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Reliability of Dynamic Multi-State Fuzzy Systems

A Thesis
Submitted to the Council of the College of Science / Al-Nahrain University in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mathematics

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بسم الله الرحمن الرحيم

يَرفَعُ ٱللَّهُ ٱلَّذِينَ آمَنُواْ مِنكُم
وَٱلَّذِينَ أُوتُواْ ٱلعِلمَ درجَٰتٍ
وٱللّهُ بِما تَعُمَلُونَ خَبِيرٌ

صدق الله العظيم

سورة المجادلة
الآية (11)

Dedication

To the martyrs of knowledge and all those who walked the path of science, but fate and circumstances cut short their endeavors ...
I dedicate my effort with much honor and respect.

إلى شهداء المعرفة وكل من كان سائرا في طريق العلم وشاء القدر وحالت الظروف بينه وبين انهائه للدرب ...
أهدي جهدي اجلالاً واحتراماً.

Ahmed

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Finally, I would like to tank my fellow students for the support they gave to me during my study.

Ahmed Yacoub Yousif

April, 2014 ✍

Supervisors Certifications

We certify that this thesis entitled ***(Reliability of Dynamic Multi-State Fuzzy Systems)*** was prepared by ***(Ahmed Yacoub Yousif)*** under our supervision at the Department of Mathematics and Computer Applications, College of Science, Al-Nahrain University, in partial fulfillment of the requirements for the degree of Master of Science in Mathematics.

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Abstract

This thesis has three main objectives:

The first objective is to study reliability theory of multi-state systems, as well as, some of its basic properties and theoretical results.

The second objective is to study the reliability of dynamic multi-state system in which the dynamic reliability indices are used to estimate the influence upon the multi-state system reliability. A practical application of the dynamic multi-state system is given, which is the oil supply system from an oil source to three station through several oil pipelines, say four. This application have not been modeled previously as a dynamic multi-state system.

The third objective is to introduce and study dynamic fuzzy reliability of fuzzy state probability and performance rate of fuzzy multi-state system that can be evaluated through aggregating the fuzzy behavior of fuzzy multi-state system.

Nomenclatures and Notations

MTTF	***Mean Time to Failure***
BSS	***Binary -State System***
MSS	***Multi-State System***
DRI	***Dynamic Reliability Indices***
CDRI	***Component Dynamic Reliability Indices***
DIRI	***Dynamic Integrated Reliability Indices***
$\boldsymbol{\Phi}(\boldsymbol{x})$	***Structure function; system state for*** $\boldsymbol{x}$***.***
$\boldsymbol{x_i}$	***State of component i***
$\boldsymbol{x}$	***Component state vector*** $(\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n})$
n	***Number of MSS components***
m	***Number of discrete levels of MSS reliability (from zero to m)***
$\boldsymbol{M_i}$	***Best state of the system;*** $\boldsymbol{M_i} \in \{\mathbf{1}, \mathbf{2}, \dots\}$
IM	***Importance Measures***
$\boldsymbol{p_{i,s_i}}$	***The i-th component state probability (s = 0, …, (m-1))***
$\frac{\partial\Phi(j\to h)}{\partial x_i(a\to b)}$	***The direct partial logic derivative of the Structure function*** $\boldsymbol{\Phi}(\boldsymbol{x})$ ***with respect to variable*** $\boldsymbol{x_i}$

$P_f(i)$	***Failure probability of MSS if the i-th component is breakdown (CDRI of failure for i-th component)***
$P_r(i)$	***Repair probability of the MSS if i-th failure component is replace (CDRI of repair for i-th component)***
P_f	***Failure probability of the MSS if one of system components breakdowns (DIRI of MSS failure)***
P_r	***Repair probability of the MSS if one of system failure components replaced (CDRI of MSSS repair)***
ρ_1	***Number of system states "1"*** $\Phi(x) = 1$
ρ_0	***Number of system states "1"*** $\Phi(x) = 0$
$\tilde{x}_i$	***Performance level of component in state i , which is represented as a fuzzy value***
FMSS	***Fuzzy Multi-State System***

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Introduction

In conventional reliability theory, binary system reliability models have been widely used to study the effectiveness and reliability of real life problems. However, for some engineering systems, the binary assumption does not accurately represent the possible states that each of the systems may experience, [29].

Compared with a binary system model, a multi-state system (MSS) model provides a more flexible tool for representing engineering systems in real life phenomenon, as first introduced in [2] and [36]. In conventional multi-state theory, it is assumed that the exact probability, and performance level of each component state are given. With the progress of modern industrial technologies, however, product development cycles have become shorter, while the lifetimes of products have become longer, [18].

In many highly reliable applications, there may be only a few available observations of the system’s failures. Therefore, it may be difficult to obtain sufficient data to estimate the precise values of the probabilities and performance levels of these systems. Moreover, the inaccuracy of system models, caused by human errors, is difficult to quantify using conventional reliability theory alone, [20]. In light of these significant challenges, new techniques are needed to solve these fundamental problems related to reliability.

In some cases, the fuzzy set theory provides a useful tool to complement conventional reliability theories. Fuzzy reliability theory, which employs the fuzzy theory introduced by Zadeh in 1965, [49] and in 1978, [48], is becoming a new methodology to study the imprecision and uncertainty phenomena in reliability engineering [31], and it has since that received increasing attention. For example, Cai, Wen and Zhang in 1991, [7] introduced the fuzzy success/failure state and the reliability model to study a gradually degrading computing system. Huang in 1995, [19] assessed the reliability a system in the presence of fuzziness in operating time. Huang, Tong and Zuo in 2004, [20] proposed to evaluate the failure possibility via posbist fault tree analysis when statistical data is scarce or failure probability is extremely small. A novel fuzzy bayesian approach was developed by Wu in 2004, [46] to create the fuzzy bayes point estimator of reliability. Huang, Zuo and Sun in 2006, [18] introduced a bayesian method to assess system reliability when lifetime data is presented as a fuzzy value. Pandey and Tyagi in 2007, [40] proposed a new method to assess the profust reliability indices. The concept of fuzzy multi-state system (FMSS) was first used by Ding and Lisnianski in 2008, [8] in a modeling study of the state probabilities and performances of a component presented as fuzzy values. Only the basic definition of a FMSS is provided instead of a general one. Further analysis and discussion of FMSS are still needed, [9].

The aim of this thesis is to study the principles of reliability of multi-state systems and solving real life problem of oil supply system and then generalize the ideas of dynamic multi-state system to introduce and study the reliability of fuzzy dynamic multi-state system.

This thesis consists of three chapters.

In chapter one, we introduce some of basic and necessary concepts, which are fundamental to the work of this thesis.

In chapter two, we study the approach for evaluation of dynamic properties of the MSS reliability by the dynamic reliability indices (DRI). The

DRI are calculated with respect to certain structure function by the direct partial logic derivatives. These indices characterize the change of the MSS reliability that is caused by the change of a component state (component efficiency). We analyze MSS reliability for different types of system structure (parallel, series and k-out-of-n). These types of systems are typically employed in reliability analysis. Finally, this chapter also contain a study of real life problem of an engineering system which has been modified and improved and therefore studied using dynamic multi-state criteria.

In chapter three, we give and introduce a new approach for studying the reliability of fuzzy multi-state system using dynamic reliability analysis.

Chapter One

Fundamental Concepts of Reliability Theory and Multi-State System

CHAPTER 1

Fundamental Concepts of Reliability Theory and Multi-State System

1.1 Introduction

Reliability plays a very important role for manufacturers and users. Thereby, the designer of reliability optimization problems seek to improve reliability at the minimum cost. The redundancy and reliability allocation problem is a classical optimization problem in the area of system reliability. In general, the objective of these problems is to optimize the system design in terms of the number of components and its reliabilities, subject to known constraints on resources as cost, weight, volume, availability, mean time to failure, etc., [35].

During the last decade, much work was devoted to study the binary state reliability analysis and optimization, where it is assumed that a system has only two possible states: one working state and one failure state. Less attention has been paid to develop methods for analyzing and optimizing the reliability of multi-state systems. Performance degradation is closer to reality than the two state performances of binary systems. Therefore, it is important to develop the theory multi-state system reliability, [1].

A binary reliability system, the system and its components are assumed to be either working or failed may not be adequate in many real-life situations. In a MSS reliability, both the system and its components may assume more than two levels of performance varying from perfect functioning to complete failure. With a discrete multi-state system model, it is often assumed that both the system and its components may be in one of M possible states, $0, 1,2, \dots, M-1; M \in \mathbb{N}$, where $M-1$ is the perfectly functioning state while 0 is the complete failure state. We use $x_i, i = 1,2, \dots, n,$ to denote the state or performance level of the component, and the vector $\mathrm{x} = (x_1, x_2, \dots, x_n)$ represents the states of all n components. The system state is denoted by Φ which is called the structures function that is a deterministic function of component states: $\Phi = \Phi(\mathrm{x}): \mathrm{S}^{\mathrm{n}} \to \mathrm{S}$, where $S = \{0,1 \dots, M-1\}$, [21].

There are numerous examples of MSS, with more than two ordered or unordered states at the system level, or the component level. As water distribution, a power plant which has five states 0,1,2,3,4 that correspond to generating electricity of 0 %, 25 %, 50 %, 75% , 100 % of its full capacity is an example of a MSS that has ordered multiple states, [3].

This chapter consist of five sections. In section 1.2, we give a literature survey for MSS, as well as, historical background related to the subject.

In section 1.3, we discuss and study in details the theoretical and practical background of system reliability and its characteristics.

In section 1.4, an axiomatic approach to the notion of the class of binary coherent system have been introduced and study the properties of its structure function.

In section 1.5, the structure function related to MSS have been studied in details and also introduces the construction of different structure functions for real-world systems. Also, we consider some properties of the structure function that determine some important properties of MSS.

1.2 Literature Survey

Many standard works on reliability theory adopt this framework in which systems and components can be in only one of two models, the mathematical and statistical theory of this case has been studied extensively by many authors, such as Birnbaum, et al. in 1961, [4].

The basic concepts of MSS reliability were primarily introduced in the mid of the 1970's by Murchland in, [36] and Ross in 1979, [43]. El-Neweihi, et al. in 1978 [12] analyzed the theoretical relationships between MSS reliability behavior and multi-state component performance. Barlow and Wu in 1978 [2] characterize component state criticality as a measure of how a particular component state affects a specific system state. Griffithin 1980 [17] formalized the concept of MSS performance, and studied the impact of component improvement on the overall system reliability behavior.

The important of MSS concepts were also discussed by Block and Savits in 1982 [5], where a decomposition theorem for MSS structure function was proved. Since that time, MSS reliability began with an intensive development. Essential achievements that were attained up to the mid of 1980's were reflected by Natvig in 1985 [38], and by El-Neveihi and Prochan in 1984 [11], where it can be found the state of the art in the field of MSS reliability at this stage.

Readers that are more interested in the history and more ideas related to the theory MSS reliability for the later work can find the corresponding overview in [29] and [39].

Lisnianski and Levitin in 2003 [29], Lisnianski et al. in 2010 [30] presented a detailed analysis of MSS reliability estimation and quantification methods in which, they considered a lot of examples as an applications of

MSS in reliability analysis of information, manufacturing, production, power generation, transportation and other systems.

Lisnianski and Levitin in 2003 [29] have considered the basic Importance Measures (IM) for systems with two performance level and multi-state components and their definitions by output performance measure. Ramirez-Marquez and Coit in 2005 [42] have generalized this result for MSS and have proposed new type of IM that is labeled as composite importance measures and then Meng in 2009 [33] has presented a review of IM's.

1.3 Basic Concepts of Reliability Theory

In this section, some fundamental concepts related to reliability theory will be given for completeness of background ideas used in this work.

1.3.1 Reliability:

Reliability is sometimes referred to as the quality in the time dimension, because it is determined by failures that may or may not occur during the life of the product, [30].

Definition (1.1) (Failure), [35]:

Failure can be defined as the termination of an item's ability to perform a required function.

Failure is regarded as a random phenomenon, since it occurs at an uncertain time.

Definition (1.2) (Reliability), [35]:

Reliability is defined as the probability that a component or system will perform a required function for a given period of time when used under stated operating conditions. It is the probability of a non-failure over time.

Probability theory has been used to analyze the reliability of components, as well as, the reliability of systems consisting of these components. Since the performance of a system usually depends on the performance of its components, the reliability of a system is a function of the reliability of its components. The intended function of the device is supposed to be understood and the degree of success of the device performance of the intended function can be measured so that we can easily conclude if the performance is satisfactory or not. Time is an important factor in the definition of reliability. If a newly purchased device can perform its intended functions satisfactorily, what is the probability that it will last (continue to perform satisfactorily) for a specified period of time?. How long will it last?, in other words, what will be the life of this device?. The life time of the device may be treated also as a random variable with a statistical distribution.

Furthermore, the operating conditions, such as stress, load, temperature, pressure, and/or other environmental factors, under which the device is expected to operate must be specified and considered by the disfigure and manufacturers.

1.3.2 Reliability Function, [28]:

In this section, a different point of view of reliability analysis will be given by considering the life time length of a system and the life time length of its components . In general, life time length of any system (or component) is a random variable, and so this lead to study of its life time distribution.

Let T be the random variable representing the life time of a device. The units of measurement for the life time may be a time unit, such as seconds, hours, days and years or any usage unit, such as miles driven and cycles of operation. The random variable T is continuous and can take only nonnegative values.

Its statistical distribution can be described by its probability density function f (t), its cumulative distribution function F(t), i.e.,

$$\mathrm{F(t) = Pr(system\ fail\ at\ time\ \leq t)}$$

$$= \mathrm{Pr}(\mathrm{T} \leq \mathrm{t}) = \int_{\mathrm{t}}^{\infty} \mathrm{f(x)dx}\,;\mathrm{t} > 0 \qquad (1.1)$$

<u>Definition (1.3) (Reliability Function), [28]:</u>

The reliability function of a system at time t if is the probability that the system will adequately perform its intended function for a specified interval of time $(0, t]$, mathematically:

$$\mathrm{R(t) = Pr(system\ function\ successefully\ throughtout\ the\ interval\ (0,t])}$$

$$= \mathrm{Pr}(\mathrm{T} > \mathrm{t})$$

$$= 1 - \mathrm{F(t)}$$

$$= \int_{\mathrm{t}}^{\infty} \mathrm{f(x)dx}\,;\mathrm{t} > 0 \qquad (1.2)$$

Where f(x) is the probability density function.

1.3.3 Mean Time to Failure:

Usually, we are interested in the expected time to next failure, and this is termed as the mean time to failure.

<u>Definition (1.4) (The Mean Time to Failure), [34]:</u>

The mean time to failure (MTTF) is defined as the expected value of the life time before a failure occurs.

Suppose that the reliability function for a system is given by R(t), the MTTF can be computed as:

$$\mathrm{MTTF} = \int_0^{\infty} \mathrm{tf(t)dt} = \int_0^{\infty} \mathrm{R(t)dt} \qquad (1.3)$$

Where f(t) is the probability density function.

1.3.4 Failure Rate Function:

The failure rate function, or hazard function, is very important in reliability analysis, because it specifies the rate of the system aging. The definition of failure rate function is given in the next definition:

Definition (1.5) (Hazard Function), [28]:

The failure rate function, or the hazard function, denoted by h(t), is defined to be the probability that a device will fail in the next time unit given that it has been working properly up to time t, that is:

$$h(t) = \frac{f(t)}{R(t)};\, t > 0 \qquad (1.4)$$

The cumulative failure rate function, or the cumulative hazard function, denoted by H(t), is defined to be:

$$H(t) = \int_0^t h(w)dw \qquad (1.5)$$

The failure rate function is often used to indicate the health condition of a working device. A high failure rate indicates a bad health condition or status, because the probability for the device to fail in the next instant of time is high.

1.3.5 Maintainability and Availability:

When a system fails to perform satisfactorily, repair is normally carried out to locate and correct the fault. The system is restored to operational effectiveness by making an adjustment or by replacing a component, [34].

Definition (1.6) (Maintainability), [34]:

Maintainability $V(t)$ is defined as the probability that a failed system will be restored to a functioning state within a given period of time when maintenance is performed according to prescribed procedures and resources.

Generally, maintainability is the probability of isolating and repairing a fault in a system within a given time. Maintenance personnel have to work with system designers to ensure that the system product can be maintained cost effectively.

Let T denote the time to repair or the total downtime. If the repair time T has a density function $g(t)$, then the maintainability, $V(t)$, is defined as the probability that the failed system will be back in service by time t, i.e.,

$$V(t) = \Pr(T \leq t) = \int_0^t g(x)dx \quad (1.6)$$

An important measure often used in maintenance studies is the mean time to repair (MTTR) or the mean downtime. The MTTR is the expected value of the repair time.

Another important related reliability concept is the system availability. This is a measure that takes both reliability and maintainability into account.

Definition (1.7) (Availability), [34]:

The availability function of a system, denoted by A(t), is the probability that the system is available at time t.

Different from the reliability that focuses on a period of time when the system is free of failures, availability concerns a time point at which the system does not stay at the failed state. Mathematically:

$$A(t) = \Pr(\text{System is up or available at time instant } t)$$

The availability function, which is a complicated function of time, has a simple steady-state or asymptotic expression. In fact, usually we are mainly concerned with systems running for a long time. The steady-state or asymptotic availability is given by:

$$A = \lim_{t \to \infty} A(t) = \frac{\text{System up time}}{\text{System up time} + \text{System down time}}$$

$$= \frac{MTTF}{MTTF + MTTR}$$

The mean time between failures (MTBF) is another important measure in repairable systems. This implies that the system has failed and has been repaired. Like MTTF and MTTR the MTBF is an expected value of the random variable time between failures. Mathematically,

MTBF=MTTR+ MTTF.

1.4 Binary Systems

This section presents a review of the structural and properties of the binary model that are most commonly used previously in reliability theory.

1.4.1 Binary Items, [35]:

An item is an entity that is not further subdivided. This imply that an item, in a given reliability study, is regarded as a self-contained unit and is not analyzed in terms of the performance of its constituents.

A binary item possesses two states: perfect functioning and complete failure. Any item is considered in perfect functioning at the starting time $t = 0$. When the item changes from functioning state to failure state, we say that it failed. The item state at time t is expressed by a binary variable $X(t)$, where:

$$X(t) = \begin{cases} 1, \text{functioning} \\ 0, \text{faild} \end{cases}$$

An important concept emerges here, which refers to the time elapsing form, when the item is given into operation until it fails for first time, called time to failure. It is not necessarily measured in time units. It can be measured by indirect time concepts, such as cycles, distances, counting, etc. The time to failure is modeled as a random variable T, because it is subject to chance variations. Figure (1.1) shows the relation between the state variable $\mathrm{X(t)}$ and the time to failure T.

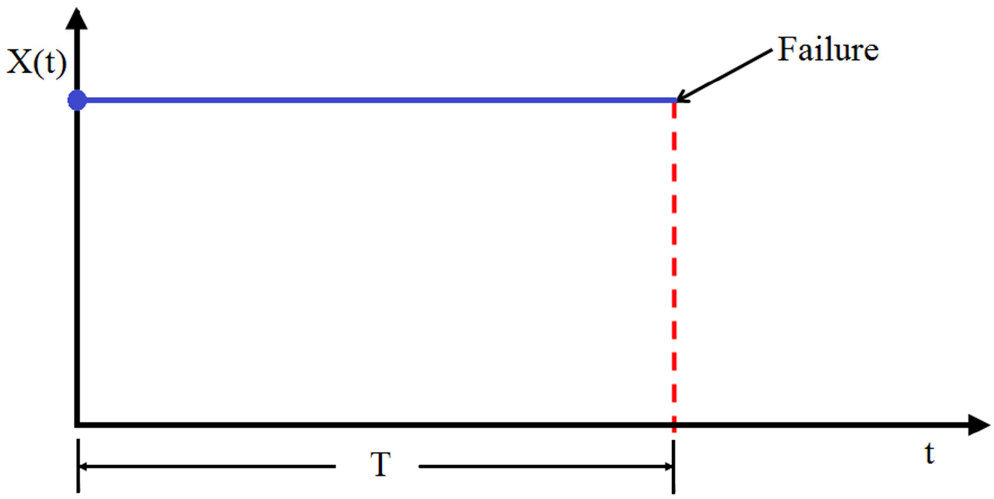


Figure (1.1) Time to failure in binary items.

