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“ Linear Programming Techniques for Network Project Management “

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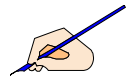
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Linear Programming Techniques for Network Project Management

A Thesis

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of Science in Mathematics*

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Abstract

In this work, Linear Programming Problems have been implemented to build four linear models for projects management. An Interior – Point Method has been implemented to solve such linear models, instead of using the usual techniques "Simplex Method", by implementing the "what's Best 9.0 " software, and obtaining the critical path in minimum completion time, minimum crashing cost and optimal total (direct & indirect) costs for a simple real project. Then we are verified the results obtained by implementing " Project 2000 " software to construct the project network and obtain the same critical path.

Finally, the Programming Evaluation Review Technique (PERT) has been used, to find the probabilities of completing the project.

الإهداء

إلى .. مثلي الأعلى ... وقدوتي في الحياة ... إلى معلمي الأول ... والنور الذي

بيضيء لي دربي ... إلى نبع المحبة الصافي ...

والذي الحبيب

إلى .. من صبرت وسهرت من أجل راحتي ... وغمرتني بالحب والحنان

وأشعرتني بالراحة والأمان ... إلى فيض الحنان الدافق ...

والدتي الحنون

إلى .. أصدقاء عمري ... وأحباء قلبي ...

إخوتي وأخواتي

الأعزاء

مع شكرى وتقديرى ...

ابلاف

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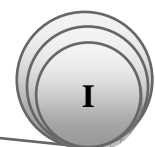
Introduction

Management is continually seeking new and better control techniques to cope with the complexities, masses of data, and tight deadlines that are characteristic of highly competitive industries. Managers also want better methods for presenting technical and cost data to customers. One of these techniques are, scheduling techniques help as to achieve these goals. The most common of scheduling techniques are:-

- (1) Gantt or Bar chart.
- (2) Milestone charts.
- (3) Network such as:-
 - Program Evaluation and Review Technique (PERT).
 - Critical Path Method (CPM).
 - Linear Programming Problem (LPP).

The advantages of network scheduling come out in helping management decisions on how to use its resources to achieve time, cost goals, evaluating alternative by answering such questions as how time delays will influence project completion? Where slack exists between elements? What elements are crucial to meet the completion data? Providing the report information, identifying the longest path or critical paths and risk analysis [11].

Gantt chart: One of the oldest and still one of the most useful methods of presenting schedule information is the Gantt chart developed around 1917 Henry L. Gantt, a pioneer in the field of scientific management. The Gantt chart shows planned and actual progress for a number of tasks displayed against a horizontal time scale. It is a particularly effective and easy – to – read method of indicating the actual current status for each of a set of tasks



compared to the planned progress for each item of the set. The charts usually contain a number of special symbols to designate or highlight items of special concern to the situation being charted. There are several advantages to the use of Gantt charts. First, even though they may contain a great deal of information, they are easily understood. While they do require frequent updating (as does any scheduling / control device), they are easy to maintain as long as task requirements are not changed or major alterations of the schedule are not made. Gantt charts, however, have a serious weakness. If a project is complex with a large set of activities, it may be very difficult to follow multiple activity paths through the project [14].

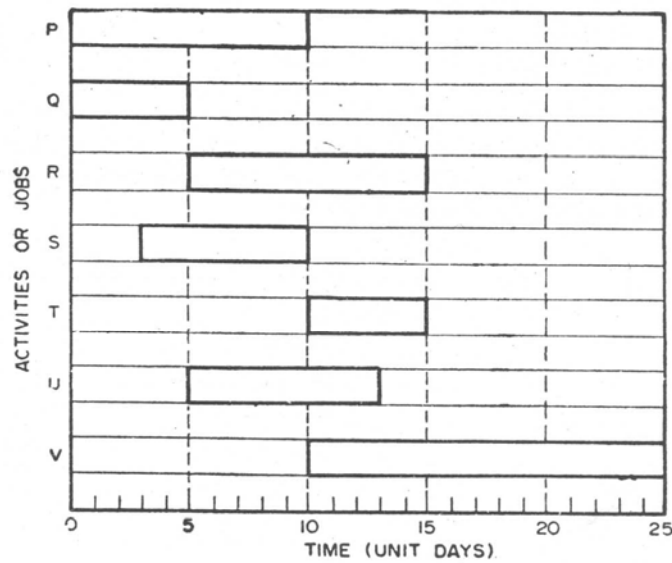


Fig. (1) Gantt chart.

Milestone charts: Milestone chart is modification over the original Gantt chart. Milestones are key events of a main activity represented by a bar: these are specific points in time which mark the completion of certain portions of the main activities. We have already seen that when a particular activity, represented by a bar on a bar –chart is very long, the details lack. If, however, the activity is broken or sub divided into a number of sub – activities, each one of which can be easily recognized during the progress of the project, controlling can be easily done and inter relationships between other similar activities can be easily established. The beginning and end of these sub – divided activities or tasks are termed as milestones [1].

Network: representation of projects: treat the project as a set of related activities that can be displayed visually in a network diagram consisting of nodes (circles) and arcs (arrows) that depict the relationships between activities. This technique is based on the basic characteristics of all projects that all work must be done in well – defined steps. The network techniques are called by various names such as PERT, CPM, which are the major network system. PERT stands for " Program Evaluation and Review Technique " which can be applied to any field requiring planning, controlling, and integrated work efforts. PERT was originally developed in 1958 and 1959 to meet the needs of "age of massive engineering " where other techniques are inapplicable. The special project office of U.S. Navy, concerned with performance trends on large military development programs, introduced PERT on its Polaris weapon system in 1958. Since that time, PERT has spread rapidly the rough out almost all industries. At the same time, similar technique known as the Critical Path Method (CPM) had been initiated, in which, no allowance is made for uncertainties in the duration

time involved. Which also has spread widely, and is particularly concentrated in the construction and processes industries[3].

A big advantage of PERT lies in its extensive planning, such as the interdependencies, other problems that are not obvious with other planning method. Keeping a project on schedule, also, one can determine the probability of meeting deadlines by developments alternative plans where the program had been change. If the decision maker is statistically sophistication, one can examine the standard deviations and the probability of accomplishment data [7].

Finally, PERT allows a large amount of sophistication data to be presented in a well – organized diagram form which contractors and customers can make joint decisions [1].

In this thesis, we are implemented, another technique called " Linear Programming Problem, LPP " by constructing the four linear models, based on the characteristics of the network project. The thesis consists of three chapters, as well as the introduction.

Chapter one, presented the basic concepts and definitions related to the network and linear programming problem.

Chapter two, presented, model construction, in which, four models are presented. Model 1: obtain the minimum completing time. Model 2: obtain the minimum crashing cost. Model 3: obtain the minimum completing time by considering the minimum crashing cost obtained from model 2.

Model 4: obtain the minimum total cost which is defined as a sum of direct cost and indirect cost.

Chapter three, presented one of the most famous techniques called "Interior- Point Method " for solving linear programming problem, which is implemented by the mathematical software " What's Best 9.0 " to identify the critical path, while the software " Project 2000 " has been used for constructing the project network, and verifying our result, by obtaining the same critical path. Also, the Programming Evaluation Review Technique " PERT " had been used to find the probabilities of completion the project.

The image features a minimalist, abstract design. It consists of several overlapping circles of varying sizes, some filled with a light gray color and others with black outlines. These circles are arranged in a way that suggests a sense of depth and movement. Two prominent black lines intersect at a point, forming a large 'V' shape that frames the central text. The overall aesthetic is clean and modern.

Chapter one

Fundamental Concepts

The background features a large, abstract geometric composition. It consists of several overlapping circles of varying sizes, some filled with a light gray color and others with black outlines. These circles are arranged in a way that suggests a sense of depth and movement. Two prominent black lines intersect at a point, forming a large 'V' shape that frames the central text. The overall aesthetic is clean, modern, and mathematical.

Chapter two

Mathematical Models

Chapter two

Mathematical Models

The process of identifying objective, variables, and constraints for a given problem is known as modeling. Construction of an appropriate model is the first step –sometimes the most important step – in the optimization process. If the model is too simple, it will not give useful insights into the practical problem, but if it is too complex, it may become too difficult to solve [11].

2.1 Mathematical Programming Models

The analysis of a simplified project management problem is useful to both PERT and CPM. In the simplified problem the completion times of all the project activities and their technological sequence are known. The management wants to determine the minimum time in which the project can be completed, and to identify the crucial jobs whose delay can delay the entire project.

The simplified project management problem can be solved by formulating it as a linear program. To illustrate this, consider the following Figure:-

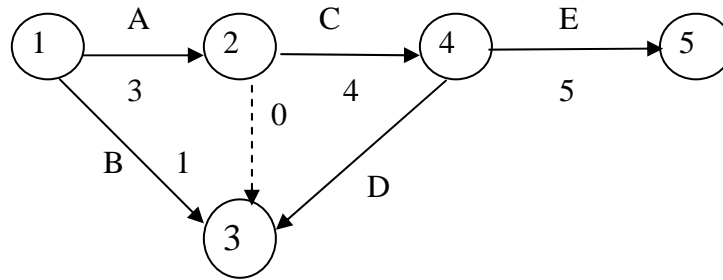


Fig. (2.1).The network.

Let t_i represent the time at which event i occurs where $i = 1, 2, 3, 4, 5$. For example, t_5 represents when the project is completed, while t_4 represents the time at which jobs C and D are completed. Thus $(t_5 - t_1)$ represents the time of completion of the entire project, and the objective is to minimize this duration. The linear programming formulation becomes:-

$$\text{Minimize: } Z = t_5 - t_1$$

Subject to

$$t_2 - t_1 \geq 3$$

$$t_3 - t_1 \geq 1$$

$$t_3 - t_2 \geq 0$$

$$t_4 - t_2 \geq 4$$

$$t_4 - t_3 \geq 2$$

$$t_5 - t_4 \geq 5$$

$$t_i \geq 0, \quad \text{for all } i = 1, 2, 3, 4, 5$$

The above linear program may be solving by any suitable method, and the optimal value of Z gives the minimum completion time for the project. As a matter of fact, the linear programming problem can be solved by inspection

by setting $t_1=0$, and choosing the value of t_i as small as possible to satisfy the constraints. Thus, an optimal solution by inspection is $t_1=0$, $t_2=3$, $t_3=3$, $t_4=7$, $t_5=12$, and minimum $Z= t_5 - t_1=12$ days.

Now, we can identify those constraints which will be satisfied as equations in the optimal solution, and the jobs corresponding to those constraints are the *critical jobs*. For example, the constraint for arc (1,2) is satisfied as an equation by $t_3=3$ and $t_1=0$. Hence A is critical job. For arc (1,3), the corresponding constraint is a strict inequality since $t_3 - t_1 = 3-0 > 1$. Hence B is not a critical job, and so on in Fig. (2.1) the critical jobs are A, C, and E, while jobs B and D are not critical [4].

Now, we can formulate a linear programming for general case:

Model 1

$$\begin{array}{ll}
 \text{Minimize: } Z = t_n - t_1 & \\
 \text{Subject to} & \\
 t_j - t_i \geq t_{ij} & \\
 t_i \geq 0 & \text{for all } i = 1, 2, \dots, n
 \end{array}
 \left. \vphantom{\begin{array}{l} \\ \\ \\ \end{array}} \right\} (2.1)$$

The above linear program may be solved by the Interior- Point method, and the optimal value of Z gives the minimum completion time for the project. As a matter of fact, the linear programming problem can be solved by setting $t_1 = 0$, and choosing the values of t as small as possible to satisfy the constraints.

For large project, the mathematical programming methods are more efficient in determining the optimal project schedule. we shall discuss some linear programming models for the critical path analysis.

Once again, it is assumed that a cost – versus – time relationship is available for every job in the project as shown in Figure (2.2).

