compton scattering. Detailed procedure for sampling this important distribution is described in Sec. (2.3.6) and implemented in the program bremsstrahlung buildup factor BBF. From the appropriate scattering law, the new energy, E_i, can be determined [16]. Also the 'local' angles of the scattering event (θ_o, ϕ) can be determined (Fig. 2.3).

The final problem in this section is how to relate the particle's old direction (θ_{i-1}, ϕ_{i-1}) and the local angles (θ_o, ϕ) to determine the new direction of the particle (θ_i, ϕ_i). The convenient method of doing this is to consider the spherical triangle defined on the unit sphere by the intersection of the surface of the sphere with the particle's old and new directions and the direction of the reference polar axis of the system, the centre of the sphere being at the scattering point. This construction is shown in Fig. (2.4).

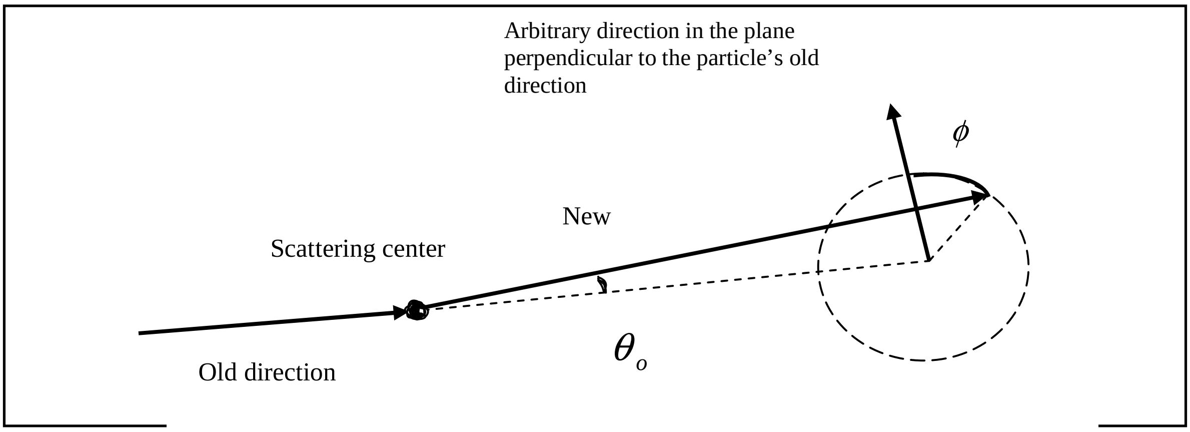

Fig. (2.3): Particle's 'local' angles of scattering. θ_o is the deflection angle, ϕ is the azimuthal angle. Usually the angle ϕ can be assumed to be randomly distributed in the range 0 to 2π

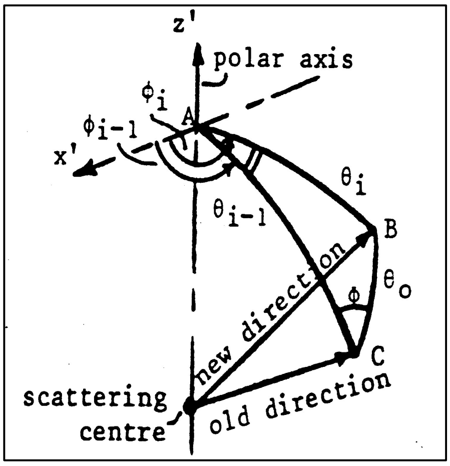

Fig. (2.4): Geometry of spherical triangle ABC defined on the unit sphere. The spherical triangle has sides $\theta_{i-1}, \theta_i, \theta_0$, and interangle ϕ and $(\phi_i - \phi_{i-1})$

Considering this spherical triangle, and by utilizing standard relationships which may be found in any text book on spherical trigonometry, it can be determined sines and cosines of θ_i and ϕ_i for use in Eq. (2.4), as follows:

From the law of cosines for spherical triangles we have

$$\cos\theta_i = \cos\theta_{i-1}\cos\theta_o + \sin\theta_{i-1}\sin\theta_o\cos\phi \qquad (2.5)$$

from which $\cos\theta_i$ can be determined. $\sin\theta_i$ can be obtained from the well-known identity

$$\sin^2\theta_i + \cos^2\theta_i = 1 \qquad (2.6)$$

Now, invoking the laws of sines, gives

$$\sin(\phi_i - \phi_{i-1}) = \frac{\sin\theta_o \sin\phi}{\sin\theta_i} \qquad (2.7)$$

Again the law of cosines enables us to write the following relationship

$$\cos\theta_o = \cos\theta_{i-1}\cos\theta_i + \sin\theta_{i-1}\sin\theta_i\cos(\phi_i - \phi_{i-1}) \qquad (2.8)$$

from which $\cos(\phi_i - \phi_{i-1})$ can be calculated.

The value of $\sin\phi_i$ and $\cos\phi_i$ can be deduced from the standard trigonometric formulae:

$$\sin\phi_i = \sin(\phi_i - \phi_{i-1})\cos\phi_{i-1} + \cos(\phi_i - \phi_{i-1})\sin\phi_{i-1} \qquad (2.9)$$

and

$$\cos\phi_i = \cos(\phi_i - \phi_{i-1})\cos\phi_{i-1} - \sin(\phi_i - \phi_{i-1})\sin\phi_{i-1} \qquad (2.10)$$

This then completes our description of a set of systematic procedures for constructing theoretical trajectories of the particles as they diffuse through the medium. Each particle is followed until its life history is terminated, for example by an absorption interaction or by escaping at the surface of a system. When one particle is terminated (and 'scored'); new particle is started from the source, and its career followed.

2.3.2 Selection techniques for given stochastic variables

A probability density function (pdf), $f(x)$ defined such the probability a randomly selected value of x is between x_1 and x_2 is given

$$P\{x_1 < x < x_2\} = \int_{x_1}^{x_2} f(x)dx. \tag{2.11}$$

from Fig. (2.5), we assumed that x may be any value between the limits a and b, either of which can be minus or plus infinity, respectively corresponding to $f(x)$, there is a cumulative probability distribution (cpd), F(x), defined by

$$F(x) \equiv \int_{a}^{x} f(x')dx'. \tag{2.12}$$

The probability that the randomly selected value of x is less than x_1, $P\{x<x_1\}$, is thus given by $F(x_1)$. It is easily seen that the function F(x) is monotonically increasing with x if $f(x)$ is properly normalized pdf between a and b, with F(a) = 0 and F(b) = 1 [9].

In the Monte Carlo process it is necessary to generate random values of some variable x (e.g., path length) which, from the physics of the particle interaction process, has some distribution $f(x)$; that is, a histogram of a large number of such randomly generated x values should approach $f(x)$ in shape.