1.4.2 Mathematical Formulation of Binary System, [6]:

Consider a system of n components, let $C = \{1,2, \dots, n\}$ denote the set of component indices. To indicate the state of the $i-th$ component $i = 1,2, \dots, n$, assign a binary indicator variable x_i to component i; $x_i = 1$ if component i is functioning and $x_i = 0$ if component i is failed .

Similarly, the binary variable Φ indicates the state of system ;$\Phi = 1$ if the system is functioning, $\Phi = 0$ if the system is failed. Let $S = \{0,1\}$, and assume that the state of the system is determined completely by the states of the components, so we may write $\Phi: S^n = S \times S \times \dots \times S \to S$.

The function Φ is called the "structure function" of the system and the vector $\text{x} = (x_1, x_2, \dots, x_n)$ is called "the state vector" of the components.

Structural properties characterizes the deterministic relationship between the state of the system and the states of the components at a fixed moment in time.

The order of a system n, is the number of distinct components that make up the system. Considering for example a system with n components, to indicate the state of the $i-th$ component, we assign a binary indicator variable x_i to component i for $i = 1, 2, \dots, n$.

$$x_i = \begin{cases} 0, & \text{if component i is failed} \\ 1, & \text{if component i is functioning} \end{cases} \tag{1.7}$$

The binary component states are summarized with the vector $\text{x} = (x_1, x_2, \dots, x_n)$. The structure function $\Phi(\text{x})$ determines the binary state of the system from the component state vector so that:

$$\Phi(\text{x}) = \begin{cases} 0, & \text{if the system has failed} \\ 1, & \text{if the system is in operating state} \end{cases} \tag{1.8}$$

1.4.3 Special Structures of Binary System, [4]:

Birnbaum, et al. in 1961 defined three basic structures for the binary case which are the series, parallel and k-out-of-n. A series system is defined so that the system is functioning if and only if each component is functioning. the structure function is defined mathematically as:

$$\Phi(\text{x}) = \prod_{i=1}^{n} x_i$$

$$= Min\{x_1, x_2, \dots, x_n\} \tag{1.9}$$

A parallel system is defined so that the system fails if and only if all the components fail, and the structure function is defined mathematically as:

$$\Phi(x) = 1 - \prod_{i=1}^{n}(1 - x_i)$$
$$= Max\{x_1, x_2, \dots, x_n\} \quad (1.10)$$

A k-out-of-n system is defined so that the system is functioning if and only if at least k-out-of-n components are functioning. The structure function is:

$$\Phi(x) = \begin{cases} 1, & if \sum_{i=1}^{n} x_i \geq k \\ 0, & if \sum_{i=1}^{n} x_i < k \end{cases} \quad (1.11)$$

Series and parallel systems are special cases of the k-out-of- n structure. A series system is an n-out-of-n structure, while a parallel system is a 1-out-of-n structure.

1.4.4 Coherent Systems of the Binary System, [1]:

A binary system of components is said to be a coherent system if its structure functions $\Phi: \{0,1\}^n \longrightarrow \{0,1\}$ satisfies the fallowing conditions:

1- $\Phi(x)$ is monotonically non-decreasing in each vector argument, x_i; $i = 1,2, \dots, n$,.

2- Each component is relevant to $\Phi(x)$.

3- $\Phi(x) = j$, for j = 0,1.

Overall, the first condition implies that a component performance improvement never causes a system failure. It ensures that the structure function Φ is a monotonically non-decreasing function of each argument. The second condition implies that each component is relevant. A component is irrelevant if it dose not matter whether or not it is working.

Finally, the last condition mentions that the entire system works when all the components work and the entire system fails when all the components fail. This condition is always satisfied with coherent systems. As a result, in the binary system context coherency means that:

1- The entire system elements are relevant.
2- The fault of all the elements causes the fault of entire system.
3- The operation of all the elements results in the entire system operation.
4- Once the system has failed, no additional failure can make the system function again.
5- When the system is working, no repair or additional of element can cause system failure.

1.5 Multi-State System, [29]

A multi-state item can perform their tasks with various distinguished levels of performance or states. The item states can vary as a result of their deterioration, or because of changing ambient conditions, from perfect functioning to complete failure.

It is assumed that at the beginning (at time t = 0) the item is in its highest performance (perfect functioning). Failures that lead to decrease the item performance are called partial failures. The item state at time t is expressed by a discrete random variable $\mathrm{x}(\mathrm{t})$ which takes its values from the state set:

$$\mathrm{x}(\mathrm{t}) = \{x_0(t), x_1(t), \dots, x_n(t)\} \qquad (1.12)$$

Generally, $x_0(t)$ represents the complete failure of the item. Whenever the item changes its performance rate, we say that there is a state transition in the item.

The probabilities associated with the different states of the item may be represented by the set:

$$p(t) = \{p_0(t), p_1(t), \dots, p_n(t)\} \qquad (1.13)$$

where $p_i(t)$ for all $i = 0,1,\dots,n$; is the probability that the variable state $x(t)$ is in the item state $x_i(t)$ at a specified time t.

$$p_i(t) = P[x(t) = x_i(t)], \forall\ i = 0,1,\dots,n \qquad (1.14)$$

An item can only be in one and only in one of the n + 1 states which means that the item states compose the complete group of mutually exclusive events, and then:

$$\sum_{i=0}^{n} p_i(t) = 1 \qquad (1.15)$$

1.5.1 The Multi-State System Structure Function, [35], [6]:

Real-world systems consist of n-components or subsystems (items) and their performance rates are unambiguously determined by the performance rates of these items. System reliability analyze the relation between the items performance of the system and the functioning of the system as a whole. The state of the entire system is determined by the states of its items.

For this model, each component and the system are allowed to have a different number of discrete states, which are assumed to be ordered, which means that the stats of each components state satisfy:

$$0 < State1 < State2 < \cdots < State\ M - 1.$$

For a multi-state system with n components, the state of the $i-th$ component is given by the discrete variable x_i, where:

$$x_i = \begin{cases} 0 & \text{, if component i is in the worst state} \\ \left.\begin{matrix} 1 \\ \vdots \\ m_i - 2 \end{matrix}\right\} & \text{, intermediate states of degradation} \\ m_i - 1 & \text{, if component i } \textit{is in} \text{ the best state} \end{cases} \quad (1.16)$$

for $i = 1,2,\ldots,n$ and $m_i < \infty$.

The state of the system is given by the variable Φ, where:

$$\Phi = \begin{cases} 0, & \text{, if the system is in the worst state} \\ \left.\begin{matrix} 1 \\ \vdots \\ M_i - 2 \end{matrix}\right\} & \text{, intermediate states of degradation} \\ M_i - 1 & \text{, if the system } \textit{is in} \text{ the best state} \end{cases} \quad (1.17)$$

where M is the best state of component.

The function Φ is called the system structure function, which represents the relation between the item state vector and the system state variable. the relationship is described by the structure function Φ(x) which can be concisely written as Φ: $\{0,1,\ldots,m_i - 1\}^n \rightarrow \{0,1,\ldots,M_i - 1\}$

The reliabilities of the system items compose the item reliability vector $\{p_1, p_2, \ldots, p_N\}$. Usually this vector is known, or can be estimated.

1.5.2 Special Structure Functions for Multi-State System [35]:

It is possible to invent an infinite number of different structure functions for real-world systems. This section presents the structures that are most commonly used in multi-state system reliability analysis.

1- Series Structure:

The series connection of system elements represents a case where a total failure of any individual item causes an overall system failure. A series structure of order n is illustrated in Figure (1.2).

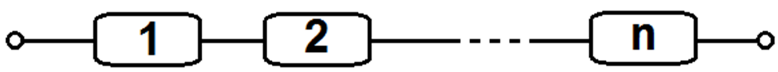


Figure (1.2) Series structure.

and when a MSS is considered, then one can distinguish between two types of series structures, namely:

• **Transmission:** a system that uses the capacity or productivity of its items as the performance measure. The operation of these systems is associated with flow continuously passing through the items. The item with the minimal transmission capacity becomes the bottleneck of the system as it is shown in Figure (1.3).

The bottleneck item determines the system performance:

$$\Phi(\mathrm{x}) = Min\{x_1(t), x_2(t), \ldots, x_n(t)\} \qquad (1.18)$$

In real-world, this kind of systems can be observed mainly in production lines.

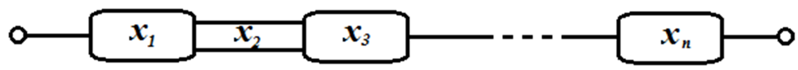


Figure (1.3) Transmission series structure.

• **Processing:** the system performance measure is characterized by an operation time or processing speed. The operation of the systems is associated with consecutive tasks performed by the ordered line of items. The total system operation time is equal to the sum of the operation times of all of its items

$$\Phi(\mathrm{x}) = \sum_{i=1}^{N} x_i(t) \qquad (1.19)$$

The complete failure state of a system item corresponds to its processing speed equal to zero, which is equivalent to an infinite operation time. In this case, the operation time of the entire system is also infinite.

Real-world systems with processing series structure can be appreciated in service companies (fast food, carwash, package shipping, etc.). Figure (1.4) presents an example of a processing series structure of order n.

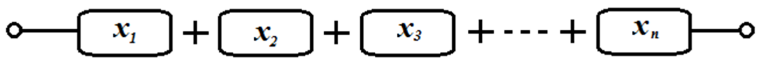


Figure (1.4) Processing series structure.

2-Parallel Structure:

The parallel connection of system elements represents a case where a system fails if and only if all of its items completely fail. A parallel structure of order n-components is shown in Figure (1.5).

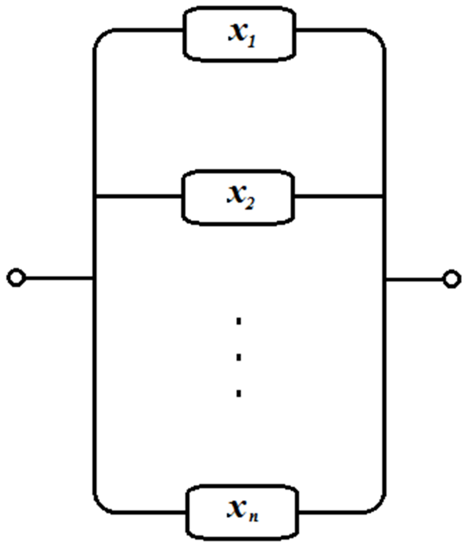


Figure (1.5) Parallel structure.

Multi-state system items connected in parallel means that some tasks can be performed by any one of the items. Thus, two basic models of parallel structures are distinguished:

• **Work sharing:** a system that shares the work among its items. The entire system performance rate is equal to the sum of the performance rates of the parallel items, given be equation (1.19)

Common work sharing systems are the queues in banks, movie theaters, supermarkets, etc. Figure (1.6) illustrates an example of a parallel structure with work sharing of order n.

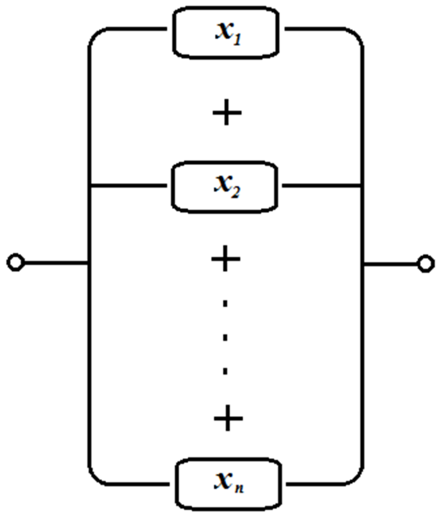
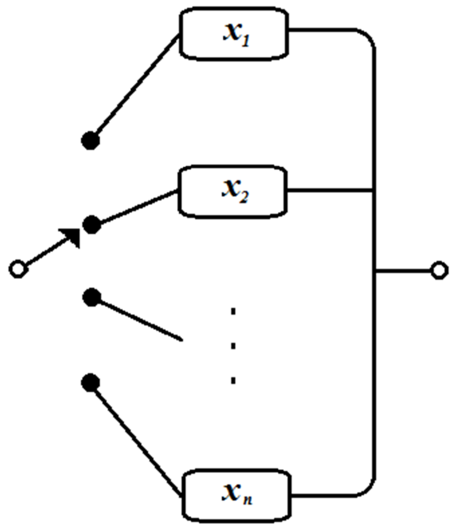


Figure (1.6) Parallel structure with work sharing.

•**Without work sharing:** represents a situation where only one item is operating at a time. The system performance rate is equal to the maximal performance rate of the available parallel items

$$\Phi(\mathrm{x}) = Max\{x_1(t)\ , x_2(t), \dots, x_n(t)\} \tag{1.20}$$

For instance, a taxicab radio service sends, from a set of free taxis, the taxi which is nearest to the call. Figure (1.7), illustrates a parallel structure without work sharing of order n.



Figure(1.7) Parallel structure without work sharing.

<u>*3- k-out-of-n Structure:*</u>

The k-out-of-n system reliability is defined as the probability that at least k elements out of n are in operable condition. As it is known, an n-out-of-n system corresponds to the series structure and a 1-out-of-n system corresponds to the parallel structure. In a multi-state generalization of the binary k-out-of-n model, the MSS is in state j if at least k_j items are in state $x_i(t)$, or above wher $i = 1,2, \dots, n$. For instance, a car with a V8 engine can walk if at least four cylinders are firing. For instance Figure (1.8) illustrates a logical representation of a 2-out-of-3 system.

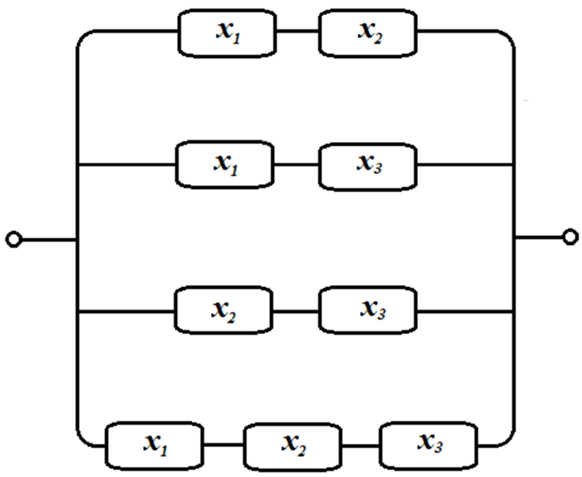


Figure (1.8) 2-out-of-3 structure.

1.5.3 The Main Properties of Multi-state Systems:

Now, some properties of the structure function will be considered that determine the important of multi-state systems properties:

1-Relevancy of system elements, [6]:

When the MSS is considered, the element is relevant if some changes in its state without changes in the states of the remaining elements cause changes in the entire system state. In terms of the MSS structure function, the relevancy of element j means that there exist such $(x_1(\mathrm{t}), x_2(\mathrm{t}), \dots, x_{\mathrm{n}}(\mathrm{t}))$, such that $j_i \neq k_i$ for all $i = 1,2,\dots,n$ there are distinct states $j \in M_i - 1$ and $k \in M_i - 1$:

$$\Phi(x_1(\mathrm{t}), \dots, x_{\mathrm{j}-1}, j_i, x_{\mathrm{j}+1}, x_n(\mathrm{t})) \neq \Phi(x_1(t), \dots, x_{j-1}, k_i, x_{j+1}, x_n(t))$$

2-Coherency, [30]:

For MSS's these requirements are met in systems with monotonic structure functions:

$$\Phi\big(x_1(t), x_2(t), \dots, x_n(t)\big) = 1 \; if \; x_i(t) = 1 \; for \; 1 \leq i \leq n$$

$$\Phi\big(x_1(t), x_2(t), \dots, x_n(t)\big) = 0 \; if \; x_i(t) = 0 \; for \; 1 \leq i \leq n$$

$$\Phi\big(x_1(t), x_2(t), \dots, x_n(t)\big) \leq \Phi\big(y_1(t), y_2(t), \dots, y_n(t)\big)$$

if there is no *i* for which $x_i(t) \leq y_i(t)$.

So, in a multi-state case, the system, is coherent if and only if its structure function is non-decreasing in each argument and all of the system elements are relevant. Note that from this structure function property it follows that the greatest system performance is achieved when the performance rates of all of the elements are in the greatest and the lowest system performance is achieved when the performance rates of all of the elements are the lowest.

3-Homogeneity, [30]:

The MSS is homogenous if all of its elements and the entire system itself have the same number of distinguished states i.e., the state spaces of component i and the system are $\{0,1,2,\dots,m_i-1\}$ and $\{0,1,2,\dots,M_i-1\}$, respectively. If $m_i-1=M_i-1$ for $1\le i\le n$, the system is considered to be homogeneous.

4-Equivalent, [21]:

Two component state vectors x and y are said to be equivalent if and only if there exists a j value such that $\Phi(\mathrm{x})=\Phi(\mathrm{y})=j, j\in\{0,1,\dots,M-1\}$.

5-The x_i are mutually independent, [6]:

The random variables representing the n-component states are assumed to be mutually independent unless specifically stated otherwise. Where the discrete random variables $\mathrm{X_1,X_2,\dots,X_n}$ are mutually independent if and only if:

$$\mathrm{pr}[\mathrm{X_1=x_1,X_2=x_2,\dots,X_n=x_n}]=\mathrm{p_1p_2\dots\ p_n}.$$

where $\mathrm{p_i}=\ \mathrm{pr}[\mathrm{X}_i=\mathrm{x}_i]$, $i=1,2,\dots,n$. Continuous random variables are mutually independent if and only if:

$$\mathrm{f(X_1,X_2,\dots,X_n)=f(X_1)\ f(X_2)\ \dots f(X_n)}$$

for every $\mathrm{(X_1,X_2,\dots,X_n)\in R^n}$ and $\mathrm{f(X_i)}$ is the marginal probability density function of $\mathrm{X_i}, \forall i=1,2,\dots,n$.

The independence assumption implies that the state of one component will have no effect on the states of the other components in the system.

Chapter Two

Reliability of Dynamic Multi-State Systems

CHAPTER 2

Reliability of Dynamic Multi-State Systems

2.1 Introduction

Multi-state system is a mathematical model that is used in reliability analysis to present a system with some level of working efficiency. A structure function allows to describe the behavior of system reliability depending on the efficiency of its components unambiguously. There are a lot of estimates of a MSS on the basic of structure function. Dynamic reliability indices belong to these estimates and characterize the changes of MSS reliability caused by changes in components efficiency. These indices are computed based on structure function and logical differential calculus, [53].

Many practical components and systems have more than two different performance levels. For example, a power generator in a power station can work at full capacity, which is its nominal capacity, say 10 MW, when there is no failures at all, [29]. Certain types of failures can cause the generator to be completely failed, while other failures will lead to the generator working at a reduced capacity, say at 4 MW. On the system level, let us consider a power generating system consisting of several power generators. The abilities of the system to meet high power load demand, normal power load demand and lower power load demand can be regarded as different system states. Another example of multi-state components is an oil transmission pipeline [44]. The pipeline is used to transmit oil from the source to spots A, B and C aligned in order along the pipeline. We say that the pipeline is in state 0 when it cannot transmit oil to any of the spots; it is in state 1 if the oil can reach spot A; it is in state 2 if the oil can reach up to spot B, i.e., spot A and B; it is in state 3 if the oil can reach up to spot C.

This chapter consists of seven sections. In section 2.2, problem formulation and description of the system model have been introduced.

In section 2.3, the direct partial logic derivatives are applied for the evaluation of dynamic characteristics of the investigation function and to reflect the changing in the value of the investigation function when the values of its variables are changed.

In section 2.4, the mathematical description of failure and repair states for the MSS is considered because, it is the most important change in system functioning.

In section 2.5, the dynamic reliability indices which define the boundary states of MSS are given and the conditions of being and changing of these states depending on the change of the system component states have been considered.