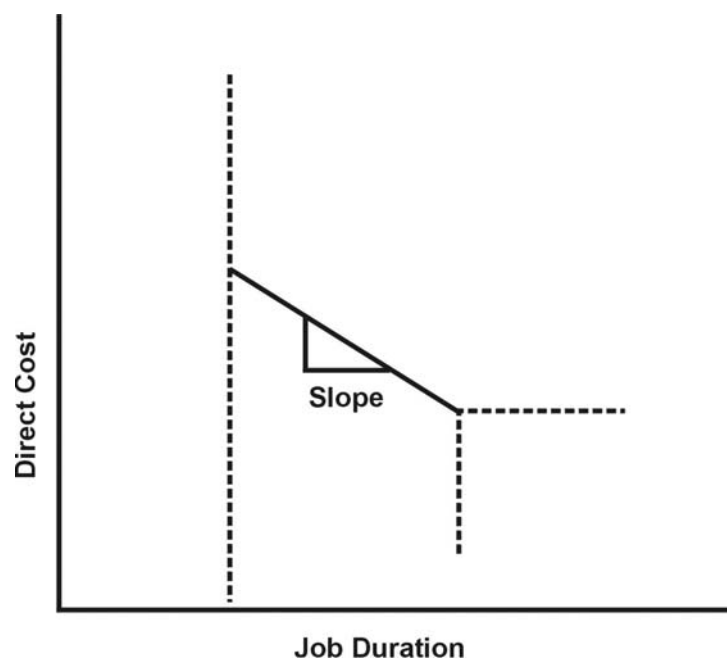


Fig .(2.2) job duration versus direct cost.

We denote by k_{ij} the normal completion time of job (i, j), while l_{ij} , denotes the crash completion time with the maximum amount of resources, C_{ij} represents the unit cost of shortening the duration of job (i, j). If t_{ij} is the completion time of job (i, j), then t_{ij} is an unknown variable between l_{ij} and k_{ij} and the cost of crashing is given by $C_{ij} (k_{ij} - t_{ij})$.

Let t_u be the unknown event times ($u = 1, 2, 3, \dots, n$) for a project consisting of n event where event 1 and n denote the start and the end of the project.

We shall now develop three important models in the critical path analysis which are useful for project management. In all these models, we will assume that the normal time, crash time, and the crashing cost are available for all the activities in the project [4].

Model 2

Given that the project must be completed by time T , we want to determine how the project activities are to be expedited such that the total cost of crashing is minimized. This problem can be formulated as a linear programming problem as follows:-

$$\begin{array}{ll} \text{Minimize: } Z = & \sum C_{ij}(k_{ij} - t_{ij}) \\ \text{Subject to} & \\ & t_j - t_i \geq t_{ij} \quad \text{for all jobs } (i, j) \\ & l_{ij} \leq t_{ij} \leq k_{ij} \\ & t_n - t_1 \leq T \\ & t_i \geq 0 \quad \text{for all } i = 1, 2, \dots, n \end{array} \quad (2.2)$$

The above problem may be solved by the Interior- Point Method. The optimal value of Z gives the minimum crashing cost. From the optimal values of t_{ij} , we can determine which jobs are expedited, and by how much.

It should be pointed out here that for the above linear program to be feasible, the value of T must be greater than or equal to the length of the critical path with all the jobs at their crash (minimum) times[4].

Model 3

Suppose an additional budget of \$ B is available for crashing the project activities. We want to determine how these additional resources may be allocated in the best possible manner so as to minimize the project completion time.

The linear programming model of this problem is given below:-

$$\begin{array}{ll}
 \text{Minimize: } Z = t_n - t_1 & \\
 \text{Subject to} & \\
 t_j - t_i \geq t_{ij} & \\
 l_{ij} \leq t_{ij} \leq k_{ij} & \text{for all jobs } (i, j) \\
 \sum C_{ij} (k_{ij} - t_{ij}) \leq B & \\
 t_i \geq 0 & \text{for all } i = 1, 2, \dots, n
 \end{array} \quad (2.3)$$

The solution to this linear program gives the least project duration that can be achieved by the additional budget B , the activities to be crashed, and their durations.

By using the linear programming Model 2 or 3 repeatedly, one could obtain a relationship between the total crashing cost and the project duration.

Figure (2.3) gives a typical plot of the direct (activity) costs against the project duration.

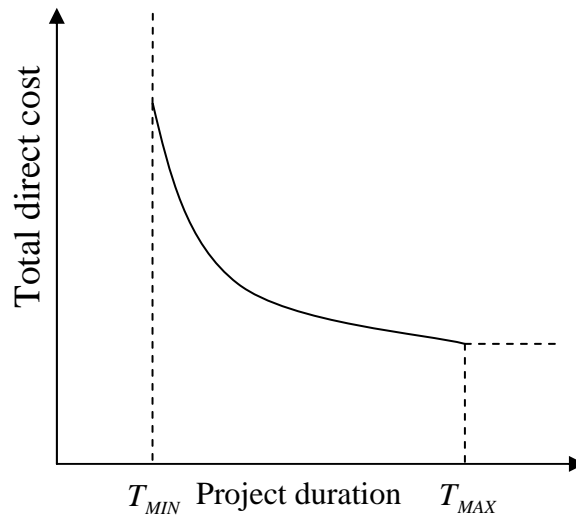


Fig. (2.3)Direct cost versus project duration.

T_{MAX} Denotes the project duration with all jobs at their normal times, while T_{MIN} denotes the project duration with all the jobs reduced to their crash times. The cost function shown in Fig. (2.3) is called a piece – wise linear function. With the help of this curve the project manager can determine:-

- (1) The minimum cost of additional resources needed to meet a given project deadline.
- (2) The optimal allocation of scarce resources to achieve the maximum reduction in project duration.

As seen in Fig. (2.3) the direct cost of completing the project activities increases when the project duration is reduced. But the indirect costs discussed earlier reduce with a reduction in project duration. Hence it will be

of interest to study how the total cost (direct – indirect costs) varies with the project duration. For various project lengths the indirect cost is added to the direct cost, and a plot of points is obtained to get a relationship between the project length and the total project cost. Such a plot is shown in Fig. (2.4).

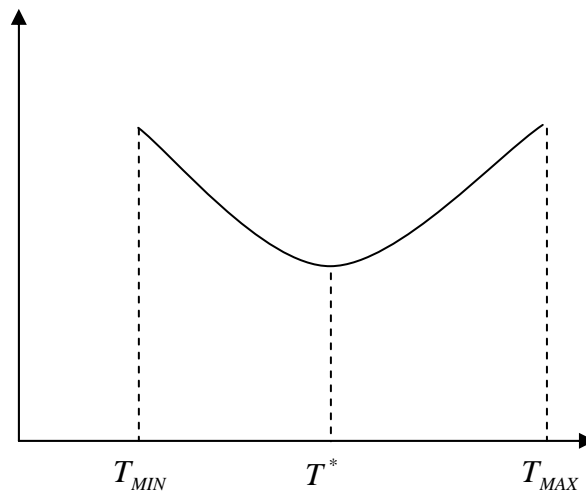


Fig. (2.4) Project cost curve.

This U-shaped curve is called a project cost curve. With the help of this curve, a project manager can select the optimal project duration (T^*) that will minimize the total costs. Corresponding to the optimal value of T , he can then determine the optimal durations of all the jobs, the cost of crashing, and the critical path. From this the optimal project schedule can be prepared[4].

Model 4

If the indirect cost of the project varies linearly with the project duration, then one can determine the optimal length of the project (T^*) and the optimal project schedule by solving a linear programming problem.

Let the indirect costs, proportional to the project duration, be denoted by F per unit time. Then the indirect cost is given by $F(t_n - t_1)$, where $(t_n - t_1)$ is the unknown length of the project. The direct cost is given by $\sum C_{ij}(k_{ij} - t_{ij})$ where t_{ij} is the unknown length of job (i, j) . The problem is to determine the optimal schedule that will minimize the total cost. The linear programming formulation becomes[4]:-

$$\begin{aligned} \text{Minimize: } Z &= F(t_n - t_1) + \sum C_{ij}(k_{ij} - t_{ij}) \\ \text{Subject to} \\ t_j - t_i &\geq t_{ij} \\ l_{ij} &\leq t_{ij} \leq k_{ij} \\ t_i &\geq 0 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \text{for all } i = 1, 2, \dots, n \end{array} \right\} (2.4)$$

2.2 Optimization Algorithms

Optimization traces its roots to the calculus of variations and the work of Euler and Lagrange. The development of linear programming in 1940s stimulated much of the progress in modern optimization theory and practice during the last century. Optimization is often called mathematical programming, a term that is somewhat confusing because it suggests the writing of computer programs with a mathematical orientation [9].

Once the model has been formulated, an optimization algorithm can be used to find its solution. Usually, the algorithm and model are complicated enough that a computer is needed to implement this process. There is no universal optimization algorithm. Rather, there are numerous algorithms, each of which is tailored to a particular type of optimization problem. It is often the user's responsibility to choose an algorithm that is appropriate for their specific application. This choice is an important one; it may determine whether the problem is solved rapidly or slowly and, indeed, whether the solution is found at all. After an optimization algorithm has been applied to the model, we must be able to recognize whether it has succeeded in its task of finding a solution. In many cases, there are elegant mathematical expressions known as optimality conditions for checking that the current set of variables is indeed the solution of the problem. If the optimality conditions are not satisfied, they may give useful information on how the current estimate of the solution can be improved. Finally, the model may be improved by applying techniques such as sensitivity analysis, which reveals the sensitivity of the solution to changes in the model and data [11].

Optimization algorithms are iterative. They begin with an initial guess of the optimal values of the variables and generate a sequence of improved estimates until they reach a solution. The strategy used to move from one iterate to the next distinguishes one algorithm from another. Most strategies make use of the value of the objective function f , the constraints c , and possibly the first and second derivatives of these functions. Some algorithms accumulate information gathered at previous iteration, while others use only local information from the current point [10].

Optimization was coined in the 1940s, before the word " programming " became inextricably linked with computer software. The original meaning of this word (and the intended one in this context) was more inclusive, with connotations of problem formulation and algorithm design and analysis [9].

In this section, we are presented two famous algorithms for solving linear programming problems. The first algorithm is a Simplex Method, and the second algorithm is an Interior- Point Algorithm. The second algorithm is presented in more details, because, it has been used implementation of the mathematical software package " What's Best 9.0 " for solving project network scheduling after formulated it into linear programming model.

2.2.1 Simplex Method, [5]

Given a system in canonical form corresponding to a basic solution, we have seen how to move to a neighboring basis solution by a pivot operation. Thus, one way to find the optimal solution of the given linear programming problem is to generate all the basic solution and pick the one which is feasible and corresponds to the optimal value of the objective function. This can be done because the optimal solution, if one exists, always occurs at an extreme point or vertex of the feasible domain. If there are m equality constraints in n variables with $n \geq m$, a basic solution can be obtained by setting any of the $(n-m)$ variables equal to zero. The number of basic solution to be inspected is thus equal to the number of ways in which m variables can be selected form a group of n variables:

$$\frac{n!}{(n-m)!m!} = \binom{n}{m} \quad (2.5)$$

Usually, we do not have to inspect all these basic solution since many of them will be infeasible. However, for large n and m , this is still every large number fore inspecting one by one. Hence, what we really need is a computation scheme that examines a sequence of basic feasible solution, each of which corresponds to a lower value of f until a minimum is reached. The simplex method is a powerful scheme for obtaining a basic feasible solution, if the solution is not optimal, the method provides for finding a neighboring basic feasible solution which has a lower or equal value of f . The process is repeated until, in a finite number of steps, an optimum is found.

2.2.2 Interior- Point Method

In the 1980 it was discovered that many large linear programs could be solved efficiently by formulating them as non linear problems and solving them with various modifications of nonlinear algorithms such as Newton's Method. One characteristic of these methods was that they required all iterates to satisfy the inequality constraints in the problem strictly, so they soon became known as Interior- Point Methods. By the early 1990, one class – Primal – Dual Methods – had distinguished itself as the most efficient practical approach and proved to be a strong competitor to the Simplex Method on large problems. The motivation for Interior – Point Methods arose from the desire to find algorithms with better theoretical properties than the Simplex Method. The Simplex Method can be quite inefficient on certain problems. Roughly speaking, the time required to solve a linear program may be exponential in the size of the problem, as measured by the number of unknowns and the amount of storage needed for the problem data. In practice, the Simplex Method is much more efficient than this bound would

suggest, but its poor worst – case complexity motivated the development of new algorithms with better guaranteed performance [2].

We consider the linear programming in standard form (1.1), (1.2) & (1.3):-
It is dual problem for is:-

$$\text{Max } b^T \lambda \quad (2.6)$$

$$\begin{aligned} \text{Subject to } A^T \lambda + s &= c \\ s &\geq 0 \end{aligned} \quad (2.7)$$

Where λ is a vector in R^m and s is a vector in R^n . The Primal – Dual solution is characterized as follows :-

$$A^T \lambda + s = c \quad (2.8a)$$

$$Ax = b \quad (2.8b)$$

$$x_i s_i = 0, \quad i = 1, 2, \dots, n \quad (2.8c)$$

$$(x, s) \geq 0 \quad (2.8d)$$

Primal – Dual Method find solution (x^*, λ^*, s^*) of this system by applying variants of Newton's Method and modifying the search directions and step lengths so that the inequalities $(x, s) \geq 0$ are satisfied strictly at every iteration.