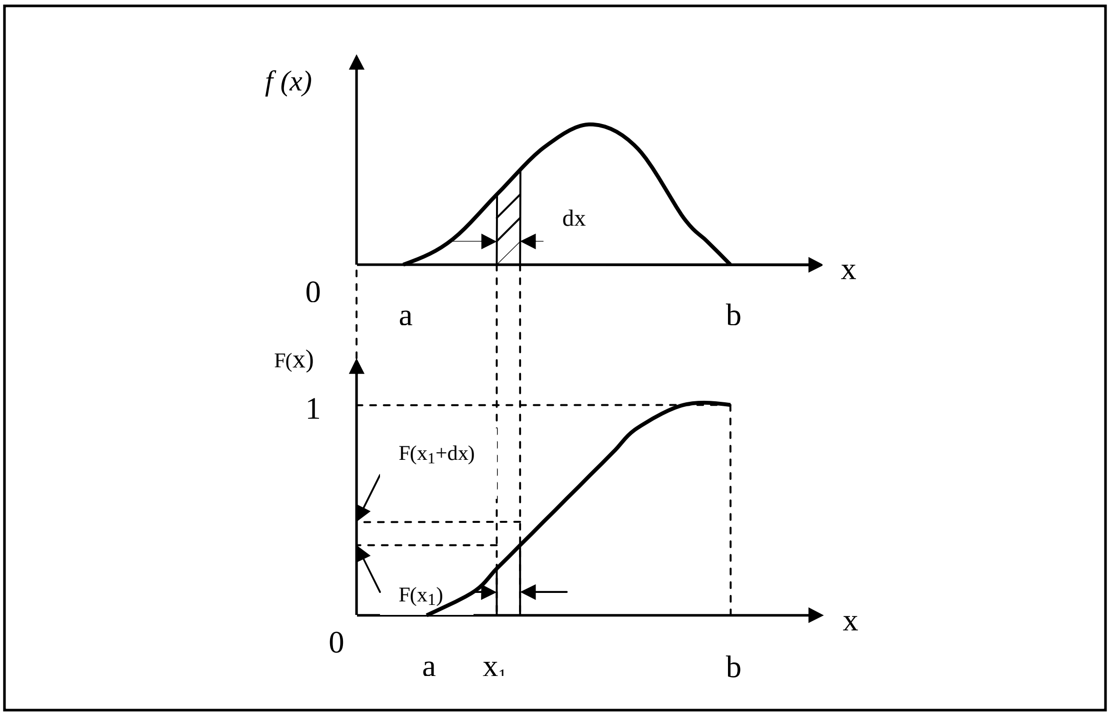

Fig. (2.5): Probability density function pdf and its corresponding cumulative probability distribution cpd [9].

2.3.3 Inverse Cumulative function Method

One of the most important methods for selecting random values for the variables relies on the fact that any random variable follow any given pdf can be expressed as a function of another random variable that is uniformly distributed between zero and 1 [16]. Then the desired value of x_i distributed as $f(x)$ is obtained by solving

$$F(x_i) = \xi_i \qquad (2.13)$$

that is,

$$x_i = F^{-1}(\xi_i) \qquad (2.14)$$

where, F^{-1} is the function obtained by solving Eq. (2.13) for x_i.

The validity of this or any other procedure can be established by showing that the probability of obtaining a value of x within the interval between some fixed value x and x + dx is $f(x)$ dx [i.e., x is distributed as $f(x)$]. For the cpd, the probability of x_i being between x and x + dx is the same as

$$P\{F(x) < \xi_i < F(x + dx)\} = F(x + dx) - F(x) \qquad (2.15)$$

$$= \int_{x}^{x+dx} f(x')dx' = f(x)dx \qquad (2.16)$$

As an example, consider a particle starting from a given interaction point (source point) in a certain direction through a medium of constant interaction properties (i.e., of constant μ). The values of the path lengths have a pdf [9].

$$f(s) = \mu e^{-\mu s} \qquad (2.17)$$

where s between 0 and s_i. Upon integration it is found that

$$F(s) = 1 - e^{-\mu s_i} \qquad (2.18)$$

Then this cpd is equal to 1- ξ_i in order to find a value of s_i, one finds that

$$s_i = \frac{-(\ln \xi_i)}{\mu} \qquad (2.19)$$

2.3.4 The Rejection Technique

This is a method of potentially great generality. In its basic form it applies to a pdf that is bounded and of finite range. It is best described by reference to the graph of a $f(x)$, as shown in Fig. (2.6).

The rejection technique proceeds as follows:

i. Select a random number, ξ_1, which is used to choose the value of x_1, namely, $\xi_1 L_1$.

ii. A second random number is chosen, gives the ordinate value, $\xi_2 L_2$.

iii. Test if the point ($\xi_1 L_1$, $\xi_2 L_2$) lies under the curve. If it does, then the value $x_1 = \xi_1 L_1$ is accepted, but if the point lies above the curve, the x_1 value is rejected and the process repeated with new values of ξ_1 and ξ_2. This procedure is repeated until an acceptable value of x_1 is obtained. now show that the values of x_1 obtained in this way are from the required distribution, $f(x)$. This follows, since the probability for accepting x_1 is

$$\int_0^{f(x_1)/L_2} d\xi_2 = \frac{f(x_1)}{L_2} \tag{2.20}$$

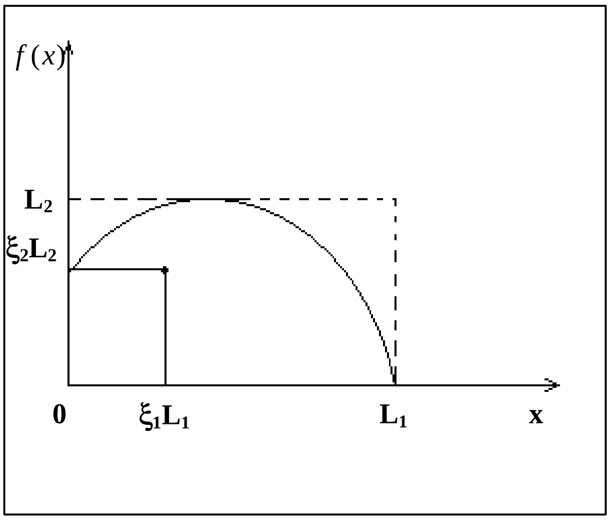

Fig. (2.6): Graph of pdf $f(x)$. L_2 is the upper bound of $f(x)$ and L_1 is the upper bound of x. In the illustrated case, $x_1 = \xi_1 L_1$, is accepted

i.e., the x_1 are from the given pdf . The efficiency of the technique can easily be assessed. The points ($\xi_1 L_1$, $\xi_2 L_2$) will be randomly distributed inside the rectangle indicated in Fig. (2.6).

Since the acceptance only occurs if the point lies under the curve, the probability of acceptance is [16].

$$\frac{1}{L_1 L_2}\int_0^{L_1} f(x)dx = \frac{1}{L_1 L_2} < 1 \tag{2.21}$$

Hence the rejection method is inefficient (i.e. too many random number pairs must be rejected) if, for a given range of x, L_2 is large. This would be the case if $f(x)$ is a 'peaked' function such as a Gaussian function.

It is clear from the preceding description that in its basic form the rejection method requires that the given pdf be bounded in magnitude, and that the range of x should also be bounded; i.e. both L_1 and L_2 should be finite. The method can be generalized to remove both these restrictions.

Suppose we begin by choosing an x from some pdf $f(x)$. Then test whether (y) the second random variable chosen from another pdf g(y) is less than a given h(x). If so, set x_1= x. This procedure may be summarised in the following flow diagram [16].