In section 2.6 presents the general model of the dynamic MSS which is considered in this chapter, it is k -out-of-n system. The k -out-of-n MSS with n components works if at least k components work.

In section 2.7 we present real life problem of an engineering system which is the problem of standing the reliability of oil supply system which has been modified and improved and therefore studied using dynamic multi-state criteria.

2.2 Problem Formulation and Description of System Model, [51]

The MSS is frequently required for applied problem, because such systems simulate the real system reliability in detail (Figure 2.1).

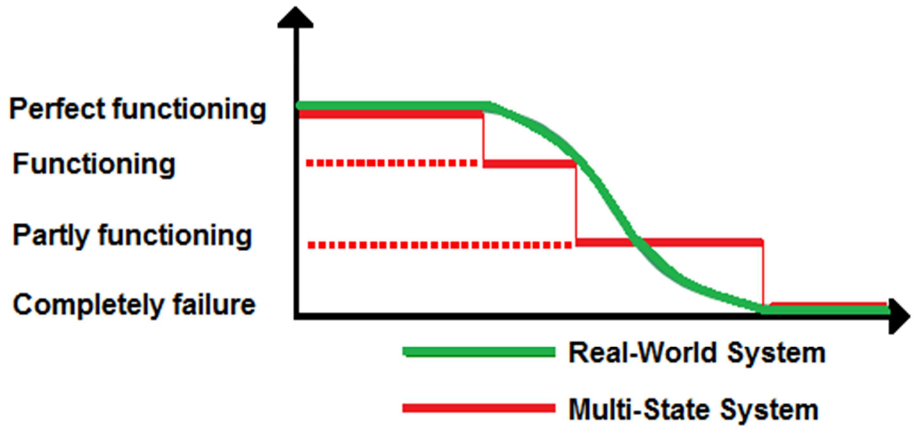


Figure (2.1) Interpretation system reliability by mathematical model of multi-state system.

The MSS and each of its n-components can be in one of m possible states; from the complete failure (it is 0) to the perfect functioning (it is m). Every system component $x_i, \forall\ i = 1,2, \dots, n$; is characterized by probability of the performance rate:

$$p_{i,s_i} = \Pr\{x_i = S_i\} \tag{2.1}$$

where $i = 1,2, \dots, n$ and $S_i = 0,1, \dots, m_i - 1$.

The system reliability (system state) depends on its components state and is defined by the structure function:

$$\Phi(x) = \Phi(x_1, x_2, \dots, x_n): \{0, \dots, m_i - 1\}^n \rightarrow \{0, \dots, m_i - 1\} \tag{2.2}$$

The structure functions of parallel, series and k-out-of-n MSS terms are declared by OR (∨) and AND (∧):

$$\Phi_P(x) = \vee_{i=1}^{n} x_i \tag{2.3}$$

$$\Phi_S(x) = \wedge_{i=1}^{n} x_i \tag{2.4}$$

$$\Phi(x) = \vee(\wedge_{i=1}^{n} x_i) \tag{2.5}$$

where $\vee_{i=1}^{n} x_i = Max\{x_1, x_2, \dots, x_n\}$; $\wedge_{i=1}^{n} x_i = Min\{x_1, x_2, \dots, x_n\}$

The mathematical model of *k*-out-of-*n* MSS (2.5) can be simplified as:

$$x_{i_1} x_{i_2} \dots x_{i_k} \vee x_{i_1} x_{i_2} \dots x_{i_k} x_{i_{k+1}} = x_{i_1} x_{i_2} \dots x_{i_k}$$

and the structure functions defined as:

$$\Phi(\mathrm{x}) = \vee(\wedge_{i=1}^{k} x_i) \tag{2.6}$$

For example, the structure functions 2-out-of-3 MSS is presented by:

$$\Phi(\mathrm{x}) = x_1 x_2 \vee x_1 x_3 \vee x_2 x_3 \vee x_1 x_2 x_3$$

The 2-out-of-3 MSS structure function in this case is given by:

$$\Phi(\mathrm{x}) = x_1 x_2 \vee x_1 x_3 \vee x_2 x_3$$

A parallel system is 1-out-of-*n* system:

$$\Phi(x) = \vee(\wedge_{i=1}^{1} x_i) = \vee_{i=1}^{n} x_i \tag{2.7}$$

And a series system is *n*-out-of-*n* MSS:

$$\Phi(x) = \vee(\wedge_{i=1}^{n} x_i) = \wedge_{i=1}^{n} x_i \tag{2.8}$$

Example (2.1), [44]:

Consider *n=3* channels in the emergency shutdown system of a nuclear power plant, detecting whether or not the operating parameters are in the safe ranges. If *k=2* channels warn that operating parameters are out of the safe ranges, the power plant will be shut down. A channel has three states:

1. State 0: unavailable, i.e., does not warn when it should.
2. State 1: warning properly.
3. State 2: warns when everything is in fact operating normally.

The *2*-out-of-*3* emergency shutdown system also has three states, namely:

1. System state 0: unavailable, i.e., does not warn when it should.
2. System state 1: warning properly.
3. System state 2: spurious operation, i.e., warns when the system is operating normally.

The structure function of the 2-out-of-3 MSS emergency shutdown system is:

$$\Phi(\mathrm{x}) = \vee\{\wedge(x_1, x_2)\wedge(x_2, x_3)\wedge(x_1, x_3)\}$$

with $m_i = 3, i = 1,2,3$ the structure functions results are indicated in table (2.1).

Table (2.1) The 2-out-of-3 MSS emergency shutdown system

x_1, x_2, x_3	$\Phi(\mathrm{x})$	x_1, x_2, x_3	$\Phi(\mathrm{x})$	x_1, x_2, x_3	$\Phi(\mathrm{x})$
0 0 0	0	1 0 0	0	2 0 0	0
0 0 1	0	1 0 1	1	2 0 1	1
0 0 2	0	1 0 2	1	2 0 2	2
0 1 0	0	1 1 0	1	2 1 0	1
0 1 1	1	1 1 1	1	2 1 1	1
0 1 2	1	1 1 2	1	2 1 2	2
0 2 0	0	1 2 0	1	2 2 0	2
0 2 1	1	1 2 1	1	2 2 1	2
0 2 2	2	1 2 2	2	2 2 2	2

2.3 Direct Partial Logic Derivative for MSS Model

The direct partial logic derivative of the structure function $\Phi(x)$ with respect to the component $x_i, \forall\, i = 1,2,\dots,n$; reflects the fact of changing of the structure function Φ from state j to state h, when the value of component x_i changing from a to b which is termed as $\frac{\partial\Phi(j\to h)}{\partial x_i(a\to b)}$ [56]:

$$\frac{\partial\Phi(j \to h)}{\partial x_i(a \to b)} = \Phi(a_i, x) \bullet \Phi(b_i, x) \tag{2.9}$$

where

$\Phi(a_i, x) = \Phi(x_1, \dots, x_{i-1}, \dots, a, x_{i+1} \dots, x_n)$; and

$\Phi(b_i, x) = \Phi(x_1, \dots, x_{i-1}, \dots, b, x_{i+1} \dots, x_n)$;

$j,\ h\ \in \{0,\ 1, \dots, M-1\}\ and\, a, b\ \in \{0,\ 1, \dots, m_i - 1\}$; and "•" is the symbol of a comparison operation defined by:

$$\frac{\partial\Phi(j \to h)}{\partial x_i\ (a \to b)} = \begin{cases} M-1, if\ \Phi(a_i, x) = j\ and\ \ \Phi(b_i, x) = h \\ 0, \quad otherwise \end{cases}$$

The analysis of the change in system reliability that is caused by a change of component states may be illustrated in Figure (2.2).

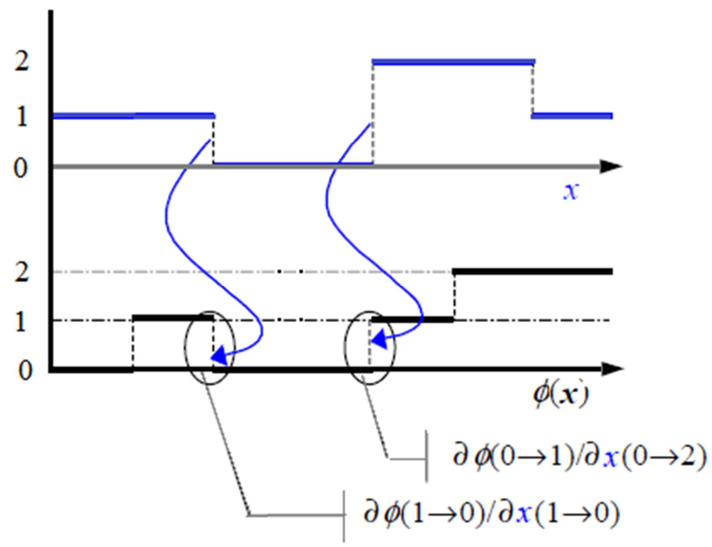


Figure (2.2) Direct partial logic derivatives and MSS states changes.

2.4 Multi-State System Failure and Repair of MSS, [54]

Direct partial logic derivative of the structure function allows to examine the influence of the $i-th$ component state change into the system reliability. In other words this derivative discovers the system states that are transformed as a result of the change of the component state.

Consequences of the direct partial logic derivatives are of interest for reliability analysis of the MSS. For this purpose, consider the following two partial derivatives:

$$\frac{\partial\Phi(\mathrm{j}\to 0)}{\partial\mathrm{x_i}(\mathrm{a}\to \mathrm{b})}\text{for } \mathrm{j}, a\in\{1,2,\dots,m_i-1\} \text{ and } b\in\{0,1,\dots,m_i-1\}$$

where $b < a$.

$$\frac{\partial\Phi(0\to h)}{\partial x_i(c\to d)} for\ h\in\{1,2,\dots,m_i-1\} \text{ and } c,d\in\{0,1,\dots,m_i-1\}$$

where $c < d$.

The first partial logic derivative is a mathematical representation for the model of the system failure if the $i\ -th$ component state changes from a to b. Because the structure function $\Phi(x)$ is non-decreasing, this derivative is $\frac{\partial\Phi(j\to 0)}{\partial x\ _i(a\to a-1)}$ where $j,a\in\{1,2,\dots,m_i-1\}$.

The second partial logic derivative permits the mathematical description of the system renewal. There are two variants of investigation for the system repairing. First it is the system repairing by the replacement of the failure component. This situation is determined by the direct partial logic derivative $\frac{\partial\Phi(0\to h)}{\partial x_i(0\to m_i-1)}$. Second, it is the increase of component state that is described as $\frac{\partial\Phi(0\to h)}{\partial x_i(c\to c+1)}$.

However, the first variant is more important for applications. Because the structure function of the MSS is non-decreasing, this derivative can be assigned as $\frac{\partial\Phi(0\to h)}{\partial x_i(0\to m_i-1)}$.

It is remarkable that direct partial logic derivatives allow to analyze dynamic properties of the MSS, which is submitted as a structural function .

2.5 Dynamic Reliability Indices

Dynamic reliability indices characterize the change of the MSS reliability that is caused by the change of a component state and include three groups of probabilistic indices, which are (DDRI's), (CDRI's) and (DIRI's). Therefore, we will explain next each of these concepts in details.

2.5.1 Dynamic Deterministic Reliability Indices:

Dynamic deterministic reliability indices evaluate the influence of a change of the component state upon system reliability. They are defined as sets of boundary states of the system. Here the boundary state of the system is the system state $s_1, \dots, s_i, \dots, s_n$ when the modification of the $i-th$ component state from s_i into $\grave{s}_i$ causes the system to fail or repair, [58].

<u>Definition (2.1) (Dynamic Deterministic Reliability Indices), [54], [55]:</u>

Dynamic deterministic reliability indices are sets of the boundary state of the system $\{G_f\}$ (for system failure) and $\{G_r\}$ (for system repairing).

The states system failure $\{G_f\}$ and the states system repairing $\{G_r\}$ are defined by:

$$\{G_f\} = \{G_f|x_1\} \cup \{G_f|x_2\} \cup \dots \cup \{G_f|x_n\} \quad (2.10)$$

$$\{G_r\} = \{G_r|x_1\} \cup \{G_r|x_2\} \cup \dots \cup \{G_r|x_n\} \quad (2.11)$$

where the subsets $\{G_f|x_i\}$ and $\{G_r|x_i\}$ are defined by:

$$\{G_f|x_i\} = \left\{G_f\left|\frac{\partial\Phi(j \to 0)}{\partial x_i(a \to 0)} \neq 0\right.\right\} \tag{2.12}$$

$$\{G_r|x_i\} = \left\{G_r\left|\frac{\partial\Phi(0 \to h)}{\partial x_i(0 \to m_i - 1)} \neq 0\right.\right\} \tag{2.13}$$

where $\{G_f|x_n\}$ and $\{G_r|x_n\}$ are subsets of the boundary state of the system for every system component $x_i, \forall\, i = 1,2, \dots, n$.

Therefore, it is necessary to analyze every component state s_i and to check the fact of MSS failure or repairing after the modification of this states. The direct partial logic derivatives (2.9) allow to formalize this procedure.

2.5.2 Component Dynamic Reliability Indices:

Component dynamic reliability indices represents the probability for evaluating the influence of the $i-th$ system component on the possibility of failure or repairing of the system. From the point of view of system reliability, unstable components are determined, [57].

Definition (2.2) (Component Dynamic Reliability Indices), [55], [50]:

Component dynamic reliability indices are probabilities of MSS failure and repairing at a modification of a state of the $i-th$ system component

$$P_f = p(i)_{a\to a-1}^{j\to 0} p_a(i) \tag{2.14}$$

$$P_r = p(i)_{0\to m_i-1}^{0\to h} p_0(i) \tag{2.15}$$

where $p(i)_{a\to a-1}^{j\to 0}$ is the probability of the $i-th$ component state modification from a to $(a-1)$ where the system fail; $p_a(i)$ is the probability of state a of the $i-th$ component; $p(i)^{0\to h}_{0\to m_i-1}$ is the probability of $i-th$ component replacement for system repairing; $p_0(i)$ is the probability of $i-th$ component failure.

$$p(i)^{1\to 0}_{1\to 0} = \frac{\rho(i)^{j\to 0}_{a\to a-1}}{m_1 m_2 \dots m_n} \tag{2.16}$$

$$p(i)^{0\to h}_{0\to m_{i-1}} = \frac{\rho(i)^{0\to h}_{0\to m_i-1}}{m_1 m_2 \dots m_n} \tag{2.17}$$

where $\rho(i)^{j\to 0}_{a\to a-1}$ is the number of system states when a change $i-th$ component state from a to $(a-1)$ forces the system failure; and $\rho(i)^{0\to h}_{0\to m_i-1}$ is the number of system states when system repairing bring about by to replacing the $i-th$ component.

Noting that, the numbers $\rho(i)^{j\to 0}_{a\to a-1}$ and $\rho(i)^{0\to h}_{0\to m_i-1}$ are obtained as number of values of direct partial logic derivative $\frac{\partial\Phi(j\to 0)}{\partial x_i(a\to a-1)}$ and $\frac{\partial\Phi(0\to h)}{\partial x_i(0\to m_i-1)}$ with respect the $i-th$ variable, which are not equal 0. In other words numbers $\rho(i)^{j\to 0}_{a\to 0}$ and $\rho(i)^{0\to h}_{0\to m_i-1}$ are the cardinality of the set $\{G_f|x_i\}$ in equation (2.12) and the set $\{G_r|x_i\}$ in equation (2.13) accordingly.

2.5.3 Dynamic Integrated Reliability Indices:

Dynamic integrated reliability indices are generalization of DDRI and are probability evaluation of a modification of the MSS reliability at a change of the system components state. In particular, the probability of the boundary of system states is estimated by these indices [32].

Definition (2.3) (Dynamic Integrated Reliability Indices), [55], [52]:

Dynamic integrated reliability indices is probability of the system failure or repairing if one of the system components fails or restores, the failure and repair probabilities are defined by:

$$P_f = \sum_{i}^{n} P_f(i) \prod_{q=1,q\neq i}^{n} \left(1 - P_f(i)\right) \tag{2.18}$$

$$P_r = \sum_{i}^{n} P_r(i) \prod_{q=1,q\neq i}^{n} \left(1 - P_r(i)\right) \tag{2.19}$$

where $P_f(i)$ and $P_r(i)$ is determined in equations (2.14) and (2.15), respectively.

Algorithm (2.1) (Calculation of the Dynamic Reliability Indices), [58]:

The DRI's are calculated using the following algorithm:

Step 1.0: Calculate $\{G_f\}$ (for system failure) and $\{G_r\}$ (for system repairing) for the MSS using the following steps:

Step 1.1: The derivatives $\frac{\partial\Phi(j\to0)}{\partial x_i(a\to0)}, \forall i = 1, \dots, n$ and $j, a \in \{1,2, \dots, m_i - 1\}$ reflects the fact of changing of the system from j to 0 when the value of component x_i changing from a to 0 are calculated by equation (2.9).

Step 1.2: The subsets failure system $\{G_f|x_i\}$ are obtained in accordance with equation (2.12).

Step 1.3: The states system failure $\{G_f\}$ in accordance with equation (2.10) is the union of subsets $\{G_f|x_i\}$ is formed.

Step 1.4: The derivative $\frac{\partial\Phi(0\to h)}{\partial x_i(0\to m_i-1)}, \forall(i=1,2,\dots,n)$ and $h \in \{1,2,\dots,m_i-1\}$ reflects the fact of changing of the system from 0 to h when the value of component x_i changing from 0 to $m_i-1$are calculated by equation (2.9).

Step 1.5: The subsets repair system $\{G_r|x_i\}$ are obtained in accordance with equation (2.13).

Step 1.6: The states system repair $\{G_r\}$ in accordance with equation (2.11) is the union of subsets $\{G_r|x_i\}$ is formed.

Step 2.0: Calculate the CDRI $P_f(i)$ and $P_r(i)$ of the MSS failure and repairing at a modification of a state using the following steps:

Step 2.1: when numbers $\rho(i)_{1\to 0}^{1\to 0}$ and $\rho(i)_{0\to m_i-1}^{0\to 1}$ are obtained. They are conformed to numbers nonzero elements of the direct partial logic derivatives $\frac{\partial\Phi(j\to 0)}{\partial x_i(a\to 0)}$ and $\frac{\partial\Phi(0\to h)}{\partial x_i(0\to m_i-1)}$ that are calculated in step 1.2 and step 1.4 accordantly.

Step 2.2: The structural probability $P(i)_{1\to 0}^{1\to 0}$ of $i-th$ component state modification from j to 0 where the system fail and the structural probability $p(i)_{0\to m_i-1}^{0\to h}$ of $i-th$ component replace for system repairing are calculated according to equations (2.16) and (2.17).

Step 2.3:The CDRI (probabilities of MSS failure or repairing at a modification of a state of $i-th$ system component) are obtained by equations (2.14) and (2.15).

Step 3:The DIRI for MSS estimation the probability of the system failure and the system repairing by equations (2.18) and (2.19).

2.6 Mathematical Simulation for the Model of Dynamic Behavior of MMS Emergency Shutdown System Model

We will consider in this section the 2-out-of-3 MSS, where the structure function Φ(x) depends on three variables, which are the number of system components $(n = 3)$ and has the best level of the components $m_i = 3, i = 1,2,3$.

The used probabilities of the component state supported by expert which are given in Table (2.2).

Table (2.2) Component state probability.

Component	***State***		
	0	**1**	**2**
x_1	0.1	0.6	0.3
x_2	0.4	0.5	0.1
x_3	0.2	0.2	0.6

System simulation will depends on algorithm (2.1), as follows:

Step 1.0: Calculate the DDRI $\{G_f\}$ of the system states, for which the failure of one component causes system failure and $\{G_r\}$ of the system failure states, which are eliminated by the replacement of a failure component for the MSS.