The equation (2.8a), (2.8b), (2.8c) are only mildly nonlinear and so are not difficult to solve by themselves. However, the problem becomes much more difficult when we add the nonnegativity requirement (2.8d).

The nonnegativity condition is the source of all the complications in the design and analysis of Interior – Point Methods.

To derive Primal – Dual Interior – Point Methods, we restate the optimality conditions (2.8) in a slightly different form by means of mapping F from \mathbb{R}^{2n+m} to \mathbb{R}^{2n+m} :-

$$F(x, \lambda, s) = \begin{bmatrix} A\lambda^T + S - C \\ AX - B \\ XSe \end{bmatrix} \quad (2.9a)$$

$$(x, s) \geq 0 \quad (2.9b)$$

Where

$$\left. \begin{aligned} X &= \text{diag}(x_1, x_2, \dots, x_n) \\ S &= \text{diag}(s_1, s_2, \dots, s_n) \end{aligned} \right\} \quad (2.9c)$$

And

$e = (1, 1, \dots, 1)^T$. Primal – Dual Method generate iteration (x^k, λ^k, s^k) that satisfy the bounds (2.9b) strictly, $x^k > 0$ and $s^k > 0$. This property is the origin of the term Interior – Point. By respecting these bounds, the methods avoid spurious solution, that is, points that satisfy:

$$F(x, \lambda, s) = 0 \text{ but not } (x, s) \geq 0.$$

Spurious solution abound, and do not provide useful information about solution (1.1) - (1.2) & (2.6) - (2.8) so it makes sense to exclude them altogether from the region of search.

Many Interior – Point Method actually require the iterates to be strictly feasible, that is, each (x^k, λ^k, s^k) must satisfy the linear equality constraints for the primal and dual problems. If we define the Primal – Dual feasible set F and strictly feasible set F° by:-

$$F = \{ (x, \lambda, s) / Ax = b, A^T \lambda + s = c, (x, s) \geq 0 \} \quad (2.10a)$$

$$F^o = \{ (x, \lambda, s) / Ax = b, A^T \lambda + s = c, (x, s) > 0 \} \quad (2.10b)$$

The strict feasibility condition can be written concisely as:-

$$(x^k, \lambda^k, s^k) \in F^o$$

Like most iterative algorithms in optimization, Primal – Dual Interior – Point Method has two basic ingredients: a procedure for determining the step and a

measure of the desirability of each point in the search space. As mentioned above, the search direction procedure has its origins in Newton's Method for the nonlinear equation (2.9a). Newton's Method forms a linear model for F around the current point and obtains the search direction $(\Delta x, \Delta \lambda, \Delta s)$ by solving the following system of linear equation:-

$$J(x, \lambda, s) \begin{bmatrix} \Delta x \\ \Delta \lambda \\ \Delta s \end{bmatrix} = -F(x, \lambda, s) \quad (2.11)$$

Where J is the Jacobin of F . If the current point is strictly feasible that is $(x, \lambda, s) \in F^o$, the Newton step equation become :

$$\begin{bmatrix} 0 & A^T & I \\ A & 0 & 0 \\ S & 0 & X \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta \lambda \\ \Delta S \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -XSe \end{bmatrix} \quad (2.12)$$

A full step along direction usually is not permissible, since it would violate the bound $(x, s) \geq 0$. To avoid this difficulty, we perform a line search along the Newton direction so that the new iterate is:

$$(x, \lambda, s) + \alpha (\Delta x, \Delta \lambda, \Delta s)$$

For some line search parameter $\alpha \in (0, 1]$. Unfortunately we often can take only a small step along the direction ($\alpha < 1$) before violating the condition $(x, s) > 0$ hence, the pure Newton direction (2.12), which is known as the affined scaling direction, often does not allow us to make much progress toward a solution [10].

Primal – Dual method modify the basic Newton procedure in two important ways:

- (1) It bias the search direction toward the interior of the nonnegative outthunt $(x, s) \geq 0$, so that we can move further along the direction before one of the components of (x, s) becomes negative.
- (2) It keep the components of (x, s) from moving " too close " to the boundary of nonnegative outthunt.

To explain, the Interior – Point Algorithm, we are presented the following example [9]:-

$$\text{Minimize } z = -6x_1 - x_2 + 4x_3$$

Subject to

$$x_1 + 4x_2 - 2x_3 \leq 1$$

$$2x_1 - 2x_2 + 6x_3 \leq 2$$

$$-2x_1 + 3x_2 + x_3 \leq 5$$

Where

$$\alpha = 0.1$$

Solution:-

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & -2 \\ 2 & -2 & 6 \\ -2 & 3 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$$\mathbf{S}^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{X}^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$-\mathbf{X}^0 \mathbf{S}^0 \mathbf{e} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & -2 & 6 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 6 & 0 & 0 & 0 & 1 \\ 1 & 4 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta \lambda_1 \\ \Delta \lambda_2 \\ \Delta \lambda_3 \\ \Delta s_1 \\ \Delta s_2 \\ \Delta s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\begin{array}{lll} \Delta x_1 = 0 & \Delta x_2 = 0 & \Delta x_3 = 0 \\ \Delta s_1 = -1 & \Delta s_2 = -1 & \Delta s_3 = -1 \\ \Delta \lambda_1 = 0.4125 & \Delta \lambda_2 = 0.23125 & \Delta \lambda_3 = 0.0625 \end{array}$$

Choosing $\alpha = 0.1$

$$\begin{aligned} (x^1, x^2, x^3, \lambda^1, \lambda^2, \lambda^3, s^1, s^2, s^3) &= (x_0^1, x_0^2, x_0^3, \lambda_0^1, \lambda_0^2, \lambda_0^3, s_0^1, s_0^2, s_0^3) + \alpha(\Delta x^1, \Delta x^2, \Delta x^3, \\ &\quad \Delta \lambda^1, \Delta \lambda^2, \Delta \lambda^3, \Delta s^1, \Delta s^2, \Delta s^3) \\ &= (1, 1, 1, 1, 0, 0, 1, 1, 1) + 0.1(0, 0, 0, \\ &\quad 0.4125, 0.23125, 0.0625, -1, -1, -1) \\ &= (1, 1, 1, 1.04125, 0.023125, 0.00625, 0.99, \\ &\quad 0.99, 0.99) \end{aligned}$$

Now we obtain new X and S

$$X^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad S^1 = \begin{bmatrix} 0.99 & 0 & 0 \\ 0 & 0.99 & 0 \\ 0 & 0 & 0.99 \end{bmatrix}$$

$$-X^1 S^1 e = \begin{bmatrix} -0.99 \\ -0.99 \\ -0.99 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & -2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 6 & 1 & 0 & 0 & 1 \\ 1 & 4 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.99 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.99 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.99 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta \lambda_1 \\ \Delta \lambda_2 \\ \Delta \lambda_3 \\ \Delta s_1 \\ \Delta s_2 \\ \Delta s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.99 \\ -0.99 \\ -0.99 \end{bmatrix}$$

$$\begin{array}{lll} \Delta x_1 = 0 & \Delta x_2 = 0 & \Delta x_3 = 0 \\ \Delta s_1 = -0.99 & \Delta s_2 = -0.99 & \Delta s_3 = -0.99 \\ \Delta \lambda_1 = 0.3982 & \Delta \lambda_2 = 0.2845 & \Delta \lambda_3 = -0.011 \end{array}$$

$$\begin{aligned} (x^1, x^2, x^3, \lambda^1, \lambda^2, \lambda^3, s^1, s^2, s^3) &= (x_0^1, x_0^2, x_0^3, \lambda_0^1, \lambda_0^2, \lambda_0^3, s_0^1, s_0^2, s_0^3) + \alpha(\Delta x^1, \Delta x^2, \Delta x^3, \\ &\quad \Delta \lambda^1, \Delta \lambda^2, \Delta \lambda^3, \Delta s^1, \Delta s^2, \Delta s^3) \end{aligned}$$

$$(x_1, x_2, x_3, \lambda_1, \lambda_2, \lambda_3, s_1, s_2, s_3) = (1, 1, 1, 1.08107, 0.051575, 0.00515, 0.891, 0.891, 0.891)$$

Then X and S equal to

$$X^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad S^2 = \begin{bmatrix} 0.891 & 0 & 0 \\ 0 & 0.891 & 0 \\ 0 & 0 & 0.891 \end{bmatrix}$$

$$-X^2 S^2 e = \begin{bmatrix} -0.891 \\ -0.891 \\ -0.891 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 2 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & -2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 6 & 1 & 0 & 0 & 1 \\ 1 & 4 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.891 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.891 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.891 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta \lambda_1 \\ \Delta \lambda_2 \\ \Delta \lambda_3 \\ \Delta s_1 \\ \Delta s_2 \\ \Delta s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.891 \\ -0.891 \\ -0.891 \end{bmatrix}$$

$$\Delta x_1 = 0$$

$$\Delta x_2 = 0$$

$$\Delta x_3 = 0$$

$$\Delta s_1 = -0.891$$

$$\Delta s_2 = -0.891$$

$$\Delta s_3 = -0.891$$

$$\Delta \lambda_1 = 0.3564$$

$$\Delta \lambda_2 = 0.2673$$

$$\Delta \lambda_3 = 0$$

$$(x^1, x^2, x^3, \lambda^1, \lambda^2, \lambda^3, s^1, s^2, s^3) = (x_0^1, x_0^2, x_0^3, \lambda_0^1, \lambda_0^2, \lambda_0^3, s_0^1, s_0^2, s_0^3) + \alpha(\Delta x^1, \Delta x^2, \Delta x^3, \Delta \lambda^1, \Delta \lambda^2, \Delta \lambda^3, \Delta s^1, \Delta s^2, \Delta s^3)$$

$$(x_1, x_2, x_3, \lambda_1, \lambda_2, \lambda_3, s_1, s_2, s_3) = (1, 1, 1, 1.11671, 0.0783, 0.00515, 0.8, 0.8, 0.8)$$

		Repeated(1)	Repeated(2)	Repeated(3)
x_1	1	1	1	1
x_2	1	1	1	1
x_3	1	1	1	1
λ_1	1	1.04125	1.08107	1.11671
λ_2	0	0.023125	0.051575	0.0783
λ_3	0	0.00625	0.00515	0.00515
s_1	1	0.99	0.891	0.8
s_2	1	0.99	0.891	0.8
s_3	1	0.99	0.891	0.8

The same procedure will be repeated for different values of α , satisfying condition (2.10b) until no significant difference value in x & s , will be appeared.

The background features a large, light gray triangle pointing downwards, with its vertex at the top center. Two thin black lines extend from the top-left and top-right corners of the triangle towards the center. Three sets of overlapping circles are positioned along these lines: a large set in the upper right, a medium set in the middle, and a large set in the lower right. Each set consists of three concentric circles with black outlines, where the innermost circle is filled with light gray and the outer two are hollow.

Chapter three

Results and Discussion

Chapter three

Result and Discussion

In this chapter we are considering a real project problem take from [6] to solved the models which are built in chapter two, by implementing " What's Best 9.0 " software The minimum completion time for the project is obtained from 1st model. The minimum total crashing cost is obtained by considering the minimum total completion time as a constraint in solving the 2nd model. While by considering the minimum total crashing cost obtained from the 2nd model, into the 3^{ed} model, we could obtain the minimum total completion time. The 4th model, obtain the minimum total cost which is defined as the sum of the direct and indirect costs. All these result obtained, are verified by implementing " Project 2000 " software showing the same critical activities. Some simple analyses are presented. Finally the Programming Evaluation and Review Technique (PERT) has been used to find the probabilities of the completing the project.

Example (3.1):

Consider the following real project network of hydroelectric power plant [6]. The details of the activities, their precedence constraints, and the estimated times are shown in the table below:-

Table (3.1).Activities durations and costs.