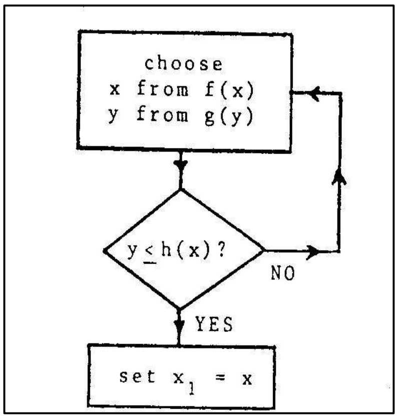

Fig. (2.7): Flow diagram for the generalized rejection sampling scheme.

The probability distribution of x_1 which arises from this procedure is

$$f(x_1) = \frac{f(x_1)\int_{-\infty}^{h(x_1)} g(y)dy}{\int_{-\infty}^{\infty} f(x_1)\int_{-\infty}^{h(x_1)} g(y)dydx_1} \tag{2.22}$$

now, if we choose g(y) to be the canonical distribution, i.e. g(y) = $0 \le y \le 1$, then,

$$f(x_1) = \frac{f(x_1)h(x_1)}{\int_{-\infty}^{\infty} f(x_1)h(x_1)dx_1} \tag{2.23}$$

where, $0 \le h(x_1) \le 1$.

2.3.5 The Klein-Nishina Formula

The relationship between photon deflection and energy loss for Compton scattering, assuming the electron to be free and stationary, is determined from the conservation laws of momentum and energy between the photon and recoiling electron. This relationship can be expressed as [16]:

$$\frac{E}{E_0} = \frac{1}{1+\left(\frac{E_0}{m_0c^2}\right)(1-\cos\theta)} \tag{2.24}$$

where E_0 and E are, respectively, the energies of the photon before and after the collision in MeV, m_0c^2 is the electron rest energy (0.511 MeV) and θ is the photon scattering angle.

In gamma ray transport calculations it is usually more convenient to use, instead of the energy variable, the wavelength of the photon in Compton units, namely, λ = 0.511/E, where E is the photon energy in MeV. The increase of the photon wavelength associated with a Compton scattering event is

$$\lambda - \lambda_0 = 1 - \cos\theta \qquad (2.25)$$

where λ_0 and λ are the wavelength of the photon before and after scattering respectively. Obviously, the maximum shift in wavelength is two Compton units, and occurs when the photon has been scattered through 180^0 (i.e., backscattered).

In Monte Carlo studies of photon transport it is necessary to know, after a Compton scattering, the angle of scattering θ, and the new value of the wavelength λ. The most commonly used procedure for obtaining these quantities is by sampling using the K-N pdf f(x) to find the value of x after a scattering, and then deduce (θ) from Eq. (2.25), where x represents λ / λ_0.

When the angular dependence of the scattered photon has been removed by integration, in terms of the dimensionless variable x, the Klein-Nishina differential cross section will take the following compact form [16]:

$$d\sigma = k\left(A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{x^3}\right)\,\mathrm{dx} \qquad 1 \le x \le 1 + 2\alpha_o \qquad (2.26)$$

where

$$\alpha_0 = \frac{1}{\lambda_0} = \frac{E_0}{0.511} \qquad (2.27)$$

$$x = \frac{\lambda}{\lambda_0} = \frac{E_0}{E} \qquad (2.28)$$

and E_0 is the incident photon energy in MeV. Additionally,

$$k = \pi \frac{r_e^2}{\alpha_0}$$
$$(r_e^2 = (\frac{\mu_0 e^2}{4\pi m_e})^2 = 7.94077 \times 10^{-26} cm^2) \quad (2.29)$$

where

$$A = \frac{1}{\alpha_0^2} \quad (2.30)$$

$$B = 1 - 2\frac{(\alpha_0 + 1)}{\alpha_0^2}, \; \mathrm{C} = \frac{(1 + 2\alpha_0)}{\alpha_0^2} \qquad \text{and } D = 1 \quad (2.31)$$

The total Compton cross section is obtained by integrating Eq. (2.26), hence,

$$\sigma_C = \int_1^{1+2\alpha_0} d\sigma$$
$$= K'\left[\frac{4}{\alpha_0} + (1 - \frac{l+\beta}{\alpha_0^2})\log_e \beta + \frac{\gamma}{2}\right] \text{ barn / electron,} \quad (2.32)$$

where

$K' = K \times 10^{24}$

$ß = 1 + \alpha_0$

and $\gamma = 1 - ß^{-2}$

It is clear from Eq.(2.32) that σ_c is only a function of the incident energy of the photon.

The pdf *f(x)* for Compton scattering process corresponds to $\frac{d\sigma / dx}{\sigma_c}$, and therfore it has the form

$$f(x) = \begin{cases} H(A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{x^3}) & \text{if } 1 \le x \le 1 + 2\alpha_0 \\ 0, & \text{else where} \end{cases} \quad (2.33)$$

where $H = K' / \sigma_c$

The pdf's for the variables E and x are simply related, as follows

$$f(E)dE = f(x)dx \quad (2.34)$$

From the preceding definitions of (σ_c) and f(x), it is immediately apparent that *f(x)* satisfies the basic requirement for any pdf, namely,

$$\int_{-\infty}^{\infty} f(x)dx = 1 \quad (2.35)$$

For completeness we need to mention at this point that to determine the photon's new direction in space, it is necessary to know, in addition to θ, the azimuthal angle ϕ. This is readily obtained by randomly sampling for any angle lying between 0^0 and 360^0 with equal likelihood, the implemention of this process presents no practical difficulty.

2.3.6 The Kahn Method for Sampling f(x)

Kahn method is the most widely used technique for sampling the K-N pdf. It does not rely on any approximation and is valid for any energy of the incident photon. But it permit repeated rejection before an acceptable value of x is found, consequently it is relatively expensive in its use of random numbers and computing time. This method is based upon the compositionrejection technique [16]. Kahn's procedure summarised in the flow diagram in Fig. (2.8).

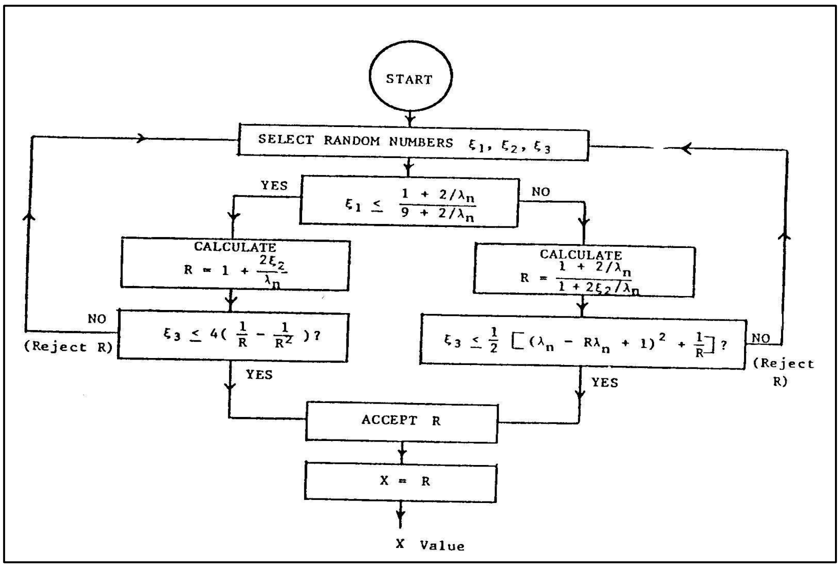

Fig. (2.8): Flow diagram for Kahn's procedure for sampling the K—N distribution to find the wavelength of a gamma photon after a Compton scattering.