Step 1.1 According to equation (2.9), Compute the direct partial logic derivatives $\frac{\partial\Phi(1\to0)}{\partial x_i(1\to0)}$,$(i = 1,2,3)$ of this function $\Phi(x)$, reflects the fact of changing of system from 1 to 0 when the value of component x_i is changing from 1 to 0:

$$\frac{\partial\Phi(1\to0)}{\partial x_1(1\to0)} = \Phi(1,x_2,x_3)\bullet\Phi(0,x_2,x_3)$$

$$\frac{\partial\Phi(1\to0)}{\partial x_2(1\to0)} = \Phi(x_1,1,x_3)\bullet\Phi(x_1,0,x_3)$$

$$\frac{\partial\Phi(1\to0)}{\partial x_3(1\to0)} = \Phi(x_1,x_2,1)\bullet\Phi(x_1,x_2,0)$$

Therefore, there elements of the direct partial logic derivative $\frac{\partial\Phi(1\to0)}{\partial x_i(1\to0)}$, $\forall\ i = 1,2,3$, are equal if both $\Phi(x) = 0$ and $\Phi(x) = 1$ for specified variables only (see Table 2.3).

Table (2.3) Direct partial logic derivatives of 2-out-of-3 failures system.

x_1, x_2, x_3	$\frac{\partial\Phi(1\to0)}{\partial x_1(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_2(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_3(1\to0)}$
0 0 0	0	0	0
0 0 1	0	0	0
0 0 2	0	0	0
0 1 0	0	0	0
0 1 1	0	2	2
0 1 2	0	2	0
0 2 0	0	0	0
0 2 1	0	0	2
0 2 2	0	0	0
1 0 0	0	0	0
1 0 1	2	0	2
1 0 2	2	0	0
1 1 0	2	2	0
1 1 1	0	0	0
1 1 2	0	0	0
1 2 0	2	0	0
1 2 1	0	0	0
1 2 2	0	0	0
2 0 0	0	0	0
2 0 1	0	0	2
2 0 2	0	0	0
2 1 0	0	2	0
2 1 1	0	0	0
2 1 2	0	0	0
2 2 0	0	0	0
2 2 1	0	0	0
2 2 2	0	0	0

Step 1.2: The subsets failure system $\{G_f|x_i\}$ are:

a) $\{G_f|x_1\} = \{101, 102, 110, 120\}$, which is if the first component is breakdown;

b) $\{G_f|x_2\} = \{011, 012, 110, 210\}$, which is if the second component is a failure;

c) $\{G_f|x_3\} = \{011, 021, 101, 201\}$, which is if the third component is not functioning.

Step 1.3: Therefore, the set of the boundary states of the system failure $\{G_f\}$ is found to be:

$$\{G_f\} = \{101, 102, 110, 120, 011, 012, 210, 021, 201\}$$

Step1.4: Similarly, the direct partial logic derivative $\frac{\partial\Phi(0\to1)}{\partial x_i(0\to m_i-1)}$ is calculated (for all $i = 1,2,3$) and the analysis of this derivative permits to obtain states of the x_1, x_2, x_3 system failure (see Table 2.4):

Table (2.4) The direct partial logic derivative2-out-of-3 MSS repairing system.

x_1, x_2, x_3	$\frac{\partial\Phi(0\to1)}{\partial x_1(0\to2)}$	$\frac{\partial\Phi(0\to1)}{\partial x_2(0\to2)}$	$\frac{\partial\Phi(0\to1)}{\partial x_3(0\to2)}$
0 0 0	0	0	0
0 0 1	2	2	0
0 0 2	0	0	0
0 1 0	2	0	2
0 1 1	0	0	0
0 1 2	0	0	0
0 2 0	0	0	0
0 2 1	0	0	0
0 2 2	0	0	0
1 0 0	0	2	2
1 0 1	0	0	0
1 0 2	0	0	0
1 1 0	0	0	0
1 1 1	0	0	0
1 1 2	0	0	0
1 2 0	0	0	0
1 2 1	0	0	0
1 2 2	0	0	0
2 0 0	0	0	0
2 0 1	0	0	0
2 0 2	0	0	0
2 1 0	0	0	0
2 1 1	0	0	0
2 1 2	0	0	0
2 2 0	0	0	0
2 2 1	0	0	0
2 2 2	0	0	0

Step 1.5: The states for repairing the system are calculated by a similar method for failure system. The subsets $\{G_r|x_i\}$, $\forall\, i = 1,2,3$; are:

a) $\{G_r|x_1\} = \{001, 010\}$, which is if the first component is replaced;
b) $\{G_r|x_2\} = \{001, 100\}$, which is if the second component is replaced;
c) $\{G_r|x_3\} = \{010, 100\}$, which is if the third component is replaced.

Step 1.6: The set $\{G_r\}$, of the boundary states of the system repairing is formed.

$\{G_r\} = \{001, 010, 100\}$.

Step 2.0: Calculate the CDRI $P_f(i)$ and $P_r(i)$ of the MSS failure and repairing at a modification of a certain state.

Step 2.1 The numbers $p(i)_{1\to 0}^{1\to 0}$ and $p(i)_{0\to m_i-1}^{0\to 1}$ are obtained. They are conformed to numbers with nonzero elements of the direct partial logic derivatives:

$\rho(1)_{1\to 0}^{1\to 0} = 4, \rho(2)_{1\to 0}^{1\to 0} = 4, \rho(3)_{1\to 0}^{1\to 0} = 4;$

$\rho(1)_{0\to 2}^{0\to 1} = 2, \rho(2)_{0\to 2}^{0\to 1} = 2, \rho(3)_{0\to 2}^{0\to 1} = 2;$

Step 2.2: The structural probability $p(i)_{1\to 0}^{1\to 0}$ of the $i-th$ component state modification from 1 to 0, where the system failure are calculated according to equation (2.16).

$$p(i)_{1\to 0}^{1\to 0} = \frac{\rho(i)_{1\to 0}^{1\to 0}}{m_1 m_2 m_3} = \frac{4}{27} = 0.148, i = 1,2,3$$

The structural probability $p(i)_{0\to m_i-1}^{0\to 1}$ of the $i-th$ component replace for system repairing are calculated according to equation (2.17).

$$p(i)_{0\to 2}^{0\to 1} = \frac{\rho(i)_{0\to 2}^{0\to 1}}{m_1\, m_2\, m_3} = \frac{2}{27} = 0.074, i = 1,2,3$$

Step 2.3: Component dynamic reliability indices probabilities of the MSS failure at a modification of a system component are found using equation (2.14).

$$P_f(1) = p(1)_{1\to 0}^{1\to 0}\, p_1(1) = 0.148 \times 0.6 = 0.089$$

$$P_f(2) = p(2)_{1\to 0}^{1\to 0}\, p_1(2) = 0.148 \times 0.5 = 0.074$$

$$P_f(3) = p(3)_{1\to 0}^{1\to 0}\, p_1(3) = 0.148 \times 0.2 = 0.029$$

Also, component dynamic reliability indices probabilities of the MSS repairing at a modification of a state of a system component are found using equation (2.15).

$$P_r(1) = p(1)_{0\to 2}^{0\to 1}\, p_0(1) = 0.074 \times 0.1 = 0.0074$$

$$P_r(2) = p(2)_{0\to 2}^{0\to 1}\, p_0(2) = 0.074 \times 0.4 = 0.0296$$

$$P_r(3) = p(3)_{0\to 2}^{0\to 1}\, p_0(3) = 0.074 \times 0.2 = 0.0148$$

Therefore the analysis of CDRI shows:

a) The system has the maximum probability of failure when the first component is in failure state because its CDRI has the largest value $P_f(1) = 0.089$.

b) The system fails with minimum probability if the third component has failed $P_f(3) = 0.029$.

c) The MSS repairs with maximum probability by replacement of the second component since CDRI , $P_r(2) = 0.0296$.

Step 3.0: The dynamic integrated reliability indices permits to obtain the probability of the system failure if one of the system components is breakdown. Hence by equation (2.18).

$$P_f = \sum_{i}^{n} P_f(i) \prod_{q=1,q\neq i}^{n} \left(1 - P_f(i)\right)$$

$$= P_f(1)\left(1 - P_f(2)\right)\left(1 - P_f(3)\right)$$

$$+ P_f(2)\left(1 - P_f(1)\right)\left(1 - P_f(3)\right)$$

$$+ P_f(3)\left(1 - P_f(1)\right)\left(1 - P_f(2)\right)$$

$$= 0.089(1 - 0.074)(1 - 0.029)$$

$$+ 0.074(1 - 0.089)(1 - 0.029)$$

$$+ 0.029(1 - 0.089)(1 - 0.074) = 0.17$$

and the probability of the system repairing if one of the failure components of the system is replaced, may be found from equation (2.19)

$$P_r = \sum_{i}^{n} P_r(i) \prod_{q=1,q\neq i}^{n} \left(1 - P_r(i)\right)$$

$$= P_r(1)\left(1 - P_r(2)\right)\left(1 - P_r(3)\right)$$

$$+ P_r(2)\left(1 - P_r(1)\right)\left(1 - P_r(3)\right)$$

$$+ P_r(3)\left(1 - P_r(1)\right)\left(1 - P_r(2)\right)$$

$$= 0.0074(1 - 0.0296)(1 - 0.0148)$$

$$+ 0.0296(1 - 0.0074)(1 - 0.0148)$$

$$+ 0.0148(1 - 0.0074)(1 - 0.0296) = 0.05$$

Now, we will calculate the dynamic reliability indices for series MSS (3-out-of-3) and parallel system (1-out-of-3) in which the basic data (the number of components, state levels of the component, etc.) are similar as for 2-out-of-3 MSS that is investigated, previously in this section.

1- For the series system: The DDRI are calculated by virtue of the direct partial logic derivatives $\frac{\partial\Phi(1\to0)}{\partial x_i(1\to0)}$ and $\frac{\partial\Phi(0\to1)}{\partial x_i(0\to m_i-1)}$ as in the above for 2-out-of 3 MSS.

Therefore the structure function for the series system is in Table (2.5).

Table (2.5) The structure function of the series system (3-out-of-3 MSS)

x_1, x_2, x_3	$\Phi(x)$	x_1, x_2, x_3	$\Phi(x)$	x_1, x_2, x_3	$\Phi(x)$
0 0 0	0	1 0 0	0	2 0 0	0
0 0 1	0	1 0 1	0	2 0 1	0
0 0 2	0	1 0 2	0	2 0 2	0
0 1 0	0	1 1 0	0	2 1 0	0
0 1 1	0	1 1 1	1	2 1 1	1`
0 1 2	0	1 1 2	1	2 1 2	1
0 2 0	0	1 2 0	0	2 2 0	0
0 2 1	0	1 2 1	1	2 2 1	1
0 2 2	0	1 2 2	2	2 2 2	2

Also, the direct partial logic derivative $\frac{\partial\Phi(1\to0)}{\partial x_i(1\to0)}$ for the failure of the component are given in (Table 2.6).

Table (2.6) Direct partial logic derivatives of the series failures system.

x_1, x_2, x_3	$\frac{\partial\Phi(1\to 0)}{\partial x_1(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_2(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_3(1\to 0)}$
0 0 0	0	0	0
0 0 1	0	0	0
0 0 2	0	0	0
0 1 0	0	0	0
0 1 1	0	0	0
0 1 2	0	0	0
0 2 0	0	0	0
0 2 1	0	0	0
0 2 2	0	0	0
1 0 0	0	0	0
1 0 1	0	0	0
1 0 2	0	0	0
1 1 0	0	0	0
1 1 1	2	2	2
1 1 2	2	2	0
1 2 0	0	0	0
1 2 1	2	0	2
1 2 2	2	0	0
2 0 0	0	0	0
2 0 1	0	0	0
2 0 2	0	0	0
2 1 0	0	0	0
2 1 1	0	2	2
2 1 2	0	2	0
2 2 0	0	0	0
2 2 1	0	0	2
2 2 2	0	0	0

The subsets failure system $\{G_f|x_i\}$, $i = 1,2,3$ are:

a) $\{G_f|x_1\}$ = {111, 112, 121, 122}, which is if the first component is breakdown;

b) $\{G_f|x_2\}$ = {111, 112, 211, 212}, which is if the second component is a failure;

c) $\{G_f|x_3\}$= {111, 121, 211, 221}, which is if the third component is not functioning.

Hence, the set of the boundary states of the system $\{G_f\}$ is given by:

$\{G_f\}$ = {111, 112, 121, 122, 211, 212, 221}

Similarly, Table (2.7) shows the direct partial logic derivative $\frac{\partial\Phi(0\to1)}{\partial x_i(0\to m_i-1)}$ for repairing calculated for $i = 1, 2, 3$ and the analysis of this derivative permits to obtain states of the x_1, x_2, x_3 system failure for which the replacement of the broken component restores the system:

Table (2.7) The direct partial logic derivative of the series repairing system.

x_1, x_2, x_3	$\frac{\partial\Phi(0\to1)}{\partial x_1(0\to2)}$	$\frac{\partial\Phi(0\to1)}{\partial x_2(0\to2)}$	$\frac{\partial\Phi(0\to1)}{\partial x_3(0\to2)}$
0 0 0	0	0	0
0 0 1	0	0	0
0 0 2	0	0	0
0 1 0	0	0	0
0 1 1	2	0	0
0 1 2	2	0	0
0 2 0	0	0	0
0 2 1	2	0	0
0 2 2	0	0	0
1 0 0	0	0	0
1 0 1	0	2	0
1 0 2	0	2	0
1 1 0	0	0	2
1 1 1	0	0	0
1 1 2	0	0	0
1 2 0	0	0	2
1 2 1	0	0	0
1 2 2	0	0	0
2 0 0	0	0	0
2 0 1	0	2	0
2 0 2	0	0	0
2 1 0	0	0	2
2 1 1	0	0	0
2 1 2	0	0	0
2 2 0	0	0	0
2 2 1	0	0	0
2 2 2	0	0	0

So, the states for repairing the system are calculated by a similar method for failure system, The subsets $\{G_r|x_i\}$, $\forall i = 1,2,3$ are:

a) $\{G_r|x_1\}$ = {011, 012, 021}, which is if the first component is replaced.
b) $\{G_r|x_2\}$ = {101, 102, 201}, which is if the second component is replaced.
c) $\{G_r|x_3\}$ = {110, 120, 210}, which is if the third component is replaced.
Thus, the set $\{G_r\}$ of the boundary states of the system is given by:

$\{G_r\}$ = {011, 012, 021, 101, 102, 201, 110, 120, 210}

In a similar manner, we can calculate the CDRI for a series system which are presented in Table (2.8).

Table (2.8) CDRI calculation for results series system (3-out-of-3).

x_i	$\rho(i)^{1\to0}_{1\to0}$	$\rho(i)^{0\to1}_{0\to2}$	$p(i)^{1\to0}_{1\to0}$	$p(i)^{0\to1}_{0\to2}$	$P_f(i)$	$P_r(i)$
x_1	4	3	0.148	0.111	0.089	0.011
x_2	4	3	0.148	0.111	0.074	0.044
x_3	4	3	0.148	0.111	0.029	0.022

So, the breakdown if the third component causes the maximum probability of the system failure $P_f(3) = 0.029$. The first component has an influence on the system failure since it is the least of all $P_f(1) = 0.089$. The system repairing is most probable by replacement of the second component, $P_r(2) = 0.044$.

Finely, the DIRI permits to obtain the probability of the system failure if one of the system components is breakdown. which is $P_f = 0.17$, while the probability of the system repairing is $P_r = 0.073$ if one of the failure components of the system is replaced.

<u>2-For the parallel system:</u> As in the series the structure function of parallel system1-out-of-3 is given in Table (2.9).

Table (2.9) The structure function of the parallel system (1-out-of-3 MSS)

x_1, x_2, x_3	$\Phi(x)$	x_1, x_2, x_3	$\Phi(x)$	x_1, x_2, x_3	$\Phi(x)$
0 0 0	0	1 0 0	0	2 0 0	2
0 0 1	1	1 0 1	1	2 0 1	2
0 0 2	2	1 0 2	2	2 0 2	2
0 1 0	1	1 1 0	1	2 1 0	2
0 1 1	1	1 1 1	1	2 1 1	2
0 1 2	2	1 1 2	2	2 1 2	2
0 2 0	2	1 2 0	2	2 2 0	2
0 2 1	2	1 2 1	2	2 2 1	2
0 2 2	2	1 2 2	2	2 2 2	2

The direct partial logic derivative $\frac{\partial\Phi(1\to0)}{\partial x_i(1\to0)}$ are given in (Table 2.10).

Table (2.10) Direct partial logic derivatives of the parallel failure system.

x_1, x_2, x_3	$\frac{\partial\Phi(1\to 0)}{\partial x_1(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_2(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_3(1\to 0)}$
0 0 0	0	0	0
0 0 1	0	0	2
0 0 2	0	0	0
0 1 0	0	2	0
0 1 1	0	0	0
0 1 2	0	0	0
0 2 0	0	0	0
0 2 1	0	0	0
0 2 2	0	0	0
1 0 0	0	0	0
1 0 1	2	0	0
1 0 2	0	0	0
1 1 0	0	0	0
1 1 1	0	0	0
1 1 2	0	0	0
1 2 0	0	0	0
1 2 1	0	0	0
1 2 2	0	0	0
2 0 0	0	0	0
2 0 1	0	0	0
2 0 2	0	0	0
2 1 0	0	0	0
2 1 1	0	0	0
2 1 2	0	0	0
2 2 0	0	0	0
2 2 1	0	0	0
2 2 2	0	0	0

Thus, the subsets $\{G_f | x_i\}, i = 1,2,3$ of the failure system are:

a) $\{G_f|x_1\}$ = {101}, which is if the first component is a breakdown.

b) $\{G_f|x_2\}$ = {010}, which is if the second component is a failure.

c) $\{G_f|x_3\}$= {001}, which is if the third component is not functioning.

Hence, the set of the boundary states $\{G_f\}$ of the system is found to be:

$\{G_f\} = \{101, 010, 001\}$

Table (2.11) shows the direct partial logic derivative $\frac{\partial\Phi(0\to1)}{\partial x_i(0\to m_i-1)}$ for repairing calculated on i = 1, 2, 3 and the analysis of this derivative permits to obtain states of the $x_1x_2x_3$ system failure for which the replacement of the broken component restores the system:

Table (2.11) The direct partial logic derivative of the parallel repairing system.

x_1, x_2, x_3	$\frac{\partial\Phi(0\to1)}{\partial x_1(0\to2)}$	$\frac{\partial\Phi(0\to1)}{\partial x_2(0\to2)}$	$\frac{\partial\Phi(0\to1)}{\partial x_3(0\to2)}$
0 0 0	0	0	0
0 0 1	0	0	0
0 0 2	0	0	0
0 1 0	0	0	0
0 1 1	0	0	0
0 1 2	0	0	0
0 2 0	0	0	0
0 2 1	0	0	0
0 2 2	0	0	0
1 0 0	0	0	0
1 0 1	0	0	0
1 0 2	0	0	0
1 1 0	0	0	0
1 1 1	0	0	0
1 1 2	0	0	0
1 2 0	0	0	0
1 2 1	0	0	0
1 2 2	0	0	0
2 0 0	0	0	0
2 0 1	0	0	0
2 0 2	0	0	0
2 1 0	0	0	0
2 1 1	0	0	0
2 1 2	0	0	0
2 2 0	0	0	0
2 2 1	0	0	0
2 2 2	0	0	0

There is no states for repairing the system are calculated by similar method for failure system.

Also, calculating the CDRI for the parallel system are presented in Table (2.12).

Table (2.12) CDRI calculation for parallel system (1-out-of-3 MSS).

x_i	$\rho(i)^{1\to0}_{1\to0}$	$\rho(i)^{0\to1}_{0\to2}$	$p(i)^{1\to0}_{1\to0}$	$p(i)^{0\to1}_{0\to2}$	$P_f(i)$	$P_r(i)$
x_1	1	-	0.083	-	0.022	-
x_2	1	-	0.250	-	0.018	-
x_3	1	-	0.125	-	0.007	-

So, the breakdown of the first component causes the maximum probability of the system failure $P_f(1) = 0.022$. The third component has an influence on the system failure since it is the least of all $P_f(3) = 0.007$.