<i>Activity</i>	<i>Predecessor</i>	<i>Normal time</i>	<i>Crash time</i>	<i>Cost of per day (\$)</i>	<i>Cost of crashing per day (\$)</i>
<i>1,2</i>	-	6	4	2	3
<i>2,3</i>	1,2	9	8	3	5
<i>2,4</i>	1,2	7	4	3	5
<i>4,5</i>	2,4	4	3	2	4
<i>4,6</i>	2,4	9	5	4	5
<i>5,6</i>	4,5	6	5	4	6
<i>3,7</i>	2,3	11	9	5	8
<i>5,8</i>	4,5	10	8	2	3
<i>10,13</i>	8,10-6,10	5	4	1	2
<i>8,11</i>	5,8-7,8	4	3	2	3
<i>7,12</i>	3,7	2	1	1	2
<i>11,12</i>	8,11	3	1	1	1
<i>8,9</i>	5,8-7,8	6	5	4	5
<i>8,10</i>	5,8-7,8	2	1	1	1

7,8	3,7	14	10	1	2
6,10	4,6-5,6	7	6	1	2
6,14	4,5-5,6	10	8	3	5
13,14	10,13	24	20	2	7
13,15	10,13	18	15	2	4
15,17	13,15	6	5	2	3
12,19	7,12-11,12	20	18	1	1
14,16	6,14-13,14	2	1	1	2
17,18	15,17	1	1	3	5
19,20	12,19	2	1	2	3

First, the problem can be reformulated as in the form of *model 1* (as described in chapter two)

$$\text{Minimize } z = t_{20} - t_1$$

S. t

$$t_2 - t_1 \geq 4$$

$$t_3 - t_2 \geq 8$$

$$t_4 - t_2 \geq 4$$

$$t_5 - t_4 \geq 3$$

$$t_5 - t_3 \geq 0$$

$$t_6 - t_4 \geq 5$$

$$t_6 - t_5 \geq 5$$

$$t_7 - t_3 \geq 9$$

$$t_8 - t_5 \geq 8$$

$$t_8 - t_7 \geq 10$$

$$t_{10} - t_6 \geq 6$$

$$t_{10} - t_8 \geq 1$$

$$t_9 - t_8 \geq 5$$

$$t_{10} - t_9 \geq 0$$

$$t_{11} - t_8 \geq 3$$

$$t_{12} - t_7 \geq 1$$

$$t_{12} - t_9 \geq 0$$

$$t_{12} - t_{11} \geq 1$$

$$t_{13} - t_{10} \geq 4$$

$$t_{14} - t_6 \geq 8$$

$$t_{14} - t_{13} \geq 20$$

$$t_{15} - t_{13} \geq 18$$

$$t_{15} - t_{12} \geq 0$$

$$t_{16} - t_{14} \geq 1$$

$$t_{17} - t_{16} \geq 0$$

$$t_{17} - t_{15} \geq 5$$

$$t_{18} - t_{17} \geq 1$$

$$t_{19} - t_{12} \geq 18$$

$$t_{19} - t_{18} \geq 0$$

$$t_{20} - t_{19} \geq 1$$

$$t_i \geq 0 \quad \text{for all } i = 1, 2, \dots, 20$$

Now, we are perform the Interior- Point Algorithm by using " What's Best 9.0 " software, to solve the above model, we get the results form as Fig. (3.1).

From the Figure (3.1) we can identify the critical jobs and the critical path:

1- 2- 3- 7- 8- 9- 10- 13- 14- 16- 17- 18- 19- 20

The results can be translated into our notation in the following table:-

Table (3.2) Nodes duration.

<i>Node</i>	<i>Duration</i>
1	0
2	4
3	12
4	8
5	23
6	30
7	21
8	31
9	36
10	36
11	35
12	36
13	40
14	60

15	55
16	61
17	61
18	62
19	62
20	63

The function is:-

$$\text{Minimize } Z = t_{20} - t_1$$

$$= 63 - 0$$

$$= 63 \text{ days}$$

The normal time of the project is 80 days, if we crash all activities; the total time which we need to complete the project is 63 days.

Now, the problem can be reformulated as in the form of *model 2* (as described in chapter two)

$$\begin{aligned}
\text{Minimize } Z = & 3(6 - t_{1,2}) + 5(9 - t_{2,3}) + 5(7 - t_{2,4}) + 4(4 - t_{4,5}) + \\
& 5(9 - t_e) + 6(6 - t_{5,6}) + 8(11 - t_{3,7}) + 3(10 - t_{4,6}) + \\
& 2(7 - t_{6,10}) + 3(4 - t_{8,11}) + 2(2 - t_{7,12}) + 1(3 - t_{11,12}) + \\
& 5(6 - t_{8,9}) + 1(2 - t_{8,10}) + 2(14 - t_{6,10}) + 2(5 - t_{10,13}) + \\
& 5(10 - t_{6,14}) + 7(24 - t_{13,14}) + 4(18 - t_{13,15}) + 3(6 - t_{15,17}) + \\
& 1(20 - t_{12,19}) + 2(2 - t_{14,16}) + 5(1 - t_{17,18}) + 3(2 - t_{19,20})
\end{aligned}$$

S. t

$$t_2 - t_1 - t_{1,2} \geq 0$$

$$t_3 - t_2 - t_{2,3} \geq 0$$

$$t_4 - t_2 - t_{2,4} \geq 0$$

$$t_5 - t_4 - t_{4,5} \geq 0$$

$$t_5 - t_3 \geq 0$$

$$t_6 - t_4 - t_e \geq 0$$

$$t_6 - t_5 - t_{5,6} \geq 0$$

$$t_7 - t_3 - t_{3,7} \geq 0$$

$$t_8 - t_5 - t_{5,8} \geq 0$$

$$t_8 - t_7 - t_{7,8} \geq 0$$

$$t_{10} - t_6 - t_{6,10} \geq 0$$

$$t_{10} - t_8 - t_{8,10} \geq 0$$

$$t_9 - t_8 - t_{8,9} \geq 0$$

$$t_{10} - t_9 \geq 0$$

$$t_{11} - t_8 - t_{8,11} \geq 0$$

$$t_{12} - t_7 - t_{7,12} \geq 0$$

$$t_{12} - t_9 \geq 0$$

$$t_{12} - t_{11} - t_{11,12} \geq 0$$

$$t_{13} - t_{10} - t_{10,13} \geq 0$$

$$t_{14} - t_6 - t_{6,14} \geq 0$$

$$t_{14} - t_{13} - t_{13,14} \geq 0$$

$$t_{15} - t_{13} - t_{13,15} \geq 0$$

$$t_{15} - t_{12} \geq 0$$

$$t_{16} - t_{14} - t_{14,16} \geq 0$$

$$t_{17} - t_{16} \geq 0$$

$$t_{17} - t_{15} - t_{15,17} \geq 0$$

$$t_{18} - t_{17} - t_{17,18} \geq 0$$

$$t_{19} - t_{12} - t_{12,19} \geq 0$$

$$t_{19} - t_{18} \geq 0$$

$$t_{20} - t_{19} - t_{19,20} \geq 0$$

$$4 \leq t_{1,2} \leq 6$$

$$6 \leq t_{2,3} \leq 9$$

$$4 \leq t_{2,4} \leq 7$$

$$3 \leq t_{4,5} \leq 4$$

$$5 \leq t_{4,6} \leq 9$$

$$5 \leq t_{5,6} \leq 6$$

$$9 \leq t_{3,7} \leq 11$$

$$8 \leq t_{5,8} \leq 10$$

$$10 \leq t_{7,8} \leq 14$$

$$6 \leq t_{6,10} \leq 7$$

$$1 \leq t_{8,10} \leq 2$$

$$5 \leq t_{8,9} \leq 6$$

$$3 \leq t_{8,11} \leq 4$$

$$1 \leq t_{7,12} \leq 2$$

$$1 \leq t_{11,12} \leq 3$$

$$4 \leq t_{10,13} \leq 5$$

$$8 \leq t_{6,14} \leq 10$$

$$20 \leq t_{13,14} \leq 24$$

$$15 \leq t_{13,15} \leq 18$$

$$1 \leq t_{14,16} \leq 2$$

$$5 \leq t_{15,17} \leq 6$$

$$t_q = 1$$

$$18 \leq t_{12,19} \leq 20$$

$$1 \leq t_{19,20} \leq 2$$

$$t_{20} - t_1 \leq 63$$

$$t_i \geq 0 \quad \text{for all } i = 1, 2, \dots, 20$$

Now, we are perform the Interior- Point Algorithm by using " What's Best 9.0 " software, to solve the above model, we get the results form as Fig. (3.2).

The results can be translated into our notation in the following:-

Table (3.3) Activity duration and Node duration.

<i>Activity</i>	<i>duration</i>
<i>1,2</i>	<i>4</i>
<i>2,3</i>	<i>8</i>
<i>2,4</i>	<i>4</i>
<i>4,5</i>	<i>3</i>
<i>4,6</i>	<i>5</i>
<i>5,6</i>	<i>5</i>
<i>3,7</i>	<i>9</i>
<i>5,8</i>	<i>8</i>
<i>7,8</i>	<i>10</i>
<i>8,10</i>	<i>1</i>
<i>6,10</i>	<i>6</i>
<i>8,11</i>	<i>3</i>
<i>7,12</i>	<i>1</i>
<i>11,12</i>	<i>1</i>
<i>8,9</i>	<i>5</i>
<i>10,13</i>	<i>4</i>
<i>6,14</i>	<i>8</i>
<i>13,14</i>	<i>20</i>

<i>13,15</i>	<i>15</i>
<i>15,17</i>	<i>5</i>
<i>12,19</i>	<i>18</i>
<i>14,16</i>	<i>1</i>
<i>17,18</i>	<i>1</i>

<i>Node</i>	<i>Duration</i>
<i>1</i>	<i>0</i>
<i>2</i>	<i>4</i>
<i>2</i>	<i>12</i>
<i>4</i>	<i>8</i>
<i>5</i>	<i>23</i>
<i>6</i>	<i>30</i>
<i>7</i>	<i>21</i>
<i>8</i>	<i>31</i>
<i>9</i>	<i>36</i>
<i>10</i>	<i>36</i>
<i>11</i>	<i>43</i>
<i>12</i>	<i>44</i>
<i>13</i>	<i>40</i>

14	60
15	56
16	61
17	61
18	62
19	62
20	63

Table (3.3)

The function is:-

$$\begin{aligned}\text{Minimize } Z &= \sum c_{ij} * k_{ij} - \sum cij * t_{ij} \\ &= \$759 - \$596 \\ &= \$163\end{aligned}$$

That mean, if we crash all activity ($t_{20} - t_1 \leq 63$), the total crashing cost is \$163.

We could obtain the relationship between the total crashing cost and the project duration:-

When $t_{20} - t_1 \leq 63$, then the total crashing cost is : \$ 163.

When $t_{20} - t_1 \leq 66$, then the total crashing cost is : \$ 54.

When $t_{20} - t_1 \leq 68$, then the total crashing cost is : \$38.

When $t_{20} - t_1 \leq 70$, then the total crashing cost is : \$26.

When $t_{20} - t_1 \leq 72$, then the total crashing cost is : \$18.

When $t_{20} - t_1 \leq 75$, then the total crashing cost is : \$10.

When $t_{20} - t_1 \leq 77$, then the total crashing cost is : \$6.

When $t_{20} - t_1 \leq 80$, then the total crashing cost is : \$0.

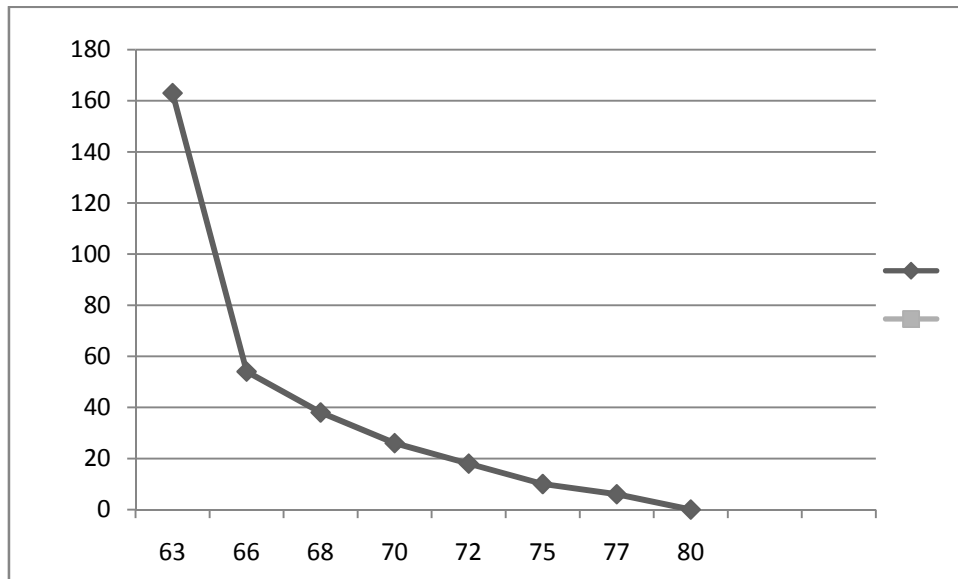


Fig. (3.3) The relation between total crashing cost and the duration.