λ_n **is photon wavelength before scattering (assumed known),**

λ_{n+1} **is photon wavelength after scattering and** $x = R = \frac{\lambda_{n+1}}{\lambda_n}$

The summarzation of this technique can be described as follows [16]:

Let us suppose that a given pdf is capable of being expressed in the form of a finite sum of terms, thus

$$f(x) = \text{constant} \times \sum_{i=1}^{N} \alpha_i g_i(x) h_i(x) \tag{2.36}$$

where

$$\sum_{i=1}^{N} \alpha_i = 1 \tag{2.37}$$

and for the range of x upon which f(x) is defined, g_i and h_i have the following properties:

$$g_i(x) \text{ is a pdf, and hence } \int g_i(x)dx = 1,$$

$$0 \le h_i(x) \le 1, \qquad \text{for i= 1,2,......, N.}$$

Hence one can sample from f(x) in the following manner. Select a value of i with probability α_i, and sample from $g_i(x)h_i(x)$ by the generalised rejection method, that is sample from the pdf $g_i(x)$ by the inverse cummulative function method and from $h_i(x)$ by the rejection method which are discussed previously [16]. For this method the random numbers are required in sets of three varibles (ξ_1, ξ_2, ξ_3).

2.4 Program Design

BBF is a computer program which solves the problem of gamma ray reflection and transmission using the basic computer program language.

A computer program “MONTRY” [46] was redesigned and improved in order to include the continuity of energy.

Using beta source, bremsstrahlung produced by striking beta particles the Lead shield at a high speed, then the continuous energy photons regarded as gamma ray continuous source uniformly incident (i.e., with fractional weight) on the left hand face of an infinite plane homogeneous slab of shielding material of variable thickness.

The method used is the Monte Carlo technique, in which the life histories of a large number of particles (photons) are followed using random sampling techniques to sample the probability laws that describe the particle's behaviour, and to trace out step by step the particle's random walk through the medium. In the particular form of the Monte Carlo method used here, there is a close analogy between the physical particles and the mathematical particles followed by the program. The only sophistication employed is the concept of survival weights. Thus one does not allow absorption of the photons as such; all collisions are forced to be Compton scatterings. The effect of absorption is accounted for by modifying the weight of the particle after each collision. That is the particular weight after a collision is the weight before collision multiplied by the survival ratio, $\mu_C(E) / \mu (E)$, which is of course the probability that a collision will be a Compton scattering. It is important to realise that in this, the basic form of the program, the pair production event is regarded as a purely absorptive interaction, the contribution from the resulting annihilation gamma rays is not considered. In this version of the program, the history of each particle is followed until the particle either escapes from the system, or due to successive scatters, its energy drops below some present minimum; in either event a 'new' photon is started with an initial weight (i.e., probability, p) of unity.

The principle quantities computed and the output by the program are:

- The buildup factor: This is essentially a measure of how the transmitted photons are enhanced by Compton scattering.
- The spectrum of photon currents at the left hand and right hand boundaries of the system. That is the number of photons which cross the surfaces of the system with energies lying within present energy intervals.

A special feature of the program is that it can consider concurrently a number of different slab thicknesses of the same material. This is a useful facility, as one usually wishes to know buildup factors for various thicknesses of slab, and it is more economical in computer time to obtain them simultaneously. Another instructive feature of the program is that a record is kept of the number of particles which suffer each of the possible fates: this enables a check to be made that all particles started from the source are correctly accounted for. The program has been deliberately written in a modular form: each subroutine or sub-program can readily be identified with a distinctive physical event in the photon's career. Possibly the greatest single aid to understanding the Monte Carlo method is provided by the flow diagram for BBF program which is shown in Fig. (2.10).

2.4.1 The essential subroutines used in the BBF program

a. MONTE

This is the segment of the program in which the calculation commences and ends. The principle function of BBF is to ensure that the main sequence of the scheme is correctly followed. In furtherance of this, MONTE monitors the number of new particles started.

b. INPT 1, INPT 2

These subroutines are concerned with reading in, the basic data and control parameters required by the program. Immediately after being read in, the data is printed out to enable the user to check that the machine did in fact receive the information intended. The data input can be separated into two sections, the core data which read in subroutine INPT 1, followed by the shield data which read in subroutine INPT 2.