Hence, the DIRI permits to obtain the probability of the system failure if one of the system components is breakdown. It is $P_f = 0.045$, while the probability of the system repairing is $P_r = 0$ if one of the failure components of the system is replaced.

Remark(2.1):

The above general examples for series, parallel and 2-out-of-3 systems reveal the main point of dynamic indices CDRI and DIRI. The CDRI's reflect the influence of the change of the specifically the $i-th$ component state upon the system reliability. In particular the system failure and system repairing depending on the $i-th$ component state modification, which are examined. Since the component state probabilities are equal to the change of the system components, they have a similar influence on the system reliability in these examples. So the second component has the largest probability of system failure if this component breaks down, [31].

Also, the DIRI describe the dynamic characteristic of the MSS which are different for series, parallel and 2-out-of-3 systems (Table 2.13). The probability of failure of the series system and 2-out-of-3 have the maximum value are $P_f = 0.17$ if one of the system components breaks down. The probability of the MSS failure P_f is minimum for the parallel system.

Table (2.13) Reliability indices for 2-out-of-3, series, and parallel systems.

P	The system 2-out-of-3	The series system	The parallel system
P_f	0.17	0.17	0.045
P_r	0.05	0.073	0

2.7 Applications of Dynamic Multi-State System Model

Many engineering systems can fit into the proposed multi-state system model. In this section, we will present on applications that have been identified by Tian, Z., Li, W. and Zuo, M. J., [44] and modified here to be dynamic MSS. Similar applications can be found in power supply systems and telecommunication systems.

Consider for example an oil supply system, as shown in Figure (2.3).

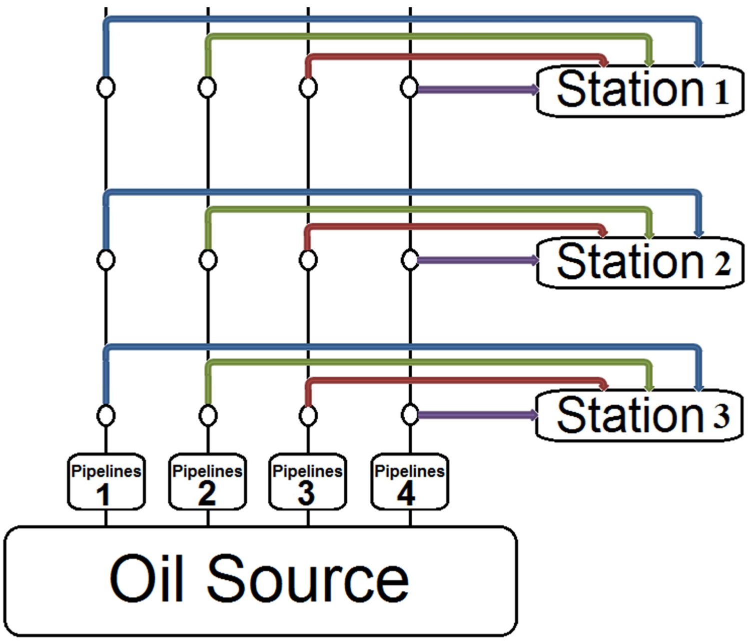


Figure (2.3) An oil supply system.

The oil is delivered from the oil source to three stations through four oil pipelines. A pipeline is considered to be a multi-state component (thus *n* = 4). A failure might occur at any part of a pipeline. Take pipeline 1 for example, if there is a failure in section S_{01}^{1}, the section of pipeline 1 between the oil source and station 1, the oil will not be able to reach any station via pipeline 1. If there is no failure in section S_{01}^{1}, but there is a failure in section S_{12}^{1}, the section of pipeline 1 between station 1 and station 2, the oil will be able to reach station 1 but will not be able to reach station 2 or beyond. Similarly, if there is no failure in section S_{01}^{1} or section S_{12}^{1}, but there is a failure in section S_{23}^{1}, the oil will be able to reach station 1 and station 2, but will not be able to reach station 3. Based on the possible failures in different sections of a pipeline, four states of a pipeline can be defined as follows:

1. State 0: oil cannot reach any stations.
2. State 1: oil can reach only station 1.
3. State 2: oil can reach station 1 and 2.
4. State 3: oil can reach station 1, 2 and 3.

Each station has different demands on the oil.

1. Station 1: requires at least one pipelines working to meet its demand.
2. Station 2: requires at least two pipelines working to meet its demand.
3. Station 3: requires at least four pipelines working to meet its demand.

At the system level, we are interested in whether the demands of up to a certain station can be met. Thus, four states of the oil supply system can be defined as follows:

1. System state 0: it cannot meet the oil demand of station 1.
2. System state 1: it can meet the oil demand of up to station 1. That is, the system can meet the demand of station 1, but cannot meet the demand of station 2.

3. System state 2: it can meet the oil demands of up to station 2. That is, the system can meet the demands of station 1 and station 2, but cannot meet the demand of station 3.
4. System state 3: it can meet the oil demands of up to station 3. That is, the demands of station 1, 2 and 3 can all be met.

In practice, we may be interested in the probability of the oil supply system in states 0, 1, 2 or 3. If $x_i, i = 1,2,3,4$ are used for the pipeline, then, the component state probability are given in Table (2.14)

Table (2.14) Component state probability of the oil source system.

component	***state***			
	0	**1**	**2**	**3**
x_1	0.0500	0.0950	0.0684	0.7866
x_2	0.0500	0.0950	0.0684	0.7866
x_3	0.0300	0.0776	0.0446	0.8478
x_4	0.0300	0.0776	0.0446	0.8478

Note, the structure function of the MSS in this example has dimension equals to:

$$\mathrm{m}^{\mathrm{n}} = 4^4 = 256.$$

The structure function released to this system and the direct partial logic derivative $\frac{\partial\Phi(1\to0)}{\partial x_i(1\to0)}$ are presented in Appendix A.

The subsets failure system $\{G_f|x_i\}$ are formed to be:

a) $\{G_f|x_1\}$ = {1000}, if the section S_{01}^1 is a failure, the oil will not be able to reach any station via pipeline 1.

b) $\{G_f|x_2\}$ = {0100}, if the section S_{01}^2 is a failure, the oil will not be able to reach any station via pipeline 2.

c) $\{G_f|x_3\}$= {0010}, if the section S_{01}^3 is a failure, the oil will not be able to reach any station via pipeline 3.

d) $\{G_f|x_4\}$= {0001}, if the section S_{01}^4 is a failure, the oil will not be able to reach any station via pipeline 4.

Appendix B shows the direct partial logic derivative $\frac{\partial\Phi(1\to2)}{\partial x_i(0\to m_i-1)}$ which are calculated for $i = 1,2,3,4$ and the analysis of this derivative permits to obtain states of the x_1, x_2, x_3, x_4 system failure for which the replacement of the broken component restores the system.

The states for repairing the system are calculated by a similar method For the failure system, The subsets of the repairing system$\{G_r|x_i\}$ are:

a) $\{G_r|x_1\}$ ={0002, 0003, 0012,0013, 0020, 0021, 0030, 0031, 0102, 0103, 0112, 0113, 0120, 0121, 0131, 0200, 0201, 0210, 0211,0300, 0301, 0310, 0311}, if there is no failure in sections S_{01}^1, S_{12}^1, and S_{23}^1.

b) $\{G_r|x_2\}$={0002, 0003, 0012, 0013, 0020, 0021, 0030, 0031, 1002, 1003, 1012, 1013, 1020, 1021, 1030, 1031, 2000, 2001, 2010, 2011, 3000, 3001, 3010, 3011}, if there is no failure in sections S_{01}^2, S_{12}^2, and S_{23}^2.

c) $\{G_r|x_3\}$={0002, 0003, 0102, 0103, 0200, 0201, 0300, 0301, 1002, 1003, 1102, 1103, 1200, 1300, 1301, 2000, 2001, 2101}, if there is no failure in section S_{01}^3, S_{12}^3, and S_{23}^3.

d) $\{G_r|x_4\}$={0020, 0030, 0120, 0130, 0200, 0210, 0300, 0310, 1120, 1130, 1200, 1210, 1300, 1310, 2000, 2010, 2100, 2110, 3000, 3010, 3100, 3110}, if there is no failure in sections S_{01}^4, S_{12}^4, and S_{23}^4.

The results of the CDRI's for the oil supply system are presented in Table (2.15).

Table (2.15) CDRI calculation for oil supply system.

x_i	$\rho(i)_{1\to0}^{1\to0}$	$\rho(i)_{0\to3}^{1\to2}$	$p(i)_{1\to0}^{1\to0}$	$p(i)_{0\to3}^{1\to2}$	$P_f(i)$	$P_r(i)$
x_1	1	23	0.003	0.089	0.00028	0.0045
x_2	1	24	0.003	0.093	0.00028	0.0047
x_3	1	18	0.003	0.07	0.00023	0.0021
x_4	1	22	0.003	0.085	0.00023	0.0026

So, the breakdown of the section S_{01}^1 and section S_{01}^2 causes the maximum probability of system failure $\mathrm{P_f}(1) = 0.00028$ and $\mathrm{P_f}(2) = 0.00028$. Section S_{01}^3 and S_{01}^4 have an influence on the system failure least of all $\mathrm{P_f}(3) = \mathrm{P_f}(4) = 0.00023$. The system repairing has its most value probable if there is no failures in sections S_{01}^2, S_{12}^2, and in S_{23}^2, $\text{i.e.}\, \mathrm{P_r}(2) = 0.0047$.

Also, the DIRI's are probabilities of the change of the system reliability if the state of one of the system components is changed. The probability of the system failure, if one of the components breaks down, is $P_f = 0.335$ in accordance to equation (2.18). The probability of system repairing obtained by equation (2.19) and is $P_r = 0.013$ if one of the failed component of the system is replaced.

Chapter Three

Reliability of Dynamic Fuzzy Multi-State Systems

CHAPTER 3

Reliability of Dynamic Fuzzy Multi-State Systems

3.1 Introduction

In conventional multi-state theory, it is assumed that the exact probability, and performance level of each component state are given. With the progress of modern industrial technologies, however, product development cycles have become shorter, while the lifetimes of products have become longer, [18]. In many highly reliable applications, there may be only a few available observations of the system's failures. Therefore, it may be difficult to obtain sufficient data to estimate the precise values of the probabilities, and performance levels of these systems. Moreover, the inaccuracy of system models, caused by human errors, is difficult to quantify using conventional reliability theory alone [20]. In light of these significant challenges, new techniques are needed to solve these fundamental problems related to reliability.

This chapter consists of four sections. In section 3.2, fundamental concepts including the definition of fuzzy sets, algebraic properties, fuzzy numbers and its operators, membership functions and α-level sets are presented.

In section 3.3, the fundamental and key definition for fuzzy multi-state system and comparison between fuzzy number are given.

Finally, in section 3.4, the introduction of re dynamic fuzzy multi-state system reliability is given as a generalization non fuzzy topic.

3.2 Basic Concepts of Fuzzy Sets

A ***classical (crisp or ordinary) set X*** is normally defined as a collection of elements or objects x, which may be finite, countable, or uncountable. Each single element can either belong to or not belong to a set A, $A \subseteq X$. In the former case, the statement "x belongs to A" is true, whereas in the latter case this statement is false. Such a classical set can be described in different ways; either one can enumerate the elements that belong to the set, one can describe the set analytically by defining a member for each element by using certain characteristic function ranging between 0 and 1, in which 1 indicates membership and 0 non-membership. For a fuzzy set, the characteristic function allows various degrees of membership for the elements of a given set, [41], [59].

Next, we will set some basic definitions and concepts related to fuzzy set theory.

<u>Definition (3.1), [59], [24]:</u>

Let X be any non-empty set of elements. A ***fuzzy set*** $\tilde{A}$ in X is the set of all $x \in X$, which are characterized by a ***membership function*** $\mu_{\tilde{A}}(x): X \to [0,1]$. The grades 0 and 1 represent respectively non-membership and full membership in a fuzzy set $\tilde{A}$. A fuzzy set $\tilde{A}$ may be written mathematically as:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X, 0 \leq \mu_{\tilde{A}}(x) \leq 1\}$$

The following concepts may be defined in fuzzy sets:

<u>Definition (3.2), [15], [25]:</u>

The ***support*** of a fuzzy set $\tilde{A}$ is the crisp set of all $x \in X$, such that $\mu_{\tilde{A}}(x) > 0$ and is denoted by supp($\tilde{A}$), i.e.,

$$\text{supp}(\tilde{A}) = \{x \in X: \mu_{\tilde{A}}(x) > 0\}$$

Definition (3.3), [10], [16]:

The ***height*** of a fuzzy set $\tilde{A}$ (denoted by hgt ($\tilde{A}$)) is the supremum value of $\mu_{\tilde{A}}(x)$ over all $x \in X$. If hgt ($\tilde{A}$) = 1, then $\tilde{A}$ is ***normal,*** otherwise it is ***subnormal***, and a fuzzy set may be always ***normalized*** by defining the scaled membership function:

$$\mu_{\tilde{A}}{}^{*}(x) = \frac{\mu_{\tilde{A}}(x)}{\operatorname*{Sup}_{x \in X} \mu_{\tilde{A}}(x)}, \forall \text{ x} \in X$$

Definitions (3.4), [10], [45]:

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy subsets of the universal set X with membership functions $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$, respectively, then:

1. $\tilde{A} \subseteq \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x), \forall\, x \in X$.
2. $\tilde{A} = \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x), \forall x \in X$.
3. $\tilde{A}^c$ is the complement of $\tilde{A}$, which is also a fuzzy set with membership function, $\mu_{\tilde{A}^c}(x) = 1 - \mu_{\tilde{A}}(x),\ \forall x \in X$.
4. The empty fuzzy set $\tilde{\emptyset}$ and the universal set X, has the membership $\mu_{\tilde{A}}(x) = 0$ and $\mu_{\tilde{A}}(x), = 1$ respectively for all $x \in X$.
5. $\tilde{C} = \tilde{A} \cap \tilde{B}$ is a fuzzy set with membership function:

   $\mu_{\tilde{C}}(x) = Min\{\mu_{\tilde{A}}(x), \mu_B(x)\}, \forall x \in X.$
6. $\tilde{D} = \tilde{A} \cup \tilde{B}$ is a fuzzy set with membership function:

   $\mu_{\tilde{D}}(x) = Max\{\mu_{\tilde{A}}(x), \mu_B(x)\}, \forall x \in X.$
7. If $\mu_{\tilde{A} \cap \tilde{B}}(x) = 0, \forall\, x \in X$, then $\tilde{A}$ and $\tilde{B}$ are said to be ***disjoint***.

3.2.1 α-Level Sets, [13]:

Because of its importance in fuzzy set theory as an intermediate concept between non fuzzy set theory and fuzzy set theory, therefore the scope of this section is to cover and discuss some basic important properties of the so called α-level sets (or α-cuts) which correspond to any fuzzy set. α-Level set is a set collect between fuzzy sets and ordinary sets, which could be used to prove most of the results that are satisfied in ordinary sets are also satisfied to fuzzy sets and vice versa.

In fuzzy set theory, if we want to exhibit an element $x \in X$ that typically belongs to a fuzzy set $\tilde{A}$, we may demand its membership value to be greater than to some threshold $\alpha \in [0, 1]$. The ordinary set of such element is called the ***α-level sets of*** $\tilde{A}$ and is denoted by A_α, i.e.,

$$A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X\}$$

It is easily to check that the following properties are satisfied for all $\alpha \in [0, 1]$.

1. $(\tilde{A} \cup \tilde{B})_\alpha = \tilde{A}_\alpha \cup \tilde{A}_\alpha$.
2. $(\tilde{A} \cap \tilde{B})_\alpha = \tilde{A}_\alpha \cap \tilde{A}_\alpha$.
3. $\tilde{A} \subseteq \tilde{B}$ gives $\tilde{A}_\alpha \subseteq \tilde{A}_\beta$, if $\alpha > \beta$.
4. $\tilde{A} = \tilde{B}$ equivalent to $\tilde{A}_\alpha = \tilde{B}_\beta$, $\forall\ \alpha \in (0, 1]$.
5. $A_\alpha \cap A_\beta = A_\beta$ and $A_\alpha \cup A_\beta = A_\alpha$, if $\alpha \leq \beta$.

3.2.2 Convex Fuzzy Set:

Convex fuzzy sets are of great importance in defining fuzzy numbers. This property is viewed as a generalization of the classical concept of convexity in crisp sets. The definition of convexity for fuzzy set does not necessarily mean that the membership function of a convex fuzzy set is also convex function, [26].

Definition (3.5), [14]:

Let X be a vector space (universal set), then a fuzzy set $\tilde{A}$ is convex if:

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq Min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \forall x_1, x_2 \in X, \lambda \in [0,1].$$

Alternatively, a fuzzy set is convex if all of its α-level sets are convex.

3.2.3 The Extension Principle:

The extension principle of fuzzy set theory may be used to generalize crisp mathematical concepts to fuzzy mathematical concepts, which may be also used to define fuzzy functions, [23].

Definition (3.6), [23]:

Let X be the Cartesian product of universes $x_1, x_2, \ldots, x_r$ and $\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_r$ be r-fuzzy sets in $x_1, x_2, \ldots, x_r$, respectively, f is a mapping from X to a universe $Y(y = f(x_1, x_2, \ldots, x_r))$. Then the fuzzy set $\tilde{B}$ in Y is defined by:

$$\tilde{B} = f(\tilde{A}) = \{(y, \mu_{\tilde{B}}(y)) | y = f(x_1, x_2, \ldots, x_r), (x_1, x_2, \ldots, x_r) \in X\}$$

Where:

$$\mu_{\tilde{B}}(y) = \begin{cases} \underset{(x_1, x_2, \ldots, x_r) \in f^{-1}(y)}{\text{Sup}} Min\{\mu_{\widetilde{A_1}}(x_1), \mu_{\widetilde{A_2}}(x_2), \ldots, \mu_{\widetilde{A_r}}(x_r)\}, if\ f^{-1}(y) \neq \emptyset \\ 0 \quad , Otherwise \end{cases}$$

where f^{-1} is the inverse image of f.

For r = 1, the extension principle, of course, reduces to:

$$\tilde{B} = f(\tilde{A}) = \{(y, \mu_{\tilde{B}}(y)) | y = f(x), x \in X\}$$

where:

$$\mu_{\tilde{B}}(y) = \begin{cases} \underset{X \in f^{-1}(y)}{\text{Sup}} \mu_{\tilde{A}}(x), if\ f^{-1}(y) \neq \emptyset \\ 0 \quad , Otherwise \end{cases}$$

which is the definition of a fuzzy function.

3.2.4 Fuzzy Number, [47], [37]:

A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set $\tilde{M}$ of the real line R, such that:

1. There exists exactly one $x_0 \in$ R, with $\mu_{\tilde{M}}(x_0) = 1$ (x_0 is called the mean value of $\tilde{M}$).
2. $\mu_{\tilde{M}}(x)$ is piecewise continuous.

Two types of fuzzy numbers may be used, which are the triangular and trapezoidal. The general form of membership function of this function is defined by, [61]:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , for\ x < a_1 \\ (x - a_1)/(a_2 - a_1) & , for\ a_1 \le x \le a_2 \\ (a_3 - x/a_3 - a_1) & , for\ a_2 \le x \le a_3 \\ 0 & , for\ x > a_3 \end{cases}$$

Also, the triangular fuzzy number may be termed by its value a_2 as a fuzzy set $\tilde{a}_2$ (see figure 3.1).

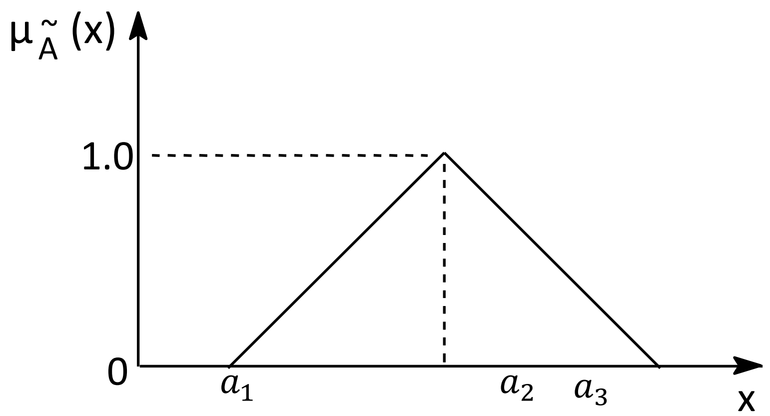


Figure (3.3) The triangular membership function.