Now, the problem can be reformulated as in the form of *model 3* (as described in chapter two)

$$\text{Minimize } Z = t_{20} - t_1$$

S. t

$$t_2 - t_1 - t_{1,2} \geq 0$$

$$t_3 - t_2 - t_{2,3} \geq 0$$

$$t_4 - t_2 - t_{2,4} \geq 0$$

$$t_5 - t_4 - t_{4,5} \geq 0$$

$$t_5 - t_3 \geq 0$$

$$t_6 - t_4 - t_{4,6} \geq 0$$

$$t_6 - t_5 - t_{5,6} \geq 0$$

$$t_7 - t_3 - t_{3,7} \geq 0$$

$$t_8 - t_5 - t_{5,8} \geq 0$$

$$t_8 - t_7 - t_{7,8} \geq 0$$

$$t_{10} - t_6 - t_{6,10} \geq 0$$

$$t_{10} - t_8 - t_{8,10} \geq 0$$

$$t_9 - t_8 - t_{8,9} \geq 0$$

$$t_{10} - t_9 \geq 0$$

$$t_{11} - t_8 - t_{8,11} \geq 0$$

$$t_{12} - t_7 - t_{7,12} \geq 0$$

$$t_{12} - t_9 \geq 0$$

$$t_{12} - t_{11} - t_{11,12} \geq 0$$

$$t_{13} - t_{10} - t_{10,13} \geq 0$$

$$t_{14} - t_6 - t_{6,14} \geq 0$$

$$t_{14} - t_{13} - t_{13,14} \geq 0$$

$$t_{15} - t_{13} - t_{13,15} \geq 0$$

$$t_{15} - t_{12} \geq 0$$

$$t_{16} - t_{14} - t_{14,16} \geq 0$$

$$t_{17} - t_{16} \geq 0$$

$$t_{17} - t_{15} - t_{15,17} \geq 0$$

$$t_{18} - t_{17} - t_{17,18} \geq 0$$

$$t_{19} - t_{12} - t_{12,19} \geq 0$$

$$t_{19} - t_{18} \geq 0$$

$$t_{20} - t_{19} - t_{19,20} \geq 0$$

$$4 \leq t_{1,2} \leq 6$$

$$6 \leq t_{2,3} \leq 9$$

$$4 \leq t_{2,4} \leq 7$$

$$3 \leq t_{4,5} \leq 4$$

$$5 \leq t_{4,6} \leq 9$$

$$5 \leq t_{5,6} \leq 6$$

$$9 \leq t_{3,7} \leq 11$$

$$8 \leq t_{5,8} \leq 10$$

$$10 \leq t_{7,8} \leq 14$$

$$6 \leq t_{6,10} \leq 7$$

$$1 \leq t_{8,10} \leq 2$$

$$5 \leq t_{8,9} \leq 6$$

$$3 \leq t_{8,11} \leq 4$$

$$1 \leq t_{7,12} \leq 2$$

$$1 \leq t_{11,12} \leq 3$$

$$4 \leq t_{10,13} \leq 5$$

$$8 \leq t_{6,14} \leq 10$$

$$20 \leq t_{13,14} \leq 24$$

$$15 \leq t_{13,15} \leq 18$$

$$1 \leq t_{14,16} \leq 2$$

$$5 \leq t_{15,17} \leq 6$$

$$t_q = 1$$

$$18 \leq t_{12,19} \leq 20$$

$$1 \leq t_{19,20} \leq 2$$

$$3(6 - t_{1,2}) + 5(9 - t_{2,3}) + 5(7 - t_{2,4}) + 4(4 - t_{4,5}) +$$

$$5(9 - t_e) + 6(6 - t_{5,6}) + 8(11 - t_{3,7}) + 3(10 - t_{4,6}) +$$

$$\begin{aligned} &2 (7 - t_{6,10}) + 3 (4 - t_{8,11}) + 2 (2 - t_{7,12}) + 1 (3 - t_{11,12}) + \\ &5 (6 - t_{8,9}) + 1 (2 - t_{8,10}) + 2 (14 - t_{6,10}) + 2 (5 - t_{10,13}) + \\ &5 (10 - t_{6,14}) + 7 (24 - t_{13,14}) + 4 (18 - t_{13,15}) + 3 (6 - t_{15,17}) + \\ &1 (20 - t_{12,19}) + 2 (2 - t_{14,16}) + 5 (1 - t_{17,18}) + 3 (2 - t_{19,20}) \leq 163 \end{aligned}$$

$$t_i \geq 0 \qquad \text{for all } i = 1, 2, \dots, 20$$

Now, we are perform the Interior- Point Algorithm by using " What's Best 9.0 " software, to solve the above model, we get the results form as Fig. (3.4).

The results can be translated into our notation in the following:-

Table (3.4) Node duration and activity duration.

<i>Node</i>	<i>Duration</i>
<i>1</i>	<i>0</i>
<i>2</i>	<i>4</i>
<i>3</i>	<i>12</i>
<i>4</i>	<i>8</i>
<i>5</i>	<i>23</i>
<i>6</i>	<i>30</i>
<i>7</i>	<i>21</i>
<i>8</i>	<i>31</i>
<i>9</i>	<i>36</i>
<i>10</i>	<i>36</i>
<i>11</i>	<i>43</i>
<i>12</i>	<i>44</i>
<i>13</i>	<i>40</i>
<i>14</i>	<i>60</i>

<i>15</i>	<i>56</i>
<i>16</i>	<i>61</i>
<i>17</i>	<i>61</i>
<i>18</i>	<i>62</i>
<i>19</i>	<i>62</i>
<i>20</i>	<i>63</i>

<i>Activity</i>	<i>Duration</i>
<i>1,2</i>	<i>4</i>
<i>2,3</i>	<i>8</i>
<i>2,4</i>	<i>4</i>
<i>4,5</i>	<i>3</i>
<i>4,6</i>	<i>5</i>
<i>5,6</i>	<i>5</i>
<i>3,7</i>	<i>9</i>

<i>5,8</i>	<i>8</i>
<i>7,8</i>	<i>10</i>
<i>8,10</i>	<i>1</i>
<i>6,10</i>	<i>6</i>
<i>8,11</i>	<i>3</i>
<i>7,12</i>	<i>1</i>
<i>11,12</i>	<i>1</i>
<i>8,9</i>	<i>5</i>
<i>10,13</i>	<i>4</i>
<i>6,14</i>	<i>8</i>
<i>13,14</i>	<i>20</i>
<i>13,15</i>	<i>15</i>
<i>15,17</i>	<i>5</i>
<i>12,19</i>	<i>18</i>
<i>14,16</i>	<i>1</i>

17,18	1
-------	---

The function is:

$$\text{Minimize } Z = t_{20} - t_1$$

$$= 63 - 0$$

$$= 63 \text{ days}$$

That mean if we add \$163 to a project, the solution gives the least project duration that can be achieved by the additional budget \$163, the activities of all the project are crash.

Now, the problem can be reformulated as in the form of *model 4* (as described in chapter two)

$$\begin{aligned} \text{Minimize } Z = & 10 (t_{20} - t_1) + 3 (6 - t_{1,2}) + 5 (9 - t_{2,3}) + 5 (7 - t_{2,4}) + \\ & 4 (4 - t_{4,5}) + 5 (9 - t_e) + 6 (6 - t_{5,6}) + 8 (11 - t_{3,7}) + \\ & 3 (10 - t_{4,6}) + 2 (7 - t_{6,10}) + 3 (4 - t_{8,11}) + 2 (2 - t_{7,12}) + \\ & 1 (3 - t_{11,12}) + 5 (6 - t_{8,9}) + 1 (2 - t_{8,10}) + 2 (14 - t_{6,10}) + \\ & 2 (5 - t_{10,13}) + 5 (10 - t_{6,14}) + 7 (24 - t_{13,14}) + \end{aligned}$$

$$4(18 - t_{13,15}) + 3(6 - t_{15,17}) + 1(20 - t_{12,19}) + 2(2 - t_{14,16}) +$$

$$5(1 - t_{17,18}) + 3(2 - t_{19,20})$$

S . t

$$t_2 - t_1 - t_{1,2} \geq 0$$

$$t_3 - t_2 - t_{2,3} \geq 0$$

$$t_4 - t_2 - t_{2,4} \geq 0$$

$$t_5 - t_4 - t_{4,5} \geq 0$$

$$t_5 - t_3 \geq 0$$

$$t_6 - t_4 - t_{4,6} \geq 0$$

$$t_6 - t_5 - t_{5,6} \geq 0$$

$$t_7 - t_3 - t_{3,7} \geq 0$$

$$t_8 - t_5 - t_{5,8} \geq 0$$

$$t_8 - t_7 - t_{7,8} \geq 0$$

$$t_{10} - t_6 - t_{6,10} \geq 0$$

$$t_{10} - t_8 - t_{8,10} \geq 0$$

$$t_9 - t_8 - t_{8,9} \geq 0$$

$$t_{10} - t_9 \geq 0$$

$$t_{11} - t_8 - t_{8,11} \geq 0$$

$$t_{12} - t_7 - t_{7,12} \geq 0$$

$$t_{12} - t_9 \geq 0$$

$$t_{12} - t_{11} - t_{11,12} \geq 0$$

$$t_{13} - t_{10} - t_{10,13} \geq 0$$

$$t_{14} - t_6 - t_{6,14} \geq 0$$

$$t_{14} - t_{13} - t_{13,14} \geq 0$$

$$t_{15} - t_{13} - t_{13,15} \geq 0$$

$$t_{15} - t_{12} \geq 0$$

$$t_{16} - t_{14} - t_{14,16} \geq 0$$

$$t_{17} - t_{16} \geq 0$$

$$t_{17} - t_{15} - t_{15,17} \geq 0$$

$$t_{18} - t_{17} - t_{17,18} \geq 0$$

$$t_{19} - t_{12} - t_{12,19} \geq 0$$

$$t_{19} - t_{18} \geq 0$$

$$t_{20} - t_{19} - t_{19,20} \geq 0$$

$$4 \leq t_{1,2} \leq 6$$

$$6 \leq t_{2,3} \leq 9$$

$$4 \leq t_{2,4} \leq 7$$

$$3 \leq t_{4,5} \leq 4$$

$$5 \leq t_{4,6} \leq 9$$

$$5 \leq t_{5,6} \leq 6$$

$$9 \leq t_{3,7} \leq 11$$

$$8 \leq t_{5,8} \leq 10$$

$$10 \leq t_{7,8} \leq 14$$

$$6 \leq t_{6,10} \leq 7$$

$$1 \leq t_{8,10} \leq 2$$

$$5 \leq t_{8,9} \leq 6$$

$$3 \leq t_{8,11} \leq 4$$

$$1 \leq t_{7,12} \leq 2$$

$$1 \leq t_{11,12} \leq 3$$

$$4 \leq t_{10,13} \leq 5$$

$$8 \leq t_{6,14} \leq 10$$

$$20 \leq t_{13,14} \leq 24$$

$$15 \leq t_{13,15} \leq 18$$

$$1 \leq t_{14,16} \leq 2$$

$$5 \leq t_{15,17} \leq 6$$

$$t_q = 1$$

$$18 \leq t_{12,19} \leq 20$$

$$1 \leq t_{19,20} \leq 2$$

$$t_i \geq 0 \quad \text{for all } i = 1, 2, \dots, 20$$

Now, we are perform the Interior- Point Algorithm by using " What's Best 9.0 " software, to solve the above model, we get the results form as Fig. (3.5).

The results can be translated into our notation in the following:-

Table (3.5).Activity duration and node duration.