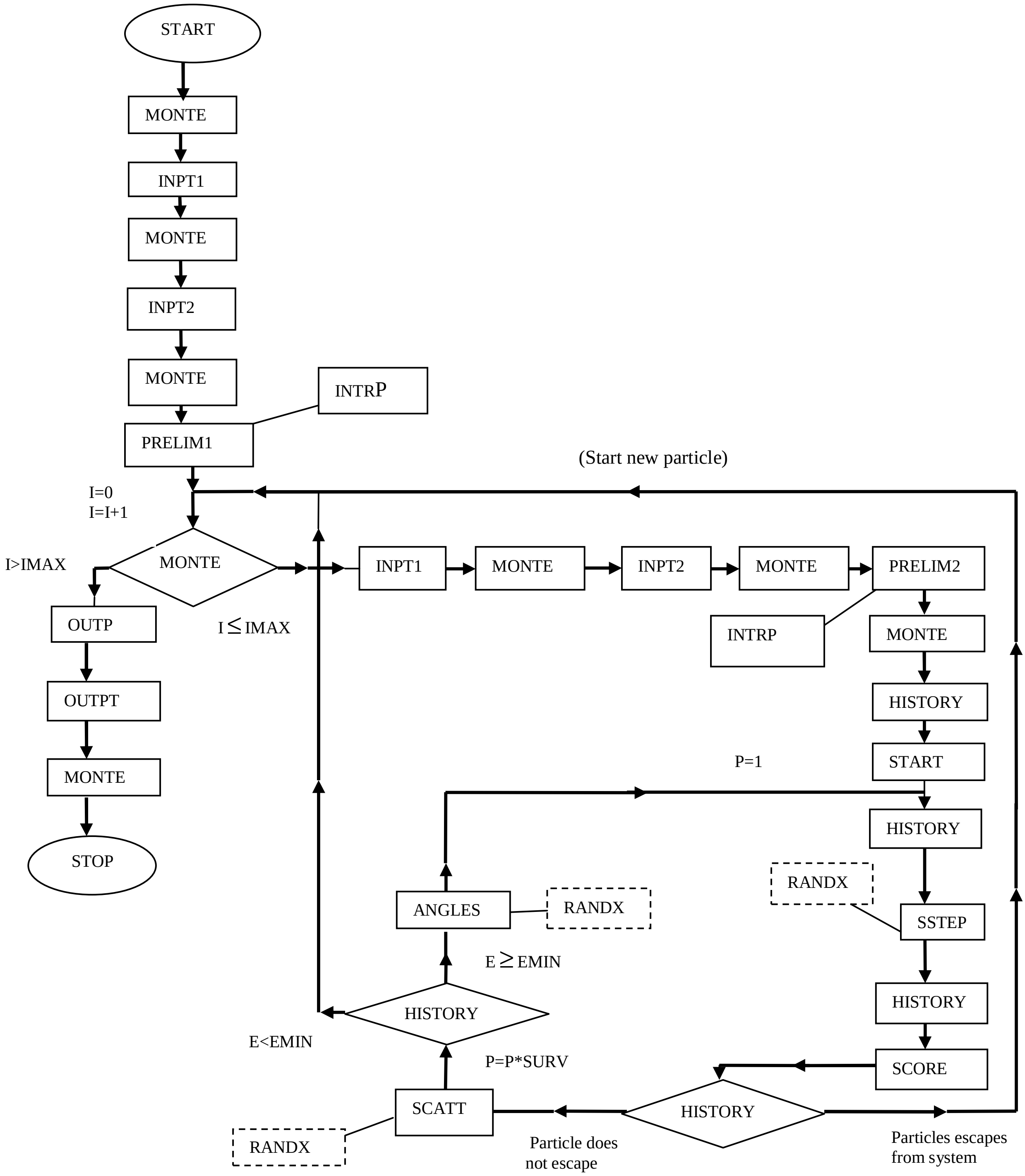

Fig. (2.10): Flow diagram of the bremsstrahlung buildup factor computer program BBF. 'RANDX' is a routine which provides random numbers, I is the current particle number, P is the statistical weight of the particle and E the energy.

c. PRELIM 1, PRELIM 2

Because of the large number of particles followed in a typical Monte Carlo calculation, it is obviously desirable to minimize the amount of computation associated with each stage of a photon's history. The efficiency of the corresponding program is greatly increased by first performing certain 'once for all' type calculations and storing the results in a convenient form for subsequent usage. It is the task of this subroutine to carry out these preparatory calculations in "BBF".

It is unrealistic to presume that the cross sections data can be input in such detail that, whatever the photon energy after a Compton scattering, there will be a corresponding value of, say, the linear attenuation coefficient immediately available from the basic data table. The procedure that we have adopted to deal with this problem is to read in the basic data for a widely spaced energy mesh, and have the program construct its own detailed data tables for a fine wavelength mesh by interpolation in the basic data. The data corresponding to any photon wavelength (in the permissible range) is then efficiently obtained from the constructed 'look up table'. The major function of PRELIM1 is to set up detailed cross section data tables. For this purpose it requires an interpolation procedure, which is implemented in the auxiliary subroutine "INTRP"

Other tables constructed in PRELIM1 are

- Values of the sine and cosine of an angle in degrees.
- The energy mesh tables for storing the photon spectra at exit from the system.

PRELIM2 repeat the same function of PRELIM1 but the difference between them is the PRELIM2 acts at continuous energies, which retain by the total results for each sub-energy as a single energy and accumulate them together under the maximum energy without removing the previous results for each energy. The PRELIM1 subroutine deals with the maximum energy only.

d. HISTORY

The particular path taken by each photon through the slab is controlled by this subroutine. The route followed depends on various decisions taken by HISTORY; these decisions depend principally on the photon's energy and whether it escapes from the system. In particular, if the photon does not escape from the system, it must be made to scatter again. To escape completely from the system, a photon traveling to the right must penetrate the thickest slab under consideration.

e. START

Those variables which define a photon's state are given their initial values here. Thus the particular type of source considered is imposed in this subroutine.

f. STEP

This subroutine performs three essential tasks:

(i) It establishes, after a collision, the position of the photon in the fine mesh wavelength table

(ii) It selects, at random, the path length of the photon to the next (postulated) collision points (by means of Eq. (2.19)).

(iii) It computes the coordinates of the next (postulated) collision point by using Eq. (2.4).

g. SCATT

In this subroutine the new energy after a Compton scattering is selected by sampling from the Klein-Nishina distribution. The particular sampling technique used is that of Kahn, the details of which are given in Sec.(2.3.5) Once the new energy is found, the Compton angle of scattering θ, (actually $\cos\theta$) is readily determined. Another function of SCATT is to modify the particle weight, p, to its new value after a scattering.

h. ANGLE

This subroutine performs the vital task of determining the variables(θ_{n+1}, ϕ_{n+1}) which define the photon's new direction after a Compton scattering. This is done, as explained in Sec. (2.3.1), solving standard spherical trigonometric relationships for the spherical triangle depicted in Fig. (2.11) The trigonometric relationships can be used to obtain the new values, because the program has the following known parameters

(a) the 'old' values of θ,φ (namely, θ_n, φ_n)

(b) the latest scattering angle (θ)

(c) the azimuthal angle of scattering (φ) which is assumed to be distributed uniformly within the range $0^0 - 360^0$.

i.SCORE

If a photon penetrates a slab thickness, its contribution to the score at this is the essential purpose of SCORE.

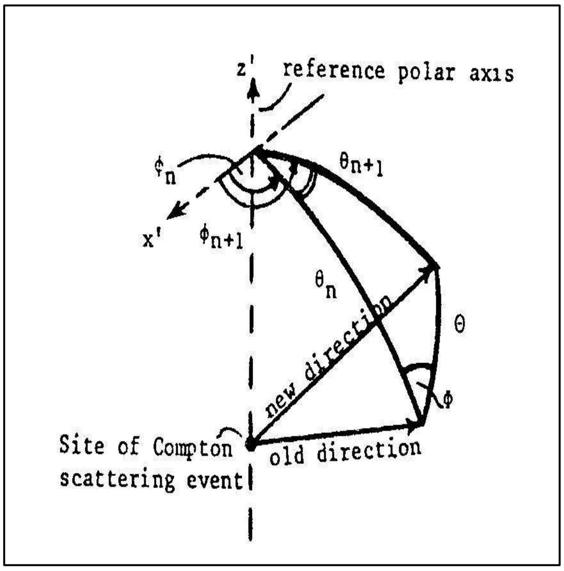

Fig.(2.11): The spherical triangle formed by the photon's previous direction and its new direction after a Compton scattering. θ the Compton angle of scattering.

Therefore, this subroutine must obtain answers to the following questions related respect to the next (postulated) collision point.

(a) Does the photon crosses an interface, if so; the score at that interface must be suitably adjusted.

(b) Can the photon still scatter again in the system, bearing in mind that a number of slab thickness are being simultaneously considered, or does the photon escape entirely.

To assist SCORE in these considerations it is important to ascertain immediately in the entry of the subroutine on the direction in which the photon is traveling, i.e. to the right or to the left.