It is a fuzzy number represented with three parameters a_1, a_2 and a_3 as follows $\tilde{A} = (a_1, a_2, a_3)$. This representation is interpreted as membership functions and hold the following conditions:

i- The membership functions from a_1 to a_2 is an increasing function
ii- The membership functions from a_2 to a_3 is decreasing function
iii- $a_1 \leq a_2 \leq a_3$.

Now, let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$, then the basic algebraic operation that can be defined and performed on triangular fuzzy number are:

$\tilde{+}, \tilde{-}, \tilde{\times}$ and $\tilde{\div}$

Addition: $\tilde{A} \tilde{+} \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction: $\tilde{A} \tilde{-} \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

Multiplication:

$$\tilde{A} \tilde{\times} \tilde{B} = (Min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, Max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$$

Division:

$$\tilde{A} \tilde{\div} \tilde{B} = (Min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), a_2/b_2, Max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3)), b_1, b_2, b_3 \neq 0$$

<u>Example (3.1), [37]:</u>

Let $\tilde{A}$ =(2, 4, 6) and $\tilde{B}$ =(1, 2, 3) be two fuzzy numbers. Then:

$\tilde{A} \tilde{+} \tilde{B} = (3, 6, 9)$.

$\tilde{A} \tilde{-} \tilde{B} = (-1, 2, 5)$.

$\tilde{A} \tilde{\times} \tilde{B} = (2, 8, 18)$.

$$\tilde{A} \tilde{\div} \tilde{B} = \left(\frac{2}{3}, \frac{4}{2}, \frac{6}{1}\right) = (0.66, 2, 6).$$

$\tilde{A} \tilde{-} \tilde{A} = (-4, 0, 4)$.

$$\tilde{A} \tilde{\div} \tilde{A} = \left(\frac{2}{6}, \frac{4}{4}, \frac{6}{2}\right) = (0.33, 1, 3).$$

Remark (3.1), [37]:

As it is mentioned earlier from example (3.1) that $\tilde{A} \tilde{-} \tilde{A} \neq 0$, $\tilde{A} \tilde{\div} \tilde{A} \neq 1$, where 0 and 1 are singletons whose fuzzy representation is (0, 0, 0) and (1, 1,1). It follows that the solution $\tilde{C}$ of the fuzzy linear equation $\tilde{A} \tilde{+} \tilde{B} = \tilde{C}$ is not as we would expect, $\tilde{B} = \tilde{C} \tilde{-} \tilde{A}$

For example, $\tilde{A} \tilde{+} \tilde{B} = (2, 4, 6) + (1, 2, 3) = (3, 6, 9) = \tilde{C}$

But $\tilde{C} \tilde{-} \tilde{A} = (3,6,9) - (2,4,6) = (-3,2,7) \neq \tilde{B}$

The same annoyance appears when solving the fuzzy equation $\tilde{A} \tilde{\times} \tilde{B} = \tilde{C}$ whose solution is not given by $\tilde{B} = \tilde{C} \tilde{\div} \tilde{A}$.

For example, $\tilde{A} \tilde{\times} \tilde{B} = (2,8,18) = \tilde{C}$

But $(2,8,18) \tilde{\div} (2,4,6) = \left(\frac{2}{6}, \frac{8}{4}, \frac{18}{2}\right) = (0.33, 2, 9) \neq \tilde{B}$

Therefore, the addition and subtraction (respectively multiplication and division) of fuzzy numbers are not reciprocal operations. According to this statement, it is not possible to solve inverse problems exactly using the standard fuzzy arithmetic operators. To overcome this in function principle operation of triangular fuzzy number a new operation is proposed that allows exact solution or inversion.

3.3 The Fuzzy Multi-State System

In this section and based on the basic concepts of fuzzy sets and fuzzy numbers and its algebraic operations given in section 3.2, we will give the basic concepts of fuzzy multi-state system reliability and introduce for the first time in the next section the so called reliability of dynamic fuzzy multi-state system.

The natural extension of the crisp definition for conventional MSS to the fuzzy set definition for FMSS is that the state probabilities and state performances of a component can be considered as fuzzy values. The general assumptions of non-dynamic FMSS are presented below, [9]:

1- The state probabilities and state performance levels of a component can be treated as fuzzy values.
2- The state index is a crisp value taking integer values only. The state spaces of component i and the system are $\{0,1,\dots,m_i-1\}$ and $\{0,1,\dots,M_i-1\}$, respectively. If $m_i-1=M_i-1$ for $1 \le i \le n$, the system is considered a homogeneous FMSS.
3- The state of a system is completely determined by the state of its components.
4- The state set of components and the system are ordered so that a lower state level represents a worse fuzzy performance level.

3.3.1 Criteria for Ordering Fuzzy Variables, [30]:

In the above fourth assumption, the methods applied in the MSS model cannot be directly used to order states in a FMSS model. In the MSS model, for a component i if $x_i-y_j>0$, then $i>j$, in which the arithmetic calculation of x_i-y_j is simple and clear. However, in the FMSS model, the performance level of state *i* being greater and less than that of the state *j* is both possible.

As an illustration consider the following example

Example(3.3), [30]:

Suppose that the fuzzy performance levels of state i and state j can be represented by triangular fuzzy numbers $(1,2,2.5)$ and $(1.8,2,2.2)$, respectively. In this case,

$(1,2,2.5)\tilde{-}(1.8,2,2.2) = (1,2,2.5)\tilde{+}(-2.2,-2,-1.8) = (-1.2,0,0.7)$ the performance level of state *i* is not definitely higher or lower than that of state *j*.

Therefore, three criterions may be used to order two fuzzy numbers. If the first criterion does not give a unique order, then the second and third criteria will be used in sequence. In this section, triangular fuzzy numbers are used to represent fuzzy variables. However the proposed definitions and characteristics are not only developed for triangular fuzzy numbers but also generally suitable for various fuzzy variables with different kinds of membership functions.

1. The first criterion for ordering (the removal), [30]:

Consider a fuzzy number $\tilde{A}$ and a crisp value *k*. The left side removal of $\tilde{A}$ with respect to *k*, (denoted by $R_l(\tilde{A},k)$) is defined as the area bounded by *k* and the left side of the fuzzy number $\tilde{A}$; and the right side removal of $\tilde{A}$ with respect to *k*, (denoted by $R_r(\tilde{A},k)$) is defined as the area bounded by *k* and the right side of the fuzzy number $\tilde{A}$. The removal of fuzzy number *A* with respect to *k* is defined as

$$R(\tilde{A},k) = \frac{1}{2}\left[R_r(\tilde{A},k) + R_l(\tilde{A},k)\right]$$

The first criterion, therefore, is set as a comparison of the removals of two different fuzzy numbers with respect to *k*. Relative to *k* = 0, the removal number $R(\tilde{A},k)$ is equivalent to an “ordinary representative” of the fuzzy number. If a fuzzy number $\tilde{A}$ is triangular and represented by a triplet (a_1,a_2,a_3), then the ordinary representative is given by:

$$\tilde{A} = \frac{a_1 + 2a_2 + a_3}{4}$$

2. The second criterion for ordering (the mode), [30]:

Different fuzzy numbers may have the same ordinary representatives. The first criterion may not be sufficient to obtain the linear ordering of these fuzzy numbers. In these cases, the second criterion, which is based on a comparison of the modes of different fuzzy numbers, is used to order these numbers. The mode of a fuzzy variable is that value which has the highest membership function. In the case of a triangular fuzzy number, it is simply a_2.

3. The third criterion for ordering (the divergence), [30]:

If the first and second criterions are not enough to obtain the ordering of fuzzy numbers, the divergences around the modes of fuzzy numbers are used to order these numbers. The divergence around a mode measures the magnitude of expansion at the given mode point. In the case of a triangular fuzzy variable, it is the value of $a_3 - a_1$.

The following example illustrates the above methods.

<u>Example(3.4), [8]:</u>

Consider a component that may be in one of four possible states. The performance levels of these states are the triangular fuzzy numbers.

$$\tilde{A}_1 = (4,6,7)\ , \tilde{A}_2 = (4,5,9\ ), \tilde{A}_3 = (\ 3,5,10)\ , \tilde{A}_4 = (0,0,0)\ .$$

Firstly, we use the first criterion of ordering:

$$\tilde{A}_1 = (4,6,7) \rightarrow \tilde{A}_1 = \frac{4 + 12 + 7}{4} = 5.75$$

$$\tilde{A}_2 = (4,5,9\ ) \rightarrow \tilde{A}_2 = \frac{4 + 10 + 9}{4} = 5.75$$

$$\tilde{A}_3 = (\ 3,5,10) \rightarrow \tilde{A}_3 = \frac{3 + 10 + 10}{4} = 5.75$$

$$\tilde{A}_4 = (0,0,0) \rightarrow \tilde{A}_4 = \frac{0 + 0 + 0}{4} = 0$$

Therefore, $\tilde{A}_4 < \tilde{A}_1, \tilde{A}_2, \tilde{A}_3$.

Secondly, the second criterion is used to order $\tilde{A}_1, \tilde{A}_2$, and $\tilde{A}_3$:

$\tilde{A}_1 = (4,6,7) \rightarrow \text{mode } 6$

$\tilde{A}_2 = (4,5,9) \rightarrow \text{mode } 5$

$\tilde{A}_3 = (3,5,10) \rightarrow \text{mode } 5$

Therefore, $\tilde{A}_1 > \tilde{A}_2, \tilde{A}_3$

Finally, the third criterion is used to order $\tilde{A}_2$ and $\tilde{A}_3$

$\tilde{A}_2 = (4,5,9) \rightarrow \text{divergence} = 9 - 4 = 5$

$\tilde{A}_3 = (3,5,10) \rightarrow \text{divergence} = 10 - 3 = 7$

Therefore, $\tilde{A}_2 < \tilde{A}_3$

We obtain the linear order, $\tilde{A}_4 < \tilde{A}_2 < \tilde{A}_3 < \tilde{A}_1$

3.4 Reliability of Dynamic Fuzzy Multi-State Systems

Consider a component that may be in one of four possible states. The performance levels of these states are the following triangular fuzzy numbers

$$\tilde{0} = (0,0,0.5)\,, \tilde{1} = (0.3,1,1.8), \tilde{2} = (1.6,2,2).$$

So we have three states for the linguistic values of basic events, which are:

State 1 (Failed): which is corresponding to the fuzzy number $\tilde{0}$

State 2 (Degraded): which is corresponding to the fuzzy number $\tilde{1}$

State 3 (Operational): which is corresponding to the fuzzy number $\tilde{2}$

In order to solve the problem we use the first criterion of ordering:

$$\tilde{0} = (0,0,0.5) \rightarrow \tilde{A}_1 = \frac{0+0+0.5}{4} = 0.125,$$

$$\tilde{1} = (0.3,1,1.8) \rightarrow \tilde{A}_2 = \frac{0.3+2+1.8}{4} = 1.025,$$

$$\tilde{2} = (1.6,2,2) \rightarrow \tilde{A}_3 = \frac{1.6+4+2}{4} = 1.9,$$

Therefore, $\tilde{0} < \tilde{1} < \tilde{2}$.

The structure function of the 2-out-of-3 FMSS system is: obtained with $m_i = \tilde{3}, i = 1,2,3$ which are given in Table (3.1).

Table (3.1) The FMSS 2-out-of-3 system

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$	$\Phi(\tilde{x})$	$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$	$\Phi(\tilde{x})$	$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$	$\Phi(\tilde{x})$
$\tilde{0}$ $\tilde{0}$ $\tilde{0}$	$\tilde{0}$	$\tilde{1}$ $\tilde{0}$ $\tilde{0}$	$\tilde{0}$	$\tilde{2}$ $\tilde{0}$ $\tilde{0}$	$\tilde{0}$
$\tilde{0}$ $\tilde{0}$ $\tilde{1}$	$\tilde{0}$	$\tilde{1}$ $\tilde{0}$ $\tilde{1}$	$\tilde{1}$	$\tilde{2}$ $\tilde{0}$ $\tilde{1}$	$\tilde{1}$
$\tilde{0}$ $\tilde{0}$ $\tilde{2}$	$\tilde{0}$	$\tilde{1}$ $\tilde{0}$ $\tilde{2}$	$\tilde{1}$	$\tilde{2}$ $\tilde{0}$ $\tilde{2}$	$\tilde{2}$
$\tilde{0}$ $\tilde{1}$ $\tilde{0}$	$\tilde{0}$	$\tilde{1}$ $\tilde{1}$ $\tilde{0}$	$\tilde{1}$	$\tilde{2}$ $\tilde{1}$ $\tilde{0}$	$\tilde{1}$
$\tilde{0}$ $\tilde{1}$ $\tilde{1}$	$\tilde{1}$	$\tilde{1}$ $\tilde{1}$ $\tilde{1}$	$\tilde{1}$	$\tilde{2}$ $\tilde{1}$ $\tilde{1}$	$\tilde{1}$
$\tilde{0}$ $\tilde{1}$ $\tilde{2}$	$\tilde{1}$	$\tilde{1}$ $\tilde{1}$ $\tilde{2}$	$\tilde{1}$	$\tilde{2}$ $\tilde{1}$ $\tilde{2}$	$\tilde{2}$
$\tilde{0}$ $\tilde{2}$ $\tilde{0}$	$\tilde{0}$	$\tilde{1}$ $\tilde{2}$ $\tilde{0}$	$\tilde{1}$	$\tilde{2}$ $\tilde{2}$ $\tilde{0}$	$\tilde{2}$
$\tilde{0}$ $\tilde{2}$ $\tilde{1}$	$\tilde{1}$	$\tilde{1}$ $\tilde{2}$ $\tilde{1}$	$\tilde{1}$	$\tilde{2}$ $\tilde{2}$ $\tilde{1}$	$\tilde{2}$
$\tilde{0}$ $\tilde{2}$ $\tilde{2}$	$\tilde{2}$	$\tilde{1}$ $\tilde{2}$ $\tilde{2}$	$\tilde{2}$	$\tilde{2}$ $\tilde{2}$ $\tilde{2}$	$\tilde{2}$

Hence, the direct partial logic derivative $\frac{\partial\Phi(\tilde{1}\rightarrow\tilde{0})}{\partial\tilde{x}_i(\tilde{1}\rightarrow\tilde{0})}$ for the failure of the component are given in Table (3.2)

Table (3.2) Direct partial logic derivatives of failure the FMSS.

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$	$\frac{\partial\Phi(\tilde{1}\to\tilde{0})}{\partial\tilde{x}_1(\tilde{1}\to\tilde{0})}$	$\frac{\partial\Phi(\tilde{1}\to\tilde{0})}{\partial\tilde{x}_2(\tilde{1}\to\tilde{0})}$	$\frac{\partial\Phi(\tilde{1}\to\tilde{0})}{\partial\tilde{x}_3(\tilde{1}\to\tilde{0})}$
$\tilde{0}\ \tilde{0}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{0}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{0}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{1}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{1}\ \tilde{1}$	$\tilde{0}$	$\tilde{2}$	$\tilde{2}$
$\tilde{0}\ \tilde{1}\ \tilde{2}$	$\tilde{0}$	$\tilde{2}$	$\tilde{0}$
$\tilde{0}\ \tilde{2}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{2}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{2}$
$\tilde{0}\ \tilde{2}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{0}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{0}\ \tilde{1}$	$\tilde{2}$	$\tilde{0}$	$\tilde{2}$
$\tilde{1}\ \tilde{0}\ \tilde{2}$	$\tilde{2}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{1}\ \tilde{0}$	$\tilde{2}$	$\tilde{2}$	$\tilde{0}$
$\tilde{1}\ \tilde{1}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{1}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{2}\ \tilde{0}$	$\tilde{2}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{2}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{2}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{0}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{0}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{2}$
$\tilde{2}\ \tilde{0}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{1}\ \tilde{0}$	$\tilde{0}$	$\tilde{2}$	$\tilde{0}$
$\tilde{2}\ \tilde{1}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{1}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{2}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{2}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{2}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$

Similarly, Table (3.3) shows the direct partial logic derivative $\frac{\partial(\tilde{0}\to\tilde{1})}{\partial x_i(\tilde{0}\to\tilde{m}_i-1)}$ for system.

Table (3.3) Direct partial logic derivatives of repair the FMSS.

x_1, x_2, x_3	$\frac{\partial\Phi(\tilde{0}\to\tilde{1})}{\partial x_1(\tilde{0}\to\tilde{2})}$	$\frac{\partial\Phi(\tilde{0}\to\tilde{1})}{\partial x_2(\tilde{0}\to\tilde{2})}$	$\frac{\partial\Phi(\tilde{0}\to\tilde{1})}{\partial x_3(\tilde{0}\to\tilde{2})}$
$\tilde{0}\ \tilde{0}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{0}\ \tilde{1}$	$\tilde{2}$	$\tilde{2}$	$\tilde{0}$
$\tilde{0}\ \tilde{0}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{1}\ \tilde{0}$	$\tilde{2}$	$\tilde{0}$	$\tilde{2}$
$\tilde{0}\ \tilde{1}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{1}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{2}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{2}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{0}\ \tilde{2}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{0}\ \tilde{0}$	$\tilde{0}$	$\tilde{2}$	$\tilde{2}$
$\tilde{1}\ \tilde{0}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{0}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{1}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{1}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{1}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{2}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{2}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{1}\ \tilde{2}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{0}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{0}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{0}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{1}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{1}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{1}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{2}\ \tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{2}\ \tilde{1}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$
$\tilde{2}\ \tilde{2}\ \tilde{2}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$

Hence, the numbers $\rho(1)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}}$ and $p(i)_{\tilde{0}\to\tilde{m}_i-1}^{\tilde{0}\to\tilde{1}}$ are obtained to be as follows:

$\rho(1)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}} = 4, \rho(2)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}} = 4, \rho(3)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}} = 4;$

$\rho(1)_{\tilde{0}\to\tilde{2}}^{\tilde{0}\to\tilde{1}} = 2,\ \rho(2)_{\tilde{0}\to\tilde{2}}^{\tilde{0}\to\tilde{1}} = 2, \rho(3)_{\tilde{0}\to\tilde{2}}^{\tilde{0}\to\tilde{1}} = 2;$

Thus, the structural probability $p(i)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}}$ of the $i-th$ component failure state modification from $\tilde{1}$ to $\tilde{0}$ are:

$$
\begin{aligned}
p(i)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}} &= \rho(1)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}} \tilde{\div} (\tilde{m}_1 \tilde{\times} \tilde{m}_2 \tilde{\times} \tilde{m}_3), i = 1,2,3 \\
&= (4,4,4) \tilde{\div} ((2.6,3,3) \tilde{\times} (2.6,3,3) \tilde{\times} (2.6,3,3)) \\
&= (4,4,4) \tilde{\div} (17.576,27,27) \\
&= (0.148,0.148,0.22)
\end{aligned}
$$

Thus, the structural probability $p(i)_{0\to\tilde{m}_i-1}^{\tilde{1}\to\tilde{0}}$ of $i-th$ component replace for system repairing are:

$$
\begin{aligned}
p(i)_{\tilde{0}\to\tilde{2}}^{\tilde{0}\to\tilde{1}} &= \rho(i)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}} \tilde{\div} (\tilde{m}_1 \times \tilde{m}_2 \times \tilde{m}_3), i = 1,2,3 \\
&= (2,2,2) \tilde{\div} ((2.6,3,3) \tilde{\times} (2.6,3,3) \tilde{\times} (2.6,3,3)) \\
&= (2,2,2) \tilde{\div} (17.576,27,27) \\
&= (0.074,0.074,0.113)
\end{aligned}
$$