<i>Activity</i>	<i>Duration</i>
<i>1,2</i>	<i>6</i>
<i>2,3</i>	<i>9</i>
<i>2,4</i>	<i>7</i>
<i>4,5</i>	<i>4</i>
<i>4,6</i>	<i>9</i>
<i>5,6</i>	<i>6</i>
<i>3,7</i>	<i>11</i>
<i>5,8</i>	<i>10</i>
<i>7,8</i>	<i>10</i>
<i>8,10</i>	<i>2</i>
<i>6,10</i>	<i>7</i>
<i>8,11</i>	<i>4</i>
<i>7,12</i>	<i>2</i>

<i>11,12</i>	<i>3</i>
<i>8,9</i>	<i>6</i>
<i>10,13</i>	<i>4</i>
<i>6,14</i>	<i>10</i>
<i>13,14</i>	<i>24</i>
<i>13,15</i>	<i>18</i>
<i>15,17</i>	<i>6</i>
<i>12,19</i>	<i>20</i>
<i>14,16</i>	<i>2</i>
<i>17,18</i>	<i>1</i>
<i>19,20</i>	<i>1</i>

<i>Node</i>	<i>Duration</i>
<i>1</i>	<i>0</i>
<i>2</i>	<i>6</i>

3	15
4	22
5	26
6	35
7	26
8	36
9	42
10	42
11	50
12	53
13	46
14	70
15	66
16	72
17	72
18	73
19	73

20	75
----	----

The function is minimize $z = 750 + 10$
 $= \$760$

If we have the per unit day of indirect cost is \$ 10, then the indirect cost of the project is $10 * 75 = \$750$.

And

The direct total crashing cost is \$ 10, and the direct cost of the project is : $418 + 10 = \$428$.

Then the cost which the project needed to complete in 75 days is :-

$$750 + 391 = \$1141.$$

To verify the results obtained from implemented " What's Best 9.0 " software we are using " Project 2000 " software, the project network show as in Figure (3.6) obtains the same critical path and show in the bold line.

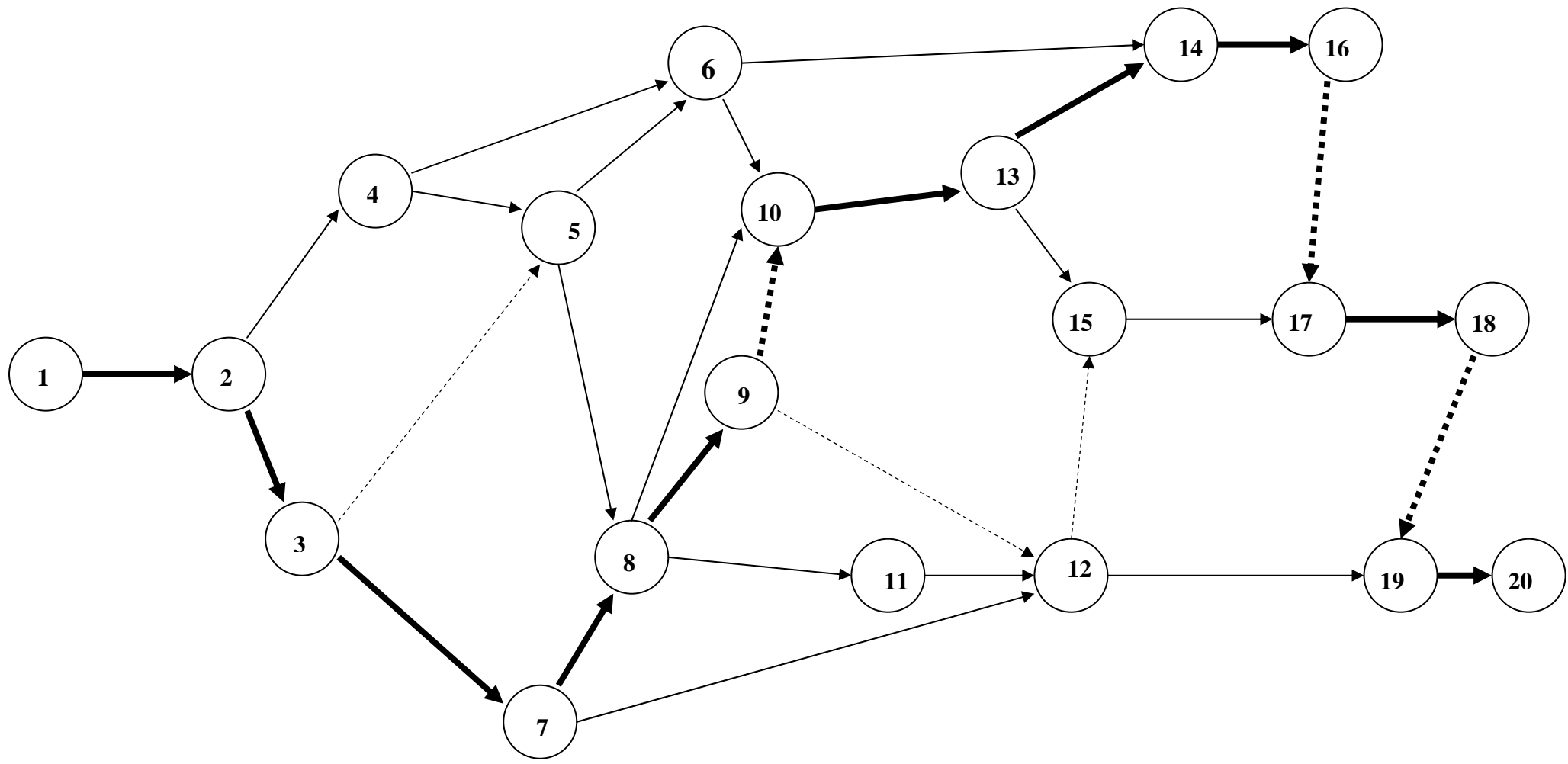


Fig. (3.6)

The probabilities of the completion project:

Consider we have the following information:-

Table (3.6).Expected time.

<i>Activity</i>	<i>Optimistic time</i>	<i>Most likely time</i>	<i>Pessimistic time</i>	<i>Expected time</i>
1,2	5	6	7	6
2,3	6	9	12	9
2,4	4	7	10	7
4,5	3	4	9	4
4,6	8	9	10	9
5,6	4	6	8	6
3,7	10	11	12	11
5,8	8	10	12	10
10,13	4	5	6	5
8,11	2	4	6	4
7,12	1	2	3	2

11,12	2	3	4	3
8,9	5	6	7	6
8,10	1	2	3	2
7,8	12	14	16	14
6,10	5	7	9	7
6,14	7	10	13	10
13,14	23	24	25	24
13,15	17	18	19	18
15,17	4	6	8	6
12,19	17	20	23	20
14,16	1	2	3	2
17,18	1	1	1	1
	1	2	3	2

To obtain the probability of completion project, first we determine standard deviation and the variance, as in the table (3.7).

Table. (3.7). The variance.

<i>Activity</i>	<i>Expected time</i>	<i>Standard deviation (σ)</i>	<i>Variance (σ)²</i>
1,2	6	1/3	1/9
2,3	9	1	1
2,4	7	1	1
4,5	4	1	1
4,6	9	1/3	1/9
5,6	6	2/3	4/9
3,7	11	1/3	1/9
5,8	10	2/3	4/9
10,13	5	1/3	1/9
8,11	4	2/3	4/9
7,12	2	1/3	1/9
11,12	3	1/3	1/9
8,9	6	1/3	1/9

8,10	2	1/3	1/9
7,8	14	2/3	4/9
6,10	7	2/3	4/9
6,14	10	1	1
13,14	24	1/3	1/9
13,15	18	1/3	1/9
15,17	6	2/3	4/9
12,19	20	1	1
14,16	2	1/3	1/9
17,18	1	0	0
19,20	2	1/3	1/9

Since, the critical path of the project is:-

1- 2- 3- 7- 8- 9- 10- 13- 14- 16- 17- 18- 19- 20

Let T denoted the project duration. The expected length of the project is:-

$E(T) =$ Sum of the expected times of the activities of the critical path

$$= 6 + 9 + 11 + 14 + 6 + 5 + 24 + 2 + 1 + 2$$

$$= 80$$

The variance of the project duration is:-

$$\begin{aligned} V(T) &= \text{Sum of the variances of the activities of the critical path} \\ &= 1/9 + 1 + 1/9 + 4/9 + 1/9 + 1/9 + 1/9 + 1/9 + 0 + 1/9 \\ &= 20/9 \end{aligned}$$

The standard deviation of the project duration is:-

$$\begin{aligned} \sigma(T) &= \sqrt{V(T)} \\ &= 0.71 \end{aligned}$$

We can also calculate the probabilities of ($T \leq 80$). This can be obtained from the tables of normal distribution: however, the tables are given for a standard normal only whose mean 0 and standard deviation is 1. From probability theory the random variable $Z = \frac{T - E(T)}{\sigma(T)}$ is distributed normally with mean 0 and standard deviation 1.

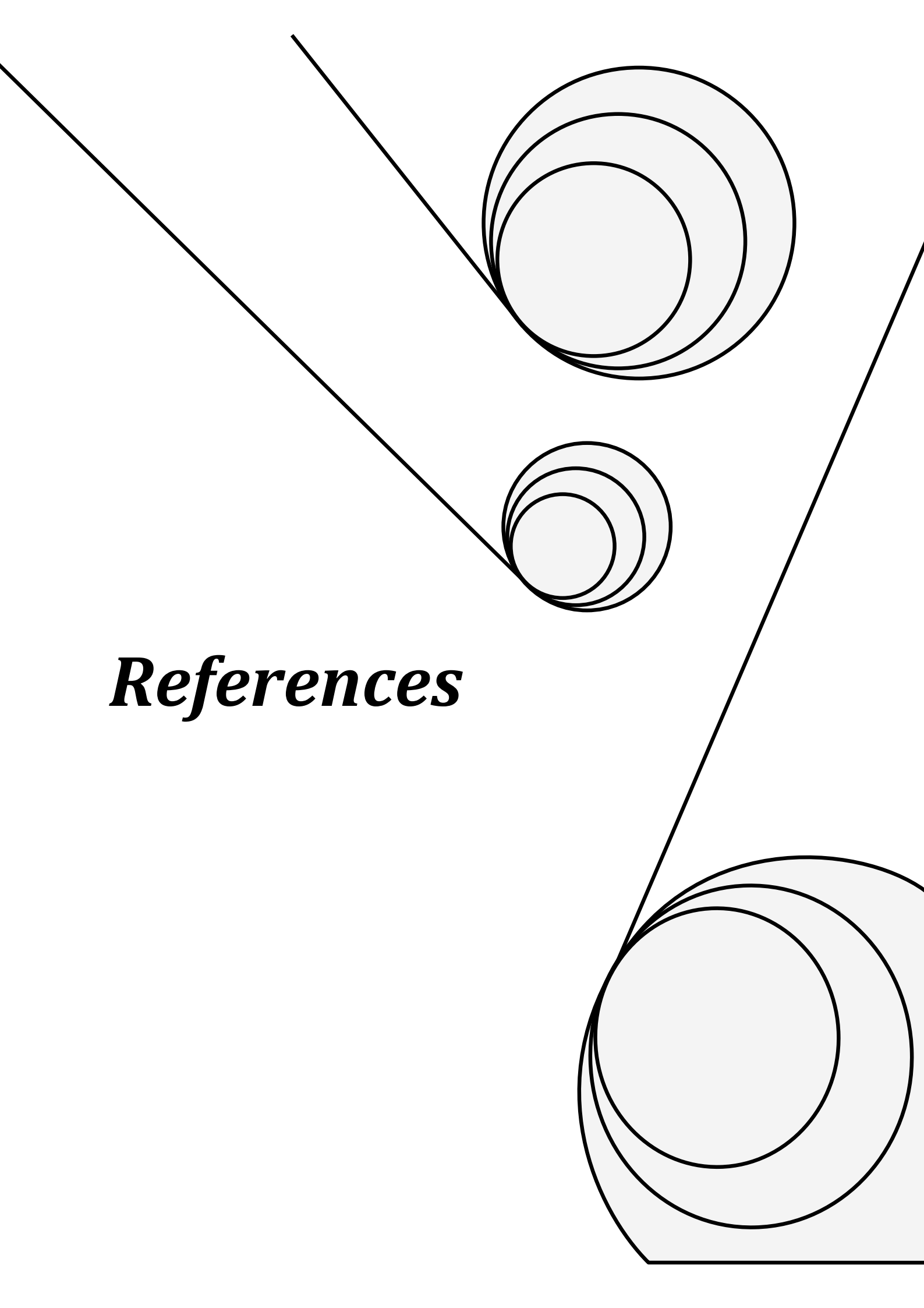
$$\text{Prob} (T \leq 85) = \text{Prob} (Z \leq \frac{85 - 80}{0.71}) = \text{Prob} (Z \leq 1.3) = 90.32\%.$$

Thus there is a 90.32% chance that the project will be completed within 80 days.