The scorings photons are classified twice: (1) according to their z-value when they cross a boundary, (2) according to their position in the energy spectrum table. This information is passed to subroutine OUTPUT for processing. Recording a particle's contribution to the quantities of interest is an important part of any Monte Carlo treatment. The flow diagram for the subroutine SCORE is shown in Fig. (2.12)

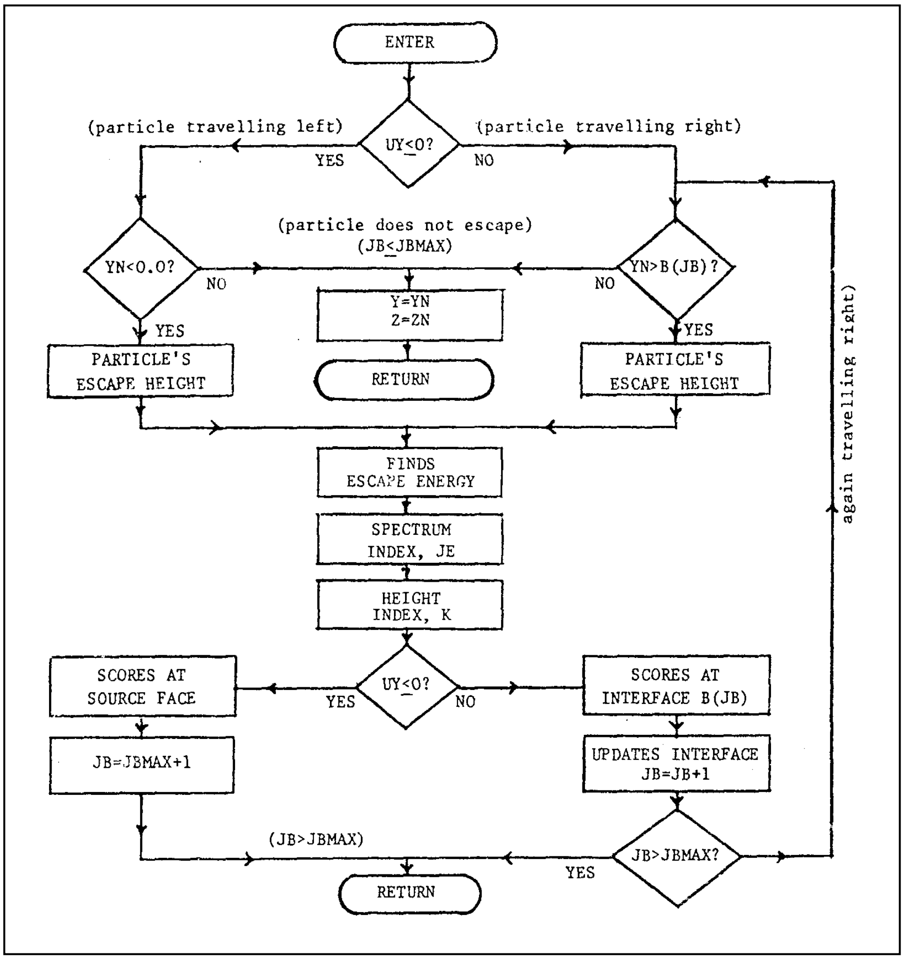

Figure (2.12): Flow diagram for subroutine SCORE in "BBF" program.

J. OUTP

When the requested maximum number of photon life histories has been simulated, the accumulated data is processed in subroutine OUTPUT, and the results printed out.

The output sections in their order of appearance are:

(i) The basic input data which are reproduced for reference.

(ii) A detailed description of the life history of the first 24-photons is listed (optional).

(iii) The results of the computation are given.

Thus, the third section of the output, begins with a list of some of the basic parameters which repeated for convenience. Next, the fractions of the photons that suffer the various possible fates are given: a necessary condition for a successful program run is that these fractions should sum to unity. Finally, the main results for each slab thickness are given. In interpreting this part of the output it is important to note that:

(a) The calculated standard deviation for the bremsstrahlung buildup factor BBF are given in the brackets adjacent to the respective parameters;

(b) The dose rate at each slab face, in rem/h, is that due to a source strength of IMAX photons per cm^2 per second uniformly incident upon the source face of the slab;

(c) The photon spectra are normalized to unit particle crossing the particular boundary.

k. INTRP

This is an auxiliary subroutine called “Prelim routine”. The Lagrange Interpolation Polynomial method was applied on the basic data requird for constructing the detailed data table of the attenuation absorption cofficients(μ).

Particular application we generally require to interpolate in two-variables (energy E and attenuation absorption coefficient μ), the interpolation process can be performed in two stages. This “double interpolation procedure is readily understood when considering of Fig. (2.13):

Stage 1:

By interpolating with $E=E_{ph}$, for successive fixed values of μ, i.e., μ_1, μ_2,

Stage 2:

The data represented by these new points are then interpolated for μ, $\bar{\mu}$ to find $\bar{\alpha}$.

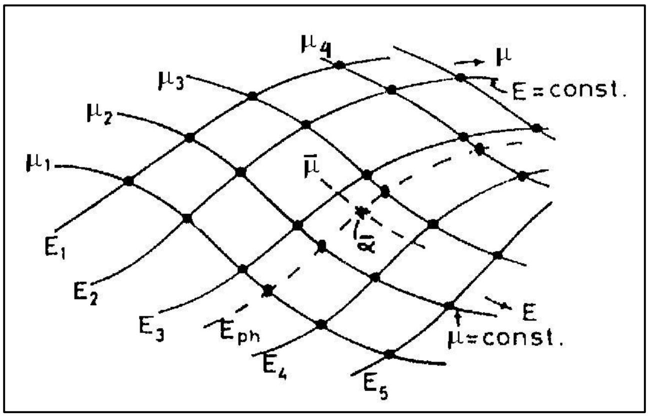

Fig.(2. 13): The surface formed by the data points The points (o) are formed by interpolating along each curve μ=constant. The curve joined the (o) points is interpolated with $\mu=\bar{\mu}$,to find $\bar{\alpha}$.

k.1. The Lagrange Interpolation Polynomial

In this section, we consider a formula for the polynomial of minimum degree, which takes specific values at a given set of points. We have already observed that there is a unique polynomial of degree at most one whose graph goes through two specified points, and this will turn out to be part of the general pattern. There is a unique polynomial of degree at most N which interpolates N+1 given values. There are many different ways of establishing this fact.

Suppose that the values {f (x_i): i= 0, 1, N} are given and let *p* be the minimal-degree interpolation polynomial. Suppose that p is of degree M; in terms of the standard basis, *p* can be written in the form

$$P(x) = a_M x^M + a_{M-1} x^{M-1} + ...+ a_1 x + a_0 \tag{2.38}$$

We require that (p) satisfy the interpolation conditions

$$P(x_i) = f(x_i) \quad (i = 0, 1, \ldots\ldots, N) \tag{2.39}$$

Substituting Eq. (2.39) into Eq.(2.38) for each x_i in turn, we obtain the system of equations.

$$\begin{aligned}
a_M x_0^M + a_{M-1} x_0^{M-1} + \ldots + a_1 x_0 + a_0 &= f(x_0) \\
a_M x_1^M + a_{M-1} x_1^{M-1} + \ldots + a_1 x_1 + a_0 &= f(x_1) \\
\vdots \qquad \vdots \qquad\qquad \vdots \quad \vdots \quad \vdots \quad &\quad \vdots \\
a_M x_N^M + a_{M-1} x_N^{M-1} + \ldots + a_1 x_N + a_0 &= f(x_N)
\end{aligned} \tag{2.40}$$

which can be written in matrix form, reversing the order of the terms on the left-hand sides of Eq.(2.40), as

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \ldots & x_0^M \\ 1 & \ldots & \ldots & \ldots \ \ldots & x_1^M \\ \ldots. & \ldots. & \ldots. & \ldots. & \ldots \\ 1 & x_N & x_N^2 & \ldots. & x_N^M \end{bmatrix} * \begin{bmatrix} a_0 \\ a_1 \\ \ldots \\ a_M \end{bmatrix} = \begin{bmatrix} f(x_0) \\ f(x_1) \\ \ldots \\ f(x_N) \end{bmatrix} \tag{2.41}$$

This is a system of (N+1) equations in the (M+1) unknown parameters (a_0, $a_{1,\ldots.,}$ a_M). In general, we except such a system to have no solution if (N > M). If, on the other hand, (M > N), then we do not except the solution to be unique. This system will be a unique solution if (M=N) and if the above matrix of coefficients is nonsingular [53].

m. RANDX

This is the auxiliary function sub-program which provides the random numbers from the canonical distribution that are essential to the Monte Carlo method. The actual sequence of the generated random numbers depends entirely on the value of QRAND data, and on the particular algorithm employed for generating the random numbers [16].