Hence, the probabilities of the component dynamic reliability indices of the FMSS failure are given by:

$$
\begin{aligned}
\tilde{P}_f(1) &= p(1)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}} \tilde{\times} p_1(1) \\
&= (0.148,0.148,0.22) \tilde{\times} 0.6 \\
&= (0.0888,0.0888,0.132) \\
\tilde{P}_f(2) &= p(2)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}} \tilde{\times} p_1(2) \\
&= (0.148,0.148,0.22) \tilde{\times} 0.5 \\
&= (0.074,0.074,0.11) \\
\tilde{P}_f(3) &= p(3)_{\tilde{1}\to\tilde{0}}^{\tilde{1}\to\tilde{0}} \tilde{\times} p_1(3) \\
&= (0.148,0.148,0.22) \tilde{\times} 0.2 \\
&= (0.0296,0.0296,0.044)
\end{aligned}
$$

Also, the probabilities of the component dynamic reliability indices of FMSS repairing at a modification of a state of a system component.

$$\tilde{P}_r(1) = p(1)_{\tilde{0}\to\tilde{2}}^{\tilde{0}\to\tilde{1}} \tilde{\times} p_0(1)$$
$$= (0.074,0.074,0.113) \tilde{\times} 0.1$$
$$= (0.0074,0.0074,0.0113)$$
$$\tilde{P}_r(2) = p(2)_{\tilde{0}\to\tilde{2}}^{\tilde{0}\to\tilde{1}} \tilde{\times} p_0(2)$$
$$= (0.074,0.074,0.113) \tilde{\times} 0.4$$
$$= (0.0296,0.0296,0.0452)$$
$$\tilde{P}_r(3) = p(3)_{\tilde{0}\to\tilde{2}}^{\tilde{0}\to\tilde{1}} \tilde{\times} p_0(3)$$
$$= (0.074,0.074,0.113) \tilde{\times} 0.2$$
$$= (0.0148,0.0148,0.0226)$$

Also, the probability of the system failure if one of the system components is breakdown is given by fuzzy probability:

$$\tilde{P}_f = \sum_{i}^{n} \tilde{P}_f(i) \tilde{\times} \prod_{q=1,q\neq i}^{n} \left(1 \tilde{-} \tilde{P}_f(i)\right)$$
$$= \tilde{P}_f(1) \tilde{\times} \left(1 \tilde{-} \tilde{P}_f(2)\right) \tilde{\times} \left(1 \tilde{-} \tilde{P}_f(3)\right) \tilde{+} \tilde{P}_f(2) \tilde{\times} \left(1 \tilde{-} \tilde{P}_f(1)\right) \tilde{\times} \left(1 \tilde{-} \tilde{P}_f(3)\right)$$
$$\tilde{+} \tilde{P}_f(3) \tilde{\times} \left(1 \tilde{-} \tilde{P}_f(1)\right) \tilde{\times} \left(1 \tilde{-} \tilde{P}_f(2)\right)$$
$$=(0.0888,0.0888,0.132) \tilde{\times} (1 \tilde{-} (0.074,0.074,0.11)) \tilde{\times} (1 \tilde{-} (0.0296,0.0296,0.044))$$
$$\tilde{+}(0.074,0.074,0.11) \tilde{\times} (1 \tilde{-} (0.0888,0.0888,0.132)) \tilde{\times} (1 \tilde{-} (0.0296,0.0296,0.044))$$
$$\tilde{+}(0.0296,0.0296,0.044) \tilde{\times} (1 \tilde{-} (0.0888,0.0888,0.132)) \tilde{\times} (1 \tilde{-} (0.074,0.074,0.11))$$
$$= (0.17,0.17,0.23)$$

and the probability of the system repairing if one of the failure components of the system is replaced is given by fuzzy probability:

$$\tilde{P}_r = \sum_{i}^{n} \tilde{P}_r(i) \,\tilde{\times} \prod_{q=1,q\neq i}^{n} \left(1 - \tilde{P}_r(i)\right)$$

$$= \tilde{P}_r(1) \,\tilde{\times} \left(1 \tilde{-} \tilde{P}_r(2)\right) \tilde{\times} \left(1 \tilde{-} \tilde{P}_r(3)\right) \tilde{+} \tilde{P}_r(2) \,\tilde{\times} \left(1 \tilde{-} \tilde{P}_r(1)\right) \tilde{\times} \left(1 \tilde{-} \tilde{P}_r(3)\right)$$

$$\tilde{+} \tilde{P}_r(3) \,\tilde{\times} \left(1 \tilde{-} \tilde{P}_r(1)\right) \tilde{\times} \left(1 \tilde{-} \tilde{P}_r(2)\right)$$

$$= (0.0074,0.0074,0.0113) \,\tilde{\times}\, (1 \tilde{-} (0.0296,0.0296,0.0452))$$

$$\tilde{\times}\, (1 \tilde{-} (0.0148,0.0148,0.0226)) \,\tilde{+}\, (0.0296,0.0296,0.0452)$$

$$\tilde{\times}\, (1 \tilde{-} (0.0074,0.0074,0.0113)) \,\tilde{\times}\, (1 \tilde{-} (0.0148,0.0148,0.0226))$$

$$\tilde{+}\, (0.0148,0.0148,0.0226) \,\tilde{\times}\, (1 \tilde{-} (0.0074,0.0074,0.0113))$$

$$\tilde{\times} (1 \tilde{-} (0.0296,0.0296,0.0452))$$

$$= (0.05,0.05,0.0756)$$

Conclusions and Recommendations

Conclusions and Recommendations

From the present study, we can conclude the following:

1- The reliability of dynamic multi-state system is an effective approach to evaluate the probability for the system failure if the efficiency of some component decreases and system repair if some of the failure components restore. Component dynamic reliability indices and dynamic integrated reliability indices can be applied to broad problems in engineering systems, supply chain and logistics, general networks for transportation and distribution, computer and communication system.

2- The dynamic reliability approach has been used successfully to evaluate the probability of the failure and repairing of the oil supply system as an application of dynamic multi-state k-out-of-n system model where the components and the system have multiple performance levels.

3- The dynamic reliability fuzzy multi-state system may be considered as a generalization to non-fuzzy or crisp multi-state system of previous investigations that have been presented when we consider the α-level to be at $\alpha = 1$. In this thesis, we consider new equations for probability of fuzzy system failure if the efficiency of some component decreases or these components are break down for k-out-of-n fuzzy multi-state system and fuzzy system repair if some of failure components restore for k-out-of-n fuzzy multi-state system.

Also, from the present study the following recommendations may be observed:

1- Modifying the reliability of dynamic fuzzy multi-state systems represent how change if one of system components impacts to the system reliability to solve analytically of a FMSS reliability change depending on fixed components efficiencies changes.

2- The application of dynamic fuzzy multi-state systems reliability In the area of multi-state system reliability, most of the reported research studies are focused on theoretical study. The contributions of this thesis work are also mainly on the theoretical side. More application study should be carried on. In the area of multi-state system reliability, most of the reported research studies are focused on theoretical study. The contributions of this thesis work are also mainly on the theoretical side and more application study should be carried on.

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Appendix A

The Direct Partial Logic Derivative of the Oil Supply Failure System

Appendix A

The Direct Partial Logic Derivative of the Oil Supply Failure System

$$\frac{\partial\Phi(1\to 0)}{\partial x_i(1\to 0)} = \Phi(x_1,\ldots,x_{i-1},1,x_{i+1},\ldots,x_n)\bullet\Phi(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_n), i = 1,2,3,4$$

No.	x_1	x_2	x_3	x_4	$\Phi(x_1,x_2,x_3,x_4)$	$\frac{\partial\Phi(1\to 0)}{\partial x_1(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_2(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_3(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_4(1\to 0)}$
1.	0	0	0	0	0	0	0	0	0
2.	0	0	0	1	1	0	0	0	3
3.	0	0	0	2	1	0	0	0	0
4.	0	0	0	3	1	0	0	0	0
5.	0	0	1	0	1	0	0	3	0
6.	0	0	1	1	1	0	0	0	0
7.	0	0	1	2	1	0	0	0	0
8.	0	0	1	3	1	0	0	0	0
9.	0	0	2	0	1	0	0	0	0
10.	0	0	2	1	1	0	0	0	0
11.	0	0	2	2	2	0	0	0	0
12.	0	0	2	3	2	0	0	0	0
13.	0	0	3	0	1	0	0	0	0
14.	0	0	3	1	1	0	0	0	0
15.	0	0	3	2	2	0	0	0	0
16.	0	0	3	3	2	0	0	0	0
17.	0	1	0	0	1	0	3	0	0
18.	0	1	0	1	1	0	0	0	0
19.	0	1	0	2	1	0	0	0	0
20.	0	1	0	3	1	0	0	0	0
21.	0	1	1	0	1	0	0	0	0
22.	0	1	1	1	1	0	0	0	0
23.	0	1	1	2	1	0	0	0	0
24.	0	1	1	3	1	0	0	0	0
25.	0	1	2	0	1	0	0	0	0
26.	0	1	2	1	1	0	0	0	0
27.	0	1	2	2	2	0	0	0	0
28.	0	1	2	3	2	0	0	0	0
29.	0	1	3	0	2	0	0	0	0
30.	0	1	3	1	1	0	0	0	0
31.	0	1	3	2	2	0	0	0	0
32.	0	1	3	3	2	0	0	0	0
33.	0	2	0	0	1	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1, x_2, x_3, x_4)$	$\frac{\partial\Phi(1\to 0)}{\partial x_1(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_2(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_3(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_4(1\to 0)}$
34.	0	2	0	1	1	0	0	0	0
35.	0	2	0	2	2	0	0	0	0
36.	0	2	0	3	2	0	0	0	0
37.	0	2	1	0	1	0	0	0	0
38.	0	2	1	1	1	0	0	0	0
39.	0	2	1	2	2	0	0	0	0
40.	0	2	1	3	2	0	0	0	0
41.	0	2	2	0	2	0	0	0	0
42.	0	2	2	1	2	0	0	0	0
43.	0	2	2	2	2	0	0	0	0
44.	0	2	2	3	2	0	0	0	0
45.	0	2	3	0	2	0	0	0	0
46.	0	2	3	1	2	0	0	0	0
47.	0	2	3	2	2	0	0	0	0
48.	0	2	3	3	2	0	0	0	0
49.	0	3	0	0	1	0	0	0	0
50.	0	3	0	1	1	0	0	0	0
51.	0	3	0	2	2	0	0	0	0
52.	0	3	0	3	2	0	0	0	0
53.	0	3	1	0	1	0	0	0	0
54.	0	3	1	1	1	0	0	0	0
55.	0	3	1	2	2	0	0	0	0
56.	0	3	1	3	2	0	0	0	0
57.	0	3	2	0	2	0	0	0	0
58.	0	3	2	1	2	0	0	0	0
59.	0	3	2	2	2	0	0	0	0
60.	0	3	2	3	2	0	0	0	0
61.	0	3	3	0	2	0	0	0	0
62.	0	3	3	1	2	0	0	0	0
63.	0	3	3	2	2	0	0	0	0
64.	0	3	3	3	2	0	0	0	0
65.	1	0	0	0	1	3	0	0	0
66.	1	0	0	1	1	0	0	0	0
67.	1	0	0	2	1	0	0	0	0
68.	1	0	0	3	1	0	0	0	0
69.	1	0	1	0	1	0	0	0	0
70.	1	0	1	1	1	0	0	0	0
71.	1	0	1	2	1	0	0	0	0
72.	1	0	1	3	1	0	0	0	0
73.	1	0	2	0	1	0	0	0	0
74.	1	0	2	1	1	0	0	0	0
75.	1	0	2	2	2	0	0	0	0
76.	1	0	2	3	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1,x_2,x_3,x_4)$	$\frac{\partial\Phi(1\to0)}{\partial x_1(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_2(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_3(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_4(1\to0)}$
77.	1	0	3	0	1	0	0	0	0
78.	1	0	3	1	1	0	0	0	0
79.	1	0	3	2	2	0	0	0	0
80.	1	0	3	3	2	0	0	0	0
81.	1	1	0	0	1	0	0	0	0
82.	1	1	0	1	1	0	0	0	0
83.	1	1	0	2	1	0	0	0	0
84.	1	1	0	3	1	0	0	0	0
85.	1	1	1	0	1	0	0	0	0
86.	1	1	1	1	1	0	0	0	0
87.	1	1	1	2	1	0	0	0	0
88.	1	1	1	3	1	0	0	0	0
89.	1	1	2	0	1	0	0	0	0
90.	1	1	2	1	1	0	0	0	0
91.	1	1	2	2	2	0	0	0	0
92.	1	1	2	3	2	0	0	0	0
93.	1	1	3	0	1	0	0	0	0
94.	1	1	3	1	1	0	0	0	0
95.	1	1	3	2	2	0	0	0	0
96.	1	1	3	3	2	0	0	0	0
97.	1	2	0	0	1	0	0	0	0
98.	1	2	0	1	1	0	0	0	0
99.	1	2	0	2	2	0	0	0	0
100.	1	2	0	3	2	0	0	0	0
101.	1	2	1	0	1	0	0	0	0
102.	1	2	1	1	1	0	0	0	0
103.	1	2	1	2	2	0	0	0	0
104.	1	2	1	3	2	0	0	0	0
105.	1	2	2	0	2	0	0	0	0
106.	1	2	2	1	2	0	0	0	0
107.	1	2	2	2	2	0	0	0	0
108.	1	2	2	3	2	0	0	0	0
109.	1	2	3	0	2	0	0	0	0
110.	1	2	3	1	2	0	0	0	0
111.	1	2	3	2	2	0	0	0	0
112.	1	2	3	3	2	0	0	0	0
113.	1	3	0	0	1	0	0	0	0
114.	1	3	0	1	1	0	0	0	0
115.	1	3	0	2	2	0	0	0	0
116.	1	3	0	3	2	0	0	0	0
117.	1	3	1	0	1	0	0	0	0
118.	1	3	1	1	1	0	0	0	0
119.	1	3	1	2	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1,x_2,x_3,x_4)$	$\frac{\partial\Phi(1\to0)}{\partial x_1(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_2(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_3(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_4(1\to0)}$
120.	1	3	1	3	2	0	0	0	0
121.	1	3	2	0	2	0	0	0	0
122.	1	3	2	1	2	0	0	0	0
123.	1	3	2	2	2	0	0	0	0
124.	1	3	2	3	2	0	0	0	0
125.	1	3	3	0	2	0	0	0	0
126.	1	3	3	1	2	0	0	0	0
127.	1	3	3	2	2	0	0	0	0
128.	1	3	3	3	2	0	0	0	0
129.	2	0	0	0	1	0	0	0	0
130.	2	0	0	1	1	0	0	0	0
131.	2	0	0	2	2	0	0	0	0
132.	2	0	0	3	2	0	0	0	0
133.	2	0	1	0	1	0	0	0	0
134.	2	0	1	1	1	0	0	0	0
135.	2	0	1	2	2	0	0	0	0
136.	2	0	1	3	2	0	0	0	0
137.	2	0	2	0	2	0	0	0	0
138.	2	0	2	1	2	0	0	0	0
139.	2	0	2	2	2	0	0	0	0
140.	2	0	2	3	2	0	0	0	0
141.	2	0	3	0	2	0	0	0	0
142.	2	0	3	1	2	0	0	0	0
143.	2	0	3	2	2	0	0	0	0
144.	2	0	3	3	2	0	0	0	0
145.	2	1	0	0	1	0	0	0	0
146.	2	1	0	1	1	0	0	0	0
147.	2	1	0	2	2	0	0	0	0
148.	2	1	0	3	2	0	0	0	0
149.	2	1	1	0	1	0	0	0	0
150.	2	1	1	1	1	0	0	0	0
151.	2	1	1	2	2	0	0	0	0
152.	2	1	1	3	2	0	0	0	0
153.	2	1	2	0	2	0	0	0	0
154.	2	1	2	1	2	0	0	0	0
155.	2	1	2	2	2	0	0	0	0
156.	2	1	2	3	2	0	0	0	0
157.	2	1	3	0	2	0	0	0	0
158.	2	1	3	1	2	0	0	0	0
159.	2	1	3	2	2	0	0	0	0
160.	2	1	3	3	2	0	0	0	0
161.	2	2	0	0	2	0	0	0	0
162.	2	2	0	1	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1,x_2,x_3,x_4)$	$\frac{\partial\Phi(1\to0)}{\partial x_1(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_2(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_3(1\to0)}$	$\frac{\partial\Phi(1\to0)}{\partial x_4(1\to0)}$
163.	2	2	0	2	2	0	0	0	0
164.	2	2	0	3	2	0	0	0	0
165.	2	2	1	0	2	0	0	0	0
166.	2	2	1	1	2	0	0	0	0
167.	2	2	1	2	2	0	0	0	0
168.	2	2	1	3	2	0	0	0	0
169.	2	2	2	0	2	0	0	0	0
170.	2	2	2	1	2	0	0	0	0
171.	2	2	2	2	2	0	0	0	0
172.	2	2	2	3	2	0	0	0	0
173.	2	2	3	0	2	0	0	0	0
174.	2	2	3	1	2	0	0	0	0
175.	2	2	3	2	2	0	0	0	0
176.	2	2	3	3	2	0	0	0	0
177.	2	3	0	0	2	0	0	0	0
178.	2	3	0	1	2	0	0	0	0
179.	2	3	0	2	2	0	0	0	0
180.	2	3	0	3	2	0	0	0	0
181.	2	3	1	0	2	0	0	0	0
182.	2	3	1	1	2	0	0	0	0
183.	2	3	1	2	2	0	0	0	0
184.	2	3	1	3	2	0	0	0	0
185.	2	3	2	0	2	0	0	0	0
186.	2	3	2	1	2	0	0	0	0
187.	2	3	2	2	2	0	0	0	0
188.	2	3	2	3	2	0	0	0	0
189.	2	3	3	0	2	0	0	0	0
190.	2	3	3	1	2	0	0	0	0
191.	2	3	3	2	2	0	0	0	0
192.	2	3	3	3	2	0	0	0	0
193.	3	0	0	0	1	0	0	0	0
194.	3	0	0	1	1	0	0	0	0
195.	3	0	0	2	2	0	0	0	0
196.	3	0	0	3	2	0	0	0	0
197.	3	0	1	0	1	0	0	0	0
198.	3	0	1	1	1	0	0	0	0
199.	3	0	1	2	2	0	0	0	0
200.	3	0	1	3	2	0	0	0	0
201.	3	0	2	0	2	0	0	0	0
202.	3	0	2	1	2	0	0	0	0
203.	3	0	2	2	2	0	0	0	0
204.	3	0	2	3	2	0	0	0	0
205.	3	0	3	0	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1,x_2,x_3,x_4)$	$\frac{\partial\Phi(1\to 0)}{\partial x_1(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_2(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_3(1\to 0)}$	$\frac{\partial\Phi(1\to 0)}{\partial x_4(1\to 0)}$
206.	3	0	3	1	2	0	0	0	0
207.	3	0	3	2	2	0	0	0	0
208.	3	0	3	3	2	0	0	0	0
209.	3	1	0	0	1	0	0	0	0
210.	3	1	0	1	1	0	0	0	0
211.	3	1	0	2	2	0	0	0	0
212.	3	1	0	3	1	0	0	0	0
213.	3	1	1	0	1	0	0	0	0
214.	3	1	1	1	1	0	0	0	0
215.	3	1	1	2	2	0	0	0	0
216.	3	1	1	3	2	0	0	0	0
217.	3	1	2	0	2	0	0	0	0
218.	3	1	2	1	2	0	0	0	0
219.	3	1	2	2	2	0	0	0	0
220.	3	1	2	3	2	0	0	0	0
221.	3	1	3	0	2	0	0	0	0
222.	3	1	3	1	2	0	0	0	0
223.	3	1	3	2	2	0	0	0	0
224.	3	1	3	3	2	0	0	0	0
225.	3	2	0	0	2	0	0	0	0
226.	3	2	0	1	2	0	0	0	0
227.	3	2	0	2	2	0	0	0	0
228.	3	2	0	3	2	0	0	0	0
229.	3	2	1	0	2	0	0	0	0
230.	3	2	1	1	2	0	0	0	0
231.	3	2	1	2	2	0	0	0	0
232.	3	2	1	3	2	0	0	0	0
233.	3	2	2	0	2	0	0	0	0
234.	3	2	2	1	2	0	0	0	0
235.	3	2	2	2	2	0	0	0	0
236.	3	2	2	3	2	0	0	0	0
237.	3	2	3	0	2	0	0	0	0
238.	3	2	3	1	2	0	0	0	0
239.	3	2	3	2	2	0	0	0	0
240.	3	2	3	3	2	0	0	0	0
241.	3	3	0	0	2	0	0	0	0
242.	3	3	0	1	2	0	0	0	0
243.	3	3	0	2	2	0	0	0	0
244.	3	3	0	3	2	0	0	0	0
245.	3	3	1	0	2	0	0	0	0
246.	3	3	1	1	2	0	0	0	0
247.	3	3	1	2	2	0	0	0	0
248.	3	3	1	3	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1, x_2, x_3, x_4)$	$\frac{\partial\Phi(1 \to 0)}{\partial x_1(1 \to 0)}$	$\frac{\partial\Phi(1 \to 0)}{\partial x_2(1 \to 0)}$	$\frac{\partial\Phi(1 \to 0)}{\partial x_3(1 \to 0)}$	$\frac{\partial\Phi(1 \to 0)}{\partial x_4(1 \to 0)}$
249.	3	3	2	0	2	0	0	0	0
250.	3	3	2	1	2	0	0	0	0
251.	3	3	2	2	2	0	0	0	0
252.	3	3	2	3	2	0	0	0	0
253.	3	3	3	0	2	0	0	0	0
254.	3	3	3	1	2	0	0	0	0
255.	3	3	3	2	2	0	0	0	0
256.	3	3	3	3	3	0	0	0	0