Suppose we want to know the probability of completing the project 4 days sooner than expected. This means we have to compute

$$\text{Prob} (T \leq 76) = \text{Prob} (Z \leq \frac{76-80}{0.71}) = \text{Prob} (Z \leq - 0.9) = 18.41\%.$$

Hence there is only a small 18.41% chance that the project will be completed in 76 days.



References

- [1] Punmia B.C. and Khandelwal K.K., " Project planning and control with PERT and CPM ", International text book company, 1987.
- [2] Katta G., " Interior Point Method for LP ", Clarendon press, oxford, 2003.
- [3] Srinath, L.S., " PERT and CPM, principles and Application ", Affiliated East west press, 1973.
- [4] Gupta P.K. and Hira D.S., " Operations Research ", S.Chand and Company Ltd, 1987.
- [5] Chandrasekaran R., " Linear Programming and Extensions " , Dallas university press, 1998.
- [6] Palmer C.F. " Quantitative Aids for Management Decision Making with applications " , Cambridge university press, 1979.
- [7] Mayers D.F., " On the Project Scheduling Under Resource Constraints " Vol. 23, No. 6, p: 494-500, 2006.
- [8] Saul I.Gass, " Linear Programming : methods & Application, forth Edition " , 1975.

- [9] Jorge N.S. and Wright j., " Numerical Optimization ", first edition, Dover publications, 1999.
- [10] Reo S.S," Optimization – Theory and Applications" 2nd edition, Wiley – Estren limited, 1984.
- [11] Brown J.W.," Project management", Journal of computational, vol.43, p: 189-193, 2006.
- [12] Phillips T. and Ravindran A., " Operation Research: Principles and Practice", Prentice- Hall, 1976.
- [13] Archibald R.D. and Villoria R.L.," Network – Based Management System (PERT / CPM) ",Affiliated East west press, 1967.
- [14] Chambers,L.M., " Project Planning with PERT / CPM ", Mir publishers, 2003.
- [15] Hillier F.S. and Liebermant G.J., " Introduction to Operation Research", S.Chand and Company Ltd, 1967.

Conclusion and future work

From this study, we concluded that there is a possible way to solve the project network by linear programming technique such as Interior –Point Method, other than the classical methods which one the CPM & PERT.

Also, the following problems may be recommended for future work, as open problems:

- (1) Studying the project scheduling problem when duration of activities (time) is random variable with probability distributions function other than Beta probability distribution duration like normal Gamma, Poisson, ... etc.
- (2) Studying the project scheduling problem when cost is a random variable due to fluctuation in prices.
- (3) Studying the project scheduling problem when building other models having cost is non linear function of activity duration (time).
- (4) Studying the scheduling of multi – project.

المستخلص

في هذه العمل, استخدمنا اسلوب النمذجة الخطية في بناء أربعة نماذج لإدارة المشاريع. واستخدمنا طريقة (Interior-Point Method) التي تتضمنه (simplex) (What's Best 9.0 software) لحل هذه النماذج بدلا من (simplex method), وإيجاد المسار الحرج (Critical path) بأقل وقت ممكن, وبأقل كلفة إضافية وبأقل كلف كلية (الكلف المباشرة – الكلف غير المباشرة) لمشروع حقيقي بسيط. وقد استخدمنا (Project 2000 software) لرسم شبكة المشروع واثبات النتائج التي حصلنا عليها سابقا. وأخيرا, استخدمنا اسلوب (PERT) لإيجاد احتمالية اكمال المشروع ضمن الوقت المحدد.



وزارة التعليم العالي والبحث العلمي
جامعة النهرين
كلية العلوم

أساليب النمذجة الخطية في إدارة شبكة المشاريع

رسالة

مقدمه إلى كلية العلوم في جامعة النهرين
وهي جزء من متطلبات نيل درجة ماجستير
علوم في الرياضيات

من قبل

إيلاف محمد عبد

(بكالوريوس علوم 4 2004)

بإشراف

د. علاء الدين نوري

DATE GENERATED:

MODEL INFORMATION:

CLASSIFICATION DATA

Numerics
Variables
Adjustables
Constraints
Integers/Binaries
Nonlinears
Coefficients

Minimum coefficient val
Minimum coefficient in
Maximum coefficient val
Maximum coefficient in

MODEL TYPE: L

SOLUTION STATUS: G

OBJECTIVE VALUE: 6

DIRECTION: M

SOLVER TYPE: .

TRIES: 5

INFEASIBILITY: 0

BEST OBJECTIVE BOUND: .

STEPS: .

ACTIVE: .

SOLUTION TIME: 0

End of Report

(model 1)

	time of node 1	time of node 2	time of node 3	time of node 4	time of node 5	time of node 6	time of node 7			
	t1	t2	t3	t4	t5	t6	t7			
	0	4	7	10	11	13	17			
	-1	0	0	0	0	0	1	min z =	17	day
(t2 -t1)	-1	1	0	0	0	0	0	4	#NAME?	4
(t3 -t1)	-1	0	1	0	0	0	0	7	#NAME?	7
(t3 -t2)	0	-1	1	0	0	0	0	3	#NAME?	2
(t4 -t3)	0	0	-1	1	0	0	0	3	#NAME?	3
(t5 -t3)	0	0	-1	0	1	0	0	4	#NAME?	3
(t6 -t4)	0	0	0	-1	0	1	0	3	#NAME?	3
(t6 -t5)	0	0	0	0	-1	1	0	2	#NAME?	2
(t7 -t6)	0	0	0	0	0	-1	1	4	#NAME?	4

the result of model 1 is :-

time of node 1 t1 = 0

time of node 2	t2 =	4
time of node 3	t3 =	7
time of node 4	t4 =	10
time of node 5	t5 =	11
time of node 6	t6 =	13
time of node 7	t7 =	17

the function is :-

$$\begin{aligned}\min z &= t_7 - t_1 \\ &= 17 - 0 \\ &= 17\end{aligned}$$

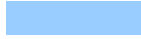
the min completion time for the project is 17 day

min z = 0 day

0	#NAME?	4
0	#NAME?	7
0	#NAME?	2
0	#NAME?	3
0	#NAME?	3
0	#NAME?	3
0	#NAME?	2
0	#NAME?	4

	time of node 1	time of node 2	time of node 3	time of node 4	time of node 5
	0	7	12	9	17
	-1	0	0	0	1
t2-t1	-1	1	0	0	0

t3-t1	-1	0	1	0	0
t4-t1	-1	0	0	1	0
t5-t2	0	-1	0	0	1
t5-t3	0	0	-1	0	1
t5-t4	0	0	0	-1	1



17

7 #NAME?

7

12	#NAME?	5
9	#NAME?	2
10	#NAME?	10
5	#NAME?	5
8	#NAME?	8

	time of node 1	time of node 2	time of node 3	time of node 4	time of node 5	time of node 6	time of node 7	time of node 8	time of node 9	time of node 10	time of node 11	time of node 12	time of node 13	time of node 14	time of node 15	time of node 16	time of node 17	time of node 18	time of node 19	time of node 20	
	0	4	12	20	23	30	21	31	36	36	43	44	40	60	55	61	61	62	62	63	
	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
(t2 -t1)	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4 #NAME?
(t3-t2)	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8 #NAME?
(t4 -t2)	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16 #NAME?
(t5 -t4)	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3 #NAME?
(t5 -t3)	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11 #NAME?
(t6 -t4)	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10 #NAME?
(t6 -t5)	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7 #NAME?
(t7 -t3)	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	9 #NAME?
(t8 -t5)	0	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	8 #NAME?
(t8 -t7)	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	10 #NAME?
(t10 -t6)	0	0	0	0	0	-1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	6 #NAME?
(t10 -t8)	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	5 #NAME?
(t9 -t8)	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	5 #NAME?
(t10 -t9)	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0 #NAME?
(t19 -t12)	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	18 #NAME?
(t11 -t8)	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	12 #NAME?
(t12 -t7)	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	0	0	0	15 #NAME?
(t12 -t9)	0	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	0	8 #NAME?
(t12 -t11)	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	1 #NAME?
(t13 -t10)	0	0	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0	0	0	4 #NAME?
(t14 -t6)	0	0	0	0	0	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	30 #NAME?
(t14 -t13)	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	20 #NAME?
(t15 -t13)	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	15 #NAME?
(t15 -t12)	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	1	0	0	0	0	0	11 #NAME?
(t16 -t14)	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	0	1 #NAME?
(t17 -t16)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0 #NAME?
(t17 -t15)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0	6 #NAME?
(t18 -t17)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	1 #NAME?
(t19 -t18)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0 #NAME?
(t20 -t19)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	1 #NAME?

min z = **63**

Fig (3.1) model 1

DATE GENERATED:

MODEL INFORMATION:

CLASSIFICATION DATA

Numerics
Variables
Adjustables
Constraints
Integers/Binaries
Nonlinears
Coefficients

Minimum coefficient val
Minimum coefficient in
Maximum coefficient val
Maximum coefficient in

MODEL TYPE: L

SOLUTION STATUS: G

OBJECTIVE VALUE: -

DIRECTION: M

SOLVER TYPE: .

TRIES: 6

INFEASIBILITY: 0

BEST OBJECTIVE BOUND: .

STEPS: .

ACTIVE: .

SOLUTION TIME: 0

End of Report

	time of node 1	time of node 2	time of node 3	time of node 4	time of node 5	time of activity1,2	time of activity1,3	time of activity 1,4	time of activity2,5	time of activity 3,5	time of activity 4,5	
	0	7	7	6	20	7	7	6	13	11	12	
	0	0	0	0	0	-10	-5	-3	-11	-4	-8	
t2-t1	-1	1	0	0	0	-1	0	0	0	0	0	0
t3-t1	-1	0	1	0	0	0	-1	0	0	0	0	0
t4-t1	-1	0	0	1	0	0	0	-1	0	0	0	0
t5-t2	0	-1	0	0	1	0	0	0	-1	0	0	0
t5-t3	0	0	-1	0	1	0	0	0	0	-1	0	2
t5-t4	0	0	0	-1	1	0	0	0	0	0	-1	2
	0	0	0	0	0	1	0	0	0	0	0	7
	0	0	0	0	0	0	1	0	0	0	0	7
	0	0	0	0	0	0	0	1	0	0	0	6
	0	0	0	0	0	0	0	0	1	0	0	13
	0	0	0	0	0	0	0	0	0	1	0	11
	0	0	0	0	0	0	0	0	0	0	1	12
	0	0	0	0	0	1	0	0	0	0	0	7
	0	0	0	0	0	0	1	0	0	0	0	7
	0	0	0	0	0	0	0	1	0	0	0	6
	0	0	0	0	0	0	0	0	1	0	0	13
	0	0	0	0	0	0	0	0	0	1	0	11
	0	0	0	0	0	0	0	0	0	0	1	12
	-1	0	0	0	1	0	0	0	0	0	0	20

#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	12
#NAME?	7
#NAME?	6
#NAME?	14
#NAME?	11
#NAME?	12
#NAME?	7
#NAME?	5
#NAME?	2
#NAME?	10
#NAME?	5
#NAME?	8
#NAME?	20

time of activity L time of activity 6,14 time of activity 13,14 time of activity 13,15 time of activity 15,17 time of activity 12,19 time of activity 14,16 time of activity 17,18 time of activity 19,20

4 8 20 15 5 18 1 1 1.00E+00

-2 -5 -7 -4 -3 -1 -2 -5 -3

min z = -596

0	0	0	0	0	0	0	0	0	0	0	#NAME?	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	12	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	11	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	17	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	2	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	4	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	-1	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	8	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	14	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	8	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
-1	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	-1	0	0	0	0	0	0	0	0	0	22	#NAME?	0
0	0	-1	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	-1	0	0	0	0	0	0	0	1	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	12	#NAME?	0
0	0	0	0	0	0	-1	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	-1	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	-1	0	0	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	-1	0	#NAME?	0
0	0	0	0	0	0	0	0	0	0	0	4	#NAME?	4
0	0	0	0	0	0	0	0	0	0	0	8	#NAME?	8
0	0	0	0	0	0	0	0	0	0	0	4	#NAME?	4
0	0	0	0	0	0	0	0	0	0	0	3	#NAME?	3
0	0	0	0	0	0	0	0	0	0	0	5	#NAME?	5
0	0	0	0	0	0	0	0	0	0	0	5	#NAME?	5
0	0	0	0	0	0	0	0	0	0	0	9	#NAME?	9
0	0	0	0	0	0	0	0	0	0	0	8	#NAME?	8
0	0	0	0	0	0	0	0	0	0	0	10	#NAME?	10
0	0	0	0	0	0	0	0	0	0	0	1	#NAME?	1
0	0	0	0	0	0	0	0	0	0	0	6	#NAME?	6
0	0	0	0	0	0	0	0	0	0	0	3	#NAME?	3
0	0	0	0	0	0	0	0	0	0	0	1	#NAME?	1
0	0	0	0	0	0	0	0	0	0	0	1	#NAME?	1
0	0	0	0	0	0	0	0	0	0	0	5	#NAME?	5
1	0	0	0	0	0	0	0	0	0	0	4	#NAME?	4
0	0	1	0	0	0	0	0	0	0	0	8	#NAME?	8
0	0	0	0	0	0	0	0	0	0	0	20	#NAME?	20
0	0	0	0	1	0	0	0	0	0	0	15	#NAME?	15
0	0	0	0	0	1	0	0	0	0	0	5	#NAME?	5
0	0	0	0	0	0	1	0	0	0	0	18	#NAME?	18
0	0	0	0	0	0	0	1	0	0	0	1	#NAME?	1
0	0	0	0	0	0	0	0	1	0	0	1	#NAME?	1
0	0	0	0	0	0	0	0	0	0	0	4	#NAME?	6
0	0	0	0	0	0	0	0	0	0	0	8	#NAME?	9
0	0	0	0	0	0	0	0	0	0	0	4	#NAME?	7
0	0	0	0	0	0	0	0	0	0	0	3	#NAME?	4
0	0	0	0	0	0	0	0	0	0	0	5	#NAME?	9
0	0	0	0	0	0	0	0	0	0	0	5	#NAME?	6
0	0	0	0	0	0	0	0	0	0	0	9	#NAME?	11
0	0	0	0	0	0	0	0	0	0	0	8	#NAME?	10
0	0	0	0	0	0	0	0	0	0	0	10	#NAME?	14
0	0	0	0	0	0	0	0	0	0	0	1	#NAME?	2
0	0	0	0	0	0	0	0	0	0	0	6	#NAME?	7
0	0	0	0	0	0	0	0	0	0	0	3	#NAME?	4
0	0	0	0	0	0	0	0	0	0	0	1	#NAME?	2
0	0	0	0	0	0	0	0	0	0	0	1	#NAME?	3
1	0	0	0	0	0	0	0	0	0	0	5	#NAME?	6
0	0	0	0	0	0	0	0	0	0	0	4	#NAME?	5
0	0	1	0	0	0	0	0	0	0	0	8	#NAME?	10
0	0	0	0	0	0	0	0	0	0	0	20	#NAME?	24
0	0	0	0	0	0	0	0	0	0	0	15	#NAME?	18
0	0	0	0	0	0	0	0	0	0	0	5	#NAME?	6
0	0	0	0	0	0	0	0	0	0	0	18	#NAME?	20
0	0	0	0	0	0	0	0	0	0	0	1	#NAME?	2
0	0	0	0	0	0	0	0	0	0	0	1	#NAME?	1
0	0	0	0	0	0	0	0	0	0	0	1	#NAME?	2
0	0	0	0	0	0	0	0	0	0	0	63	#NAME?	63

DATE GENERATED:

MODEL INFORMATION:

CLASSIFICATION DATA

Numerics
Variables
Adjustables
Constraints
Integers/Binaries
Nonlinears
Coefficients

Minimum coefficient val
Minimum coefficient in
Maximum coefficient val
Maximum coefficient in

MODEL TYPE: L

SOLUTION STATUS: G

OBJECTIVE VALUE: 1

DIRECTION: M

SOLVER TYPE: .

TRIES: 0

INFEASIBILITY: 0

BEST OBJECTIVE BOUND: .

STEPS: .

ACTIVE: .

SOLUTION TIME: 0

End of Report

the result of model 2 is :-

time of node 1	t1 =	0
time of node 2	t2 =	0
time of node 3	t3 =	0
time of node 4	t4 =	0
time of node 5	t5 =	0
time of node 6	t6 =	0
time of node 7	t7 =	0

time of job a	t8 =	0
time of job b	t9 =	0
time of job c	t10 =	0
time of job d	t11 =	0
time of job e	t12 =	0
time of job f	t13 =	0
time of job g	t14 =	0
time of job h	t15 =	0

(model 3)

time of node 1 t1	time of node 2 t2	time of node 3 t3	time of node 4 t4	time of node 5 t5	time of node 6 t6	time of node 7 t7	time of job a t8	time of job b t9	time of job c t10	time of job d t11	time of job e t12	time of job f t13	time of job g t14	time of job h t15	sum of cij *kij t16	
0	4	7	10	10	13	17	7	4	2	3	3	3	2	4	138	
-1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	
-1	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
-1	1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
0	-1	1	0	0	0	0	0	0	-1	0	0	0	0	0	0	1
0	0	-1	1	0	0	0	0	0	0	-1	0	0	0	0	0	0
0	0	-1	0	1	0	0	0	0	0	0	-1	0	0	0	0	0
0	0	0	-1	0	1	0	0	0	0	0	0	-1	0	0	0	0
0	0	0	0	-1	1	0	0	0	0	0	0	0	-1	0	0	1
0	0	0	0	0	-1	1	0	0	0	0	0	0	0	-1	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	7
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	4
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	2
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	3
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	3
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	4
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	7
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	4
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	2
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	3
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	3
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	3
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	4

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	138
0	0	0	0	0	0	0	-4	-2	-2	-3	-3	-5	-1	-4	1	47

the result of model 3 is :-

time of node 1	t1 =	0
time of node 2	t2 =	4
time of node 3	t3 =	7
time of node 4	t4 =	7
time of node 5	t5 =	10
time of node 6	t6 =	10
time of node 7	t7 =	13

time of job a	t8 =	17
time of job b	t9 =	7
time of job c	t10 =	4
time of job d	t11 =	2
time of job e	t12 =	3
time of job f	t13 =	3
time of job g	t14 =	3
time of job h	t15 =	2

the function is :-

$$\begin{aligned} \min z &= t7 - t1 \\ &= 17 - 0 \\ &= 17 \end{aligned}$$

min z = 17

#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	0
#NAME?	10
#NAME?	5
#NAME?	3
#NAME?	4
#NAME?	5
#NAME?	6
#NAME?	5
#NAME?	5
#NAME?	7
#NAME?	4
#NAME?	2
#NAME?	3
#NAME?	3
#NAME?	3
#NAME?	2
#NAME?	4

#NAME? 138
#NAME? 47

DATE GENERATED:

MODEL INFORMATION:

CLASSIFICATION DATA

Numerics
Variables
Adjustables
Constraints
Integers/Binaries
Nonlinears
Coefficients

Minimum coefficient val
Minimum coefficient in
Maximum coefficient val
Maximum coefficient in

MODEL TYPE: L

SOLUTION STATUS: G

OBJECTIVE VALUE: 1

DIRECTION: M

SOLVER TYPE: .

TRIES: 2

INFEASIBILITY: 0

BEST OBJECTIVE BOUND: .

STEPS: .

ACTIVE: .

SOLUTION TIME: 0

End of Report

time of node 1	time of node 2	time of node 3	time of node 4	time of node 5
t1	t2	t3	t4	t5
0	5	8	11	13
5	0	0	0	0
-1	0	1	0	0
-1	1	0	0	0
0	-1	1	0	0
0	0	-1	1	0
0	0	-1	0	1
0	0	0	-1	0
0	0	0	0	-1
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

the result of model 2

time of node 1
time of node 2
time of node 3
time of node 4
time of node 5
time of node 6
time of node 7

the function is :-

$$\begin{aligned} \min z &= 13\text{€} \\ &= 138 \\ &= 121 \end{aligned}$$

that mean t
project lear

(model 4)

time of node 6 t6	time of node 7 t7	time of job a t8	time of job b t9	time of job c t10	time of job d t11	time of job e t12	time of job f t13	time of job g t14
17	21	8	5	3	3	5	6	4
0	-5	4	2	2	3	3	5	1
0	0	-1	0	0	0	0	0	0
0	0	0	-1	0	0	0	0	0
0	0	0	0	-1	0	0	0	0
0	0	0	0	0	-1	0	0	0
0	0	0	0	0	0	-1	0	0
1	0	0	0	0	0	0	-1	0
1	0	0	0	0	0	0	0	-1
-1	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0

is :-

t1 = 0
 t2 = 5
 t3 = 8
 t4 = 11
 t5 = 13
 t6 = 17
 t7 = 21

time of job a t8 = 8
 time of job b t9 = 5
 time of job c t10 = 3
 time of job d t11 = 3
 time of job e t12 = 5
 time of job f t13 = 6
 time of job g t14 = 4
 time of job h t15 = 4

$$\begin{aligned}
 & 3 - (5t_1 - 5t_7 + 4t_8 + 2t_9 + 2t_{10} + 3t_{11} + 3t_{12} + 5t_{13} + t_{14} + 4t_{15}) \\
 & i - 17 \\
 & \$
 \end{aligned}$$

The minimum cost of the project is 121 \$ when the optimal
 length is 21 day

time of
job h
t15
4

min z = 17

0	0	#NAME?	0
0	0	#NAME?	0
0	0	#NAME?	0
0	0	#NAME?	0
0	0	#NAME?	0
0	0	#NAME?	0
0	0	#NAME?	0
-1	0	#NAME?	0
0	8	#NAME?	10
0	5	#NAME?	5
0	3	#NAME?	3
0	3	#NAME?	4
0	5	#NAME?	5
0	6	#NAME?	6
0	4	#NAME?	5
1	4	#NAME?	5
0	8	#NAME?	7
0	5	#NAME?	4
0	3	#NAME?	2
0	3	#NAME?	3
0	5	#NAME?	3
0	6	#NAME?	3
0	4	#NAME?	2
1	4	#NAME?	4

t1	t2	t3	t4	t5	t6	t7	t8	t9
0	0	0	0	0	0	0	0	0
-5	0	0	0	0	0	5	-4	-2
-1	0	1	0	0	0	0	-1	0
-1	1	0	0	0	0	0	0	-1
0	-1	1	0	0	0	0	0	0
0	0	-1	1	0	0	0	0	0
0	0	-1	0	1	0	0	0	0
0	0	0	-1	0	1	0	0	0
0	0	0	0	-1	1	0	0	0
0	0	0	0	0	-1	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

$\min z = 138$

t10	t11	t12	t13	t14	t15	t16	
0	0	0	0	0	0	1.38E+02	
-2	-3	-3	-5	-1	-4	1	
0	0	0	0	0	0	0	0 =>=
0	0	0	0	0	0	0	0 =>=
-1	0	0	0	0	0	0	0 >=
0	-1	0	0	0	0	0	0 =>=
0	0	-1	0	0	0	0	0 >=
0	0	0	-1	0	0	0	0 =>=
0	0	0	0	-1	0	0	0 =>=
0	0	0	0	0	-1	0	0 =>=
0	0	0	0	0	0	0	0 <=
0	0	0	0	0	0	0	0 <=
1	0	0	0	0	0	0	0 <=
0	1	0	0	0	0	0	0 <=
0	0	1	0	0	0	0	0 <=
0	0	0	1	0	0	0	0 <=
0	0	0	0	1	0	0	0 <=
0	0	0	0	0	1	0	0 <=
0	0	0	0	0	0	0	0 =>=
0	0	0	0	0	0	0	0 =>=
1	0	0	0	0	0	0	0 =>=
0	1	0	0	0	0	0	0 =>=
0	0	1	0	0	0	0	0 =>=
0	0	0	1	0	0	0	0 =>=
0	0	0	0	1	0	0	0 =>=
0	0	0	0	0	1	0	0 =>=
0	0	0	0	0	0	1	0 =>=
0	0	0	0	0	0	1	138 #NAME?

0
0
0
0
0
0
0
0
10
5
3
4
5
6
5
5
7
4
2
3
3
3
2
4
138