2.5 Test results for the present BBF program

In this section the present BBF program was tested. The input and output data are expressed for 2 cm water shield thickness, effective atomic number 7.89 ≈ 8 for 1000000 particles studied, minimum energy 0.019 MeV and maximum bremsstrahlung energy 2.2 MeV.

RUN	IMAX	EMAX	EMIN	DENSITY
8	1000000	2.2	.019	1

FR. ESC. L. = 1.654977E-03 FR. ESC. R. = .7356824 FR. ABS. = .2627106

FR. Less than E. Cut = 3.090109E-32 sum = 1.00004 Width of slab = 2 cm

Histogram of Particle Current

Energy (MeV)		Spectrum R.	Spectrum L.
1 -	2.2	5.433169E-06	0
2 -	2.15	0	0
3 -	2.1	0	0
4 -	2.05	0	0
5 -	2	0	0
6 -	1.95	6.791461E-06	0
7 -	1.9	5.433169E-06	0

8 -	1.85	6.815961E-06	0
9 -	1.8	6.791461E-06	0
10 -	1.75	1.222463E-05	0
11 -	1.7	1.901609E-05	0
12 -	1.65	2.580755E-05	0
13 -	1.6	2.444926E-05	0
14 -	1.55	3.667389E-05	0
15 -	1.5	4.754023E-05	0
16 -	1.45	5.025681E-05	0
17 -	1.4	1.222463E-04	0
18 -	1.35	3.803218E-05	0
19 -	1.3	8.288238E-05	0
20 -	1.25	1.290378E-04	0
21 -	1.2	1.440321E-04	0
22 -	1.15	2.798865E-04	0
23 -	1.1	4.157138E-04	0
24 -	1.05	7.227393E-04	0
25 -	1	3.492057E-04	0
26 -	.95	8.085909E-04	0
27 -	.9	3.603286E-04	0
28 -	.85	4.718616E-04	0
29 -	.8	6.96072E-04	0
30 -	.75	2.990415E-03	0
31 -	.7	3.90226E-03	0
32 -	.65	3.086478E-03	0
33 -	.6	3.312451E-03	0
34 -	.55	4.536806E-03	0
35 -	.5	2.096356E-02	0
36 -	.45	1.294438E-02	0
37 -	.4	2.127371E-02	0
38 -	.35	1.312362E-02	0
39 -	.3	4.277361E-02	1.401054E-04
40 -	.25	.0103403	1.381567E-03
41 -	.2	3.992816E-02	1.227708E-02
42 -	.15	8.144335E-02	4.578224E-02
43 -	.1	.7319269	.0930927
44 -	.05	2.585999E-03	.8473256
45 -	0	4.577997E-11	6.812418E-07

Bremsstrahlung buildup factor (BBUF) = 1.007671 ∓ 0.000826

Contents

الاسم الثلاثي: ميلاد جذلان علي

عنوان السكن: الغزالية محلة ٥٦٥ زقاق ٦١ دار٧

الايميل : milad-alansari@hotmail.com

تاريخ المناقشة: ٢٠٠٦/٥/٢٥

اسم الاطروحة : **محاكاة عامل التراكم لاشعة الكبح الناتجة من الامتصاص الكلي لاشعة بيتا**

Examination Committee Certification

We certify that we have read the thesis entitled **"Simulation of Buildup Factor for Bremsstrahlung Produced by Complete Absorption of Beta Rays"** and as an Examining Committee, examined the student

Milad Jathlan Ali Al-Ansari

in its contents, and that in our opinion it is adequate as standard of thesis, with Excellent standing of Degree of Master of Science in Physics

Signature:
Name: **Dr. Hazim L. Mansur**
Title: Professor (Chairman)
Date: / 6/2006

Signature:
Name: **Dr. Adel K. Hamodi**
Title: Assist. Professor (Member)
Date: / 6 /2006

Signature:
Name: **Dr. Nada F. Tawfiq**
Title: Assist. Professor (Member)
Date: / 6 /2006

Signature:
Name: **Dr. Mazin M. Elias**
Title: Professor (Supervisor)
Date: / 6/2006

Approved by the University Committee of Postgraduate Studies

Signature:
Name: **Dr. Laith A. Al-Ani**
(Dean of the College of Science)
Date: / 6/2006

References

[1] E. Duffy, Project Report, University of Ireland, Maynooth, 2002

[2] Health Physics Society, 2003 "Radiation Effects"
Retrieved on June 6, 2005 from:
http://www.hps.org/publicinformation / ate/q 1474.html.

[3] G. F. Knoll, "**Radiation Detection and Measurement**", John Wily &Sons, 2nd Ed., N. Y., 1989.

[4] C. Jensen, "**Controversy and Consensus**", Birkhauser, Berlin, 2000.

[5] E. M. A. Hussein, "**Handbook on Radiation Probing, Gauging, Imaging and Analysis**", Springer, N. Y. , 2003.

[6] E. Haug and W. Nakel, "**The Elementary Process of Bremsstrahlung**", World Scientific Lecture Notes in Physics, Danvers, 2004.

[7] N. Tsoulfanidis, "**Measurement and Detection of Radiation**", McGraw-Hill, N. Y. , 1983.

[8] M. F. L'annunziata, "**Radioactivity Analysis**", Elsevier, 2nd Ed., California, 2003.

[9] A. Chilton; "**Principles of radiation shielding**"; Prentice Hall, Inc., London, 1984.

[10] F. Wissmann, "**Compton Scattering**" Springer, N. Y., 2004.

[11] R. A. Baily, H. M. Clark,J.P. Ferris,S. Krause and R. Strong, "**Chemistry of the Environment**", Elsevier, 2nd Ed., California, 2002.

[12] J. T. Bushberg, J. N. Seibert, E. M. Leidholdt and J. M. Boone, "**The Essential Physics of Medical Imagining**", Williams &Wilkins, 2nd Ed., Feladephia, 2002.

[13] J. Sabol and P. H. Weng, "**Introduction to Radiation Protection Dosimetry**", World Scientific, N. Y. , 1995.

[14] F. H. Attix, "**Introduction to Radiological Physics and Radiation Dosimetry**" Wiley-VCH, Weinheim, 2004.

[15] A.A. Kadhum, Ph.D Thesis, College of Science, University of Baghdad, 2004.

[16] J. Wood; "**Computational Methods in Reactor Shielding**", Pergamon Press, London, 1981.

[17] K. Shure; Nucl. Sci. and Eng., 69 (1979) 432.

[18] A.B. Chilton, C. M. Eisenhauer and G.L.Simmons, Nucl. Sci. and Eng., 73 (1980) 97.

[19] M. A. Capo; "**Polynomial Approximation of Gamma Ray Buildup Factors for Point Isotropic Source**", Appex-510 (1958).

[20] D. L. Border, Soviet J. of Atomic Energy, 12 (1962) 30.

[21] G.R. White; Phys. Rev., 80 (1950) 154.

[22] W. R. Dixon, Phys. Rev. , 85 (1952)498.

[23] C. D. Zerby and H. S. Moran; J. App. Phys., 34 (1963) 2445.

[24] J. H. Hubbell, "Journal of Research of the National Bureau of Standards- C. Engineering and Instrumentation", 67C (1963) 291.

[25] E.E. Morris and A. B. Chilton; Nucl. Sci. And Eng., 40 (1970) 128.

[26] E. E. Morris, A. B. Chilton and A. F. Vetter; Nucl. Sci. And Eng., 56 (1975) 171.

[27] A. M. Eisenhaure and G. L. Simmon; Nucl. Sci. and Eng., 56 (1975) 263.

[28] K. Shure and O. J. Wallace; Nucl. Sci. and Eng., 62 (1977) 736.

[29] M. Metghalchi; Nucl. Sci. and Eng., 67 (1978) 341.

[30] A. B. Chilton; Nucl. Sci. and Eng., 69 (1979) 436.

[31] A. Foderaro and R. J. Hall; Nucl. Sci. and Eng., 74 (1980) 78.

[32] H. Hirayama; Nucl. Tech., 77 (1987) 60.

[33] G. B. Bishop; Nucl. Ins. Meth., A225 (1987) 165.

[34] G. Herbold , G. Hartmann, H Treuer and W J Lorenz; 33 No.9 (1988)1037.

[35] E. M. Fourine and A.B. Chilton, Nucl. Sci. and Eng., 76(1989)66.

[36] L. A. Al- Ani; M. Sc. Thesis, Baghdad University, 1989.

[37] K. H. Hattif; Ph. D. Thesis, Baghdad University; 1994.

[38] G. C. Bakos; Ann. Nucl. Eng., 21(1994)651.

[39] G. C. Bakos; Ann. Nucl. Eng., 22, No. 5(1995)297.

[40] H. Al- Ammar M. Sc. Thesis, Baghdad University, 1996.

[41] G. S. Sidhu, P. S. Singh and G. S. Mudahar, 20 No. 1 (2000) 53.

[42] K. Al- Baite, M. Sc. Thesis, University of Babylon, 2001.

[43] A. Al – Samaraey, M. Sc. Thesis, Baghdad University, 2002.

[44] A. Shimizu; Nucl. Sci. And Tech. , 39, No. 5, (2002) 477.

[45] A. Shimizu and H. Hirayama; Nucl. Sci. Tech. , 40, No. 4, (2003) 192.

[46] A. A. Al- Rawi, M. Sc. Thesis, Al- Nahrain University, 2004.

[47] A. Shimizu, T. Onda and Y. Sakamoto; Nucl. Sci. Tech. , 41 No. 4, (2004) 413.

[48] M. M. Elias, A. A. Abdulla and A. A. Kadhum, *to be puplished*, 2006.

[49] R. Y. Rubinstein; “**Simulation and the Monte Carlo Method”**, John Wiley & Sons, N. Y. , 1981.

[50] T. J. Naylor, J. L. Balintfy, D. S. Burdick and K. Chu, “**Computer Simulation Techniques**”, Wiley, N. Y., 1966.

[51] I. M. Sobol; “**The Monte Carlo Method**”, the University of Chicago press, 1974.

[52] M. Dapor, “**Electron Beam Interactions with Solids**”, Springer, N. Y., 2003.

[53] J. L. Buchanan, “**Numerical Method and analysis**”, McGraw-Hill, Inc., Amsterdam, 1992.

[54] XCOM, 2004 “Partial Interaction Coefficients and Total Attenuation Coefficients”
Retrieved on March 15, 2005 from:
http://physics.nist.gov/cgi-bin/xcom/xcom3_1.

الخلاصة

تم تطبيق المحاكاة بطريقة مونتي كارلو لدراسة عامل التراكم لاشعة الكبح المتولدة من الامتصاص الكلي لجسيمات بيتا، حيث لم يسبق دراسته من قبل رغم اهميتها الكبيرة في الدراسات المتعلقة بحساب الجرعة الاشعاعية.

حسب عامل تراكم اشعة الكبح لاسماك مختلفة من الماء و الكونكريت و الالمنيوم و القصدير و الرصاص لمصدر السترونتيوم/ اتريوم $^{90}Sr/^{90}Y$ لانتاج اشعة بيتا بطاقة عظمى مقدارها ٢,٢ مليون الكترون فولت، وقد تم ذلك من خلال تصميم وتنفيذ برنامج حاسوبي للمحاكاة اطلقنا عليه تسمية "BBF" لتمثيل حلول المسائل التقليدية (الكلاسيكية) لانعكاسات و عبور اشعة الكبح. حيث يتظمن البرنامج دوال لمتابعة حياة او مسيرة الفوتونات نظريا باعتبار نظرية الاحتمالات التي تصف السلوك الحقيقي للفوتونات.

في هذا البحث تم دراسة عدد من المعاملات المتعلقة بتصميم البرنامج و المسماة بعوامل المحاكاة وكذلك دراسة المعاملات المتعلقة بمادة الدرع و المسماة بالعوامل الفيزيائية.

تم حساب عامل تراكم اشعة الكبح لدرع مفرد و دلت نتائج المحاكاة على ما يلي:

- يتزايد عامل التراكم لاشعة الكبح مع زيادة سمك الدرع.
- عامل تراكم اشعة الكبح للدروع ذات العدد الذري الواطئ اقل منها في الدروع ذات العدد الذري العالي لنفس طاقة المصدر.
- تم استنتاج علاقة شبه تجريبية لعامل تراكم اشعة الكبح كدالة لكل من العدد الذري Z والسمك X لمادة الدرع كالاتي:

$$\mathbf{BBUF = 1 + (a_1\, Z + a_2)\, X^{(a_3 Z + a_4)}}$$

حيث a_4, a_3, a_2, a_1 هي معاملات الملائمة و تعتمد على التوزيع الطاقي لاشعة الكبح. لمصدر كبح السترونتيوم / اتريوم فان قيم هذه المعاملات هي :

$$a_1 = 0.0007,\ a_2 = 0.0022,\ a_3 = 0.0072,\ a_4 = 0.4204$$

Republic of IRAQ
Ministry of Higher Education
& Scientific Research
Al-Nahrain University /College of Science

Simulation of Buildup Factor for Bremsstrahlung Produced by Complete Absorption of Beta Rays

A Thesis
Submitted to the College of Science of
Al-Nahrain University in Partial Fulfillment of
the Requirements for the Degree of

Master of Science in

Physics

by

Milad Jathlan Ali Al- Ansari
(B.Sc.2001)

March 2006 A.D. Safar 1427 A.H.

جمهورية العراق
وزارة التعليم العالي و البحث العلمي
جامعة النهرين /كلية العلوم

محاكاة عامل التراكم لاشعة الكبح الناتجة من الامتصاص الكلي لاشعة بيتا

رسالة
مقدمة إلى كلية العلوم في جامعة النهرين
كجزء من متطلبات نيل درجة الماجستير علوم في

الفيزياء

من قبل

ميلاد جذلان علي الانصاري
(بكالوريوس ٢٠٠١)

صفر ١٤٢٧
اذار ٢٠٠٦