Appendix B

The Direct Partial Logic Derivative of the Oil Supply repairing System

Appendix B

The Direct Partial Logic Derivative of the Oil Supply repairing System

$$\frac{\partial\Phi(1\rightarrow 2)}{\partial x_i(0\rightarrow 3)} = \Phi(x_1,\dots,x_{i-1},0,x_{i+1},\dots,x_n)\bullet\Phi(x_1,\dots,x_{i-1},3,x_{i+1},\dots,x_n), i = 1,2,3,4$$

No.	x_1	x_2	x_3	x_4	$\Phi(x_1,x_2,x_3,x_4)$	$\frac{\partial\Phi(1\rightarrow 2)}{\partial x_1(0\rightarrow 3)}$	$\frac{\partial\Phi(1\rightarrow 2)}{\partial x_2(0\rightarrow 3)}$	$\frac{\partial\Phi(1\rightarrow 2)}{\partial x_3(0\rightarrow 3)}$	$\frac{\partial\Phi(1\rightarrow 2)}{\partial x_4(0\rightarrow 3)}$
1.	0	0	0	0	0	0	0	0	0
2.	0	0	0	1	1	0	0	0	0
3.	0	0	0	2	1	3	3	3	0
4.	0	0	0	3	1	3	3	3	0
5.	0	0	1	0	1	0	0	0	0
6.	0	0	1	1	1	0	0	0	0
7.	0	0	1	2	1	3	3	0	0
8.	0	0	1	3	1	3	3	0	0
9.	0	0	2	0	1	3	3	0	3
10.	0	0	2	1	1	3	3	0	0
11.	0	0	2	2	2	0	0	0	0
12.	0	0	2	3	2	0	0	0	0
13.	0	0	3	0	1	3	3	0	3
14.	0	0	3	1	1	3	3	0	0
15.	0	0	3	2	2	0	0	0	0
16.	0	0	3	3	2	0	0	0	0
17.	0	1	0	0	1	0	0	0	0
18.	0	1	0	1	1	0	0	0	0
19.	0	1	0	2	1	3	0	3	0
20.	0	1	0	3	1	3	0	3	0
21.	0	1	1	0	1	0	0	0	0
22.	0	1	1	1	1	0	0	0	0
23.	0	1	1	2	1	3	0	0	0
24.	0	1	1	3	1	3	0	0	0
25.	0	1	2	0	1	3	0	0	3
26.	0	1	2	1	1	3	0	0	0
27.	0	1	2	2	2	0	0	0	0
28.	0	1	2	3	2	0	0	0	0
29.	0	1	3	0	2	0	0	0	3
30.	0	1	3	1	1	3	0	0	0
31.	0	1	3	2	2	0	0	0	0
32.	0	1	3	3	2	0	0	0	0
33.	0	2	0	0	1	3	0	3	3

No.	x_1	x_2	x_3	x_4	$\Phi(x_1,x_2,x_3,x_4)$	$\frac{\partial\Phi(1\to2)}{\partial x_1(0\to3)}$	$\frac{\partial\Phi(1\to2)}{\partial x_2(0\to3)}$	$\frac{\partial\Phi(1\to2)}{\partial x_3(0\to3)}$	$\frac{\partial\Phi(1\to2)}{\partial x_4(0\to3)}$
34.	0	2	0	1	1	3	0	3	0
35.	0	2	0	2	2	0	0	0	0
36.	0	2	0	3	2	0	0	0	0
37.	0	2	1	0	1	3	0	0	3
38.	0	2	1	1	1	3	0	0	0
39.	0	2	1	2	2	0	0	0	0
40.	0	2	1	3	2	0	0	0	0
41.	0	2	2	0	2	0	0	0	0
42.	0	2	2	1	2	0	0	0	0
43.	0	2	2	2	2	0	0	0	0
44.	0	2	2	3	2	0	0	0	0
45.	0	2	3	0	2	0	0	0	0
46.	0	2	3	1	2	0	0	0	0
47.	0	2	3	2	2	0	0	0	0
48.	0	2	3	3	2	0	0	0	0
49.	0	3	0	0	1	3	0	3	3
50.	0	3	0	1	1	3	0	3	0
51.	0	3	0	2	2	0	0	0	0
52.	0	3	0	3	2	0	0	0	0
53.	0	3	1	0	1	3	0	0	3
54.	0	3	1	1	1	3	0	0	0
55.	0	3	1	2	2	0	0	0	0
56.	0	3	1	3	2	0	0	0	0
57.	0	3	2	0	2	0	0	0	0
58.	0	3	2	1	2	0	0	0	0
59.	0	3	2	2	2	0	0	0	0
60.	0	3	2	3	2	0	0	0	0
61.	0	3	3	0	2	0	0	0	0
62.	0	3	3	1	2	0	0	0	0
63.	0	3	3	2	2	0	0	0	0
64.	0	3	3	3	2	0	0	0	0
65.	1	0	0	0	1	0	0	0	0
66.	1	0	0	1	1	0	0	0	0
67.	1	0	0	2	1	0	3	3	0
68.	1	0	0	3	1	0	3	3	0
69.	1	0	1	0	1	0	0	0	0
70.	1	0	1	1	1	0	0	0	0
71.	1	0	1	2	1	0	3	0	0
72.	1	0	1	3	1	0	3	0	0
73.	1	0	2	0	1	0	3	0	3
74.	1	0	2	1	1	0	3	0	0
75.	1	0	2	2	2	0	0	0	0
76.	1	0	2	3	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1, x_2, x_3, x_4)$	$\frac{\partial\Phi(1\to2)}{\partial x_1(0\to3)}$	$\frac{\partial\Phi(1\to2)}{\partial x_2(0\to3)}$	$\frac{\partial\Phi(1\to2)}{\partial x_3(0\to3)}$	$\frac{\partial\Phi(1\to2)}{\partial x_4(0\to3)}$
77.	1	0	3	0	1	0	3	0	3
78.	1	0	3	1	1	0	3	0	0
79.	1	0	3	2	2	0	0	0	0
80.	1	0	3	3	2	0	0	0	0
81.	1	1	0	0	1	0	0	0	0
82.	1	1	0	1	1	0	0	0	0
83.	1	1	0	2	1	0	0	3	0
84.	1	1	0	3	1	0	0	3	0
85.	1	1	1	0	1	0	0	0	0
86.	1	1	1	1	1	0	0	0	0
87.	1	1	1	2	1	0	0	0	0
88.	1	1	1	3	1	0	0	0	0
89.	1	1	2	0	1	0	0	0	3
90.	1	1	2	1	1	0	0	0	0
91.	1	1	2	2	2	0	0	0	0
92.	1	1	2	3	2	0	0	0	0
93.	1	1	3	0	1	0	0	0	3
94.	1	1	3	1	1	0	0	0	0
95.	1	1	3	2	2	0	0	0	0
96.	1	1	3	3	2	0	0	0	0
97.	1	2	0	0	1	0	0	3	3
98.	1	2	0	1	1	0	0	3	0
99.	1	2	0	2	2	0	0	0	0
100.	1	2	0	3	2	0	0	0	0
101.	1	2	1	0	1	0	0	0	3
102.	1	2	1	1	1	0	0	0	0
103.	1	2	1	2	2	0	0	0	0
104.	1	2	1	3	2	0	0	0	0
105.	1	2	2	0	2	0	0	0	0
106.	1	2	2	1	2	0	0	0	0
107.	1	2	2	2	2	0	0	0	0
108.	1	2	2	3	2	0	0	0	0
109.	1	2	3	0	2	0	0	0	0
110.	1	2	3	1	2	0	0	0	0
111.	1	2	3	2	2	0	0	0	0
112.	1	2	3	3	2	0	0	0	0
113.	1	3	0	0	1	0	0	3	3
114.	1	3	0	1	1	0	0	3	0
115.	1	3	0	2	2	0	0	0	0
116.	1	3	0	3	2	0	0	0	0
117.	1	3	1	0	1	0	0	0	3
118.	1	3	1	1	1	0	0	0	0
119.	1	3	1	2	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1,x_2,x_3,x_4)$	$\frac{\partial\Phi(1\to 2)}{\partial x_1(0\to 3)}$	$\frac{\partial\Phi(1\to 2)}{\partial x_2(0\to 3)}$	$\frac{\partial\Phi(1\to 2)}{\partial x_3(0\to 3)}$	$\frac{\partial\Phi(1\to 2)}{\partial x_4(0\to 3)}$
120.	1	3	1	3	2	0	0	0	0
121.	1	3	2	0	2	0	0	0	0
122.	1	3	2	1	2	0	0	0	0
123.	1	3	2	2	2	0	0	0	0
124.	1	3	2	3	2	0	0	0	0
125.	1	3	3	0	2	0	0	0	0
126.	1	3	3	1	2	0	0	0	0
127.	1	3	3	2	2	0	0	0	0
128.	1	3	3	3	2	0	0	0	0
129.	2	0	0	0	1	0	3	3	3
130.	2	0	0	1	1	0	3	3	0
131.	2	0	0	2	2	0	0	0	0
132.	2	0	0	3	2	0	0	0	0
133.	2	0	1	0	1	0	3	0	3
134.	2	0	1	1	1	0	3	0	0
135.	2	0	1	2	2	0	0	0	0
136.	2	0	1	3	2	0	0	0	0
137.	2	0	2	0	2	0	0	0	0
138.	2	0	2	1	2	0	0	0	0
139.	2	0	2	2	2	0	0	0	0
140.	2	0	2	3	2	0	0	0	0
141.	2	0	3	0	2	0	0	0	0
142.	2	0	3	1	2	0	0	0	0
143.	2	0	3	2	2	0	0	0	0
144.	2	0	3	3	2	0	0	0	0
145.	2	1	0	0	1	0	0	0	3
146.	2	1	0	1	1	0	0	3	0
147.	2	1	0	2	2	0	0	0	0
148.	2	1	0	3	2	0	0	0	0
149.	2	1	1	0	1	0	0	0	3
150.	2	1	1	1	1	0	0	0	0
151.	2	1	1	2	2	0	0	0	0
152.	2	1	1	3	2	0	0	0	0
153.	2	1	2	0	2	0	0	0	0
154.	2	1	2	1	2	0	0	0	0
155.	2	1	2	2	2	0	0	0	0
156.	2	1	2	3	2	0	0	0	0
157.	2	1	3	0	2	0	0	0	0
158.	2	1	3	1	2	0	0	0	0
159.	2	1	3	2	2	0	0	0	0
160.	2	1	3	3	2	0	0	0	0
161.	2	2	0	0	2	0	0	0	0
162.	2	2	0	1	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1,x_2,x_3,x_4)$	$\frac{\partial\Phi(1\to 2)}{\partial x_1(0\to 3)}$	$\frac{\partial\Phi(1\to 2)}{\partial x_2(0\to 3)}$	$\frac{\partial\Phi(1\to 2)}{\partial x_3(0\to 3)}$	$\frac{\partial\Phi(1\to 2)}{\partial x_4(0\to 3)}$
163.	2	2	0	2	2	0	0	0	0
164.	2	2	0	3	2	0	0	0	0
165.	2	2	1	0	2	0	0	0	0
166.	2	2	1	1	2	0	0	0	0
167.	2	2	1	2	2	0	0	0	0
168.	2	2	1	3	2	0	0	0	0
169.	2	2	2	0	2	0	0	0	0
170.	2	2	2	1	2	0	0	0	0
171.	2	2	2	2	2	0	0	0	0
172.	2	2	2	3	2	0	0	0	0
173.	2	2	3	0	2	0	0	0	0
174.	2	2	3	1	2	0	0	0	0
175.	2	2	3	2	2	0	0	0	0
176.	2	2	3	3	2	0	0	0	0
177.	2	3	0	0	2	0	0	0	0
178.	2	3	0	1	2	0	0	0	0
179.	2	3	0	2	2	0	0	0	0
180.	2	3	0	3	2	0	0	0	0
181.	2	3	1	0	2	0	0	0	0
182.	2	3	1	1	2	0	0	0	0
183.	2	3	1	2	2	0	0	0	0
184.	2	3	1	3	2	0	0	0	0
185.	2	3	2	0	2	0	0	0	0
186.	2	3	2	1	2	0	0	0	0
187.	2	3	2	2	2	0	0	0	0
188.	2	3	2	3	2	0	0	0	0
189.	2	3	3	0	2	0	0	0	0
190.	2	3	3	1	2	0	0	0	0
191.	2	3	3	2	2	0	0	0	0
192.	2	3	3	3	2	0	0	0	0
193.	3	0	0	0	1	0	3	0	3
194.	3	0	0	1	1	0	3	0	0
195.	3	0	0	2	2	0	0	0	0
196.	3	0	0	3	2	0	0	0	0
197.	3	0	1	0	1	0	3	0	3
198.	3	0	1	1	1	0	3	0	0
199.	3	0	1	2	2	0	0	0	0
200.	3	0	1	3	2	0	0	0	0
201.	3	0	2	0	2	0	0	0	0
202.	3	0	2	1	2	0	0	0	0
203.	3	0	2	2	2	0	0	0	0
204.	3	0	2	3	2	0	0	0	0
205.	3	0	3	0	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1, x_2, x_3, x_4)$	$\frac{\partial\Phi(1\to2)}{\partial x_1(0\to3)}$	$\frac{\partial\Phi(1\to2)}{\partial x_2(0\to3)}$	$\frac{\partial\Phi(1\to2)}{\partial x_3(0\to3)}$	$\frac{\partial\Phi(1\to2)}{\partial x_4(0\to3)}$
206.	3	0	3	1	2	0	0	0	0
207.	3	0	3	2	2	0	0	0	0
208.	3	0	3	3	2	0	0	0	0
209.	3	1	0	0	1	0	0	0	3
210.	3	1	0	1	1	0	0	0	0
211.	3	1	0	2	2	0	0	0	0
212.	3	1	0	3	1	0	0	0	0
213.	3	1	1	0	1	0	0	0	3
214.	3	1	1	1	1	0	0	0	0
215.	3	1	1	2	2	0	0	0	0
216.	3	1	1	3	2	0	0	0	0
217.	3	1	2	0	2	0	0	0	0
218.	3	1	2	1	2	0	0	0	0
219.	3	1	2	2	2	0	0	0	0
220.	3	1	2	3	2	0	0	0	0
221.	3	1	3	0	2	0	0	0	0
222.	3	1	3	1	2	0	0	0	0
223.	3	1	3	2	2	0	0	0	0
224.	3	1	3	3	2	0	0	0	0
225.	3	2	0	0	2	0	0	0	0
226.	3	2	0	1	2	0	0	0	0
227.	3	2	0	2	2	0	0	0	0
228.	3	2	0	3	2	0	0	0	0
229.	3	2	1	0	2	0	0	0	0
230.	3	2	1	1	2	0	0	0	0
231.	3	2	1	2	2	0	0	0	0
232.	3	2	1	3	2	0	0	0	0
233.	3	2	2	0	2	0	0	0	0
234.	3	2	2	1	2	0	0	0	0
235.	3	2	2	2	2	0	0	0	0
236.	3	2	2	3	2	0	0	0	0
237.	3	2	3	0	2	0	0	0	0
238.	3	2	3	1	2	0	0	0	0
239.	3	2	3	2	2	0	0	0	0
240.	3	2	3	3	2	0	0	0	0
241.	3	3	0	0	2	0	0	0	0
242.	3	3	0	1	2	0	0	0	0
243.	3	3	0	2	2	0	0	0	0
244.	3	3	0	3	2	0	0	0	0
245.	3	3	1	0	2	0	0	0	0
246.	3	3	1	1	2	0	0	0	0
247.	3	3	1	2	2	0	0	0	0
248.	3	3	1	3	2	0	0	0	0

No.	x_1	x_2	x_3	x_4	$\Phi(x_1, x_2, x_3, x_4)$	$\frac{\partial\Phi(1\to 2)}{\partial x_1(0\to 3)}$	$\frac{\partial\Phi(1\to 2)}{\partial x_2(0\to 3)}$	$\frac{\partial\Phi(1\to 2)}{\partial x_3(0\to 3)}$	$\frac{\partial\Phi(1\to 2)}{\partial x_4(0\to 3)}$
249.	3	3	2	0	2	0	0	0	0
250.	3	3	2	1	2	0	0	0	0
251.	3	3	2	2	2	0	0	0	0
252.	3	3	2	3	2	0	0	0	0
253.	3	3	3	0	2	0	0	0	0
254.	3	3	3	1	2	0	0	0	0
255.	3	3	3	2	2	0	0	0	0
256.	3	3	3	3	3	0	0	0	0

المستخلص

لهذه الرسالة ثلاثة أهداف رئيسية:

الهدف الأول هو دراسة نظرية المعولية (Reliability Theory) للأنظمة متعددة الحالات، فضلا عن، بعض الخصائص الأساسية والنتائج النظرية.

الهدف الثاني هو دراسة المعولية الديناميكية للأنظمة متعددة الحالات (Reliability of Dynamic Multi-State System) حيث تم استخدام معولية المؤشرات الديناميكية (Dynamic Reliability Indices) لتقدير تأثير هذه المؤشرات على معولية للأنظمة متعددة الحالات. كما و تم إعطاء تطبيق عملي للنظام الديناميكي متعدد الحالات وهو نظام تجهيز النفط (Oil Supply System) من مصدر الانتاج النفط الى ثلاث محطات فرعية من خلال عدد من انابيب نقل النفط، مثلا أربعة، حيث ان هذا التطبيق لم تتم دراسته من قبل كنظام ديناميكي متعدد الحالات.

الهدف الثالث هو تقديم ودراسة معولية للأنظمة متعددة الحالات الضبابية (Fuzzy Multi-State System) من خلال دراسة السلوك الضبابي للنظام الديناميكي متعدد الحالات.

جمهـورية العـراق
وزارة التعليم العالي والبحث العلمي
جـــامعة النهـــــريـــن
كليــة العلوم
قسم الرياضيات وتطبيقات الحاسوب

معولية الأنظمة الديناميكية متعددة الحالات الضبابية

رسالة
مقدمة إلى مجلس كلية العلوم – جامعة النهرين
وهي جزء من متطلبات نيل درجة ماجستير علوم
في الرياضيات

من قبل

أحمد يعقوب يوسف
(بكالوريوس علوم، جامعة النهرين، 2008)

إشراف

